Homework #12: ECE 461/661

Gain & Lead Compensation with Bode Plots. Due Monday, November 30th

Problem 1 & 2: Assume

$$G(s) = \left(\frac{1.4427}{(s+0.1617)(s+1.04)(s+2.719)(s+5.05)} \right)$$

1) Design a gain compensator, K(s) = k, that results in

- A 50 degree phase margin

Also determine

- The resulting 0dB gain frequency,
- The closed-loop frequency response: $\left(\frac{Gk}{1+Gk} \right)$

Search G(jw) until the angle is -130 degrees

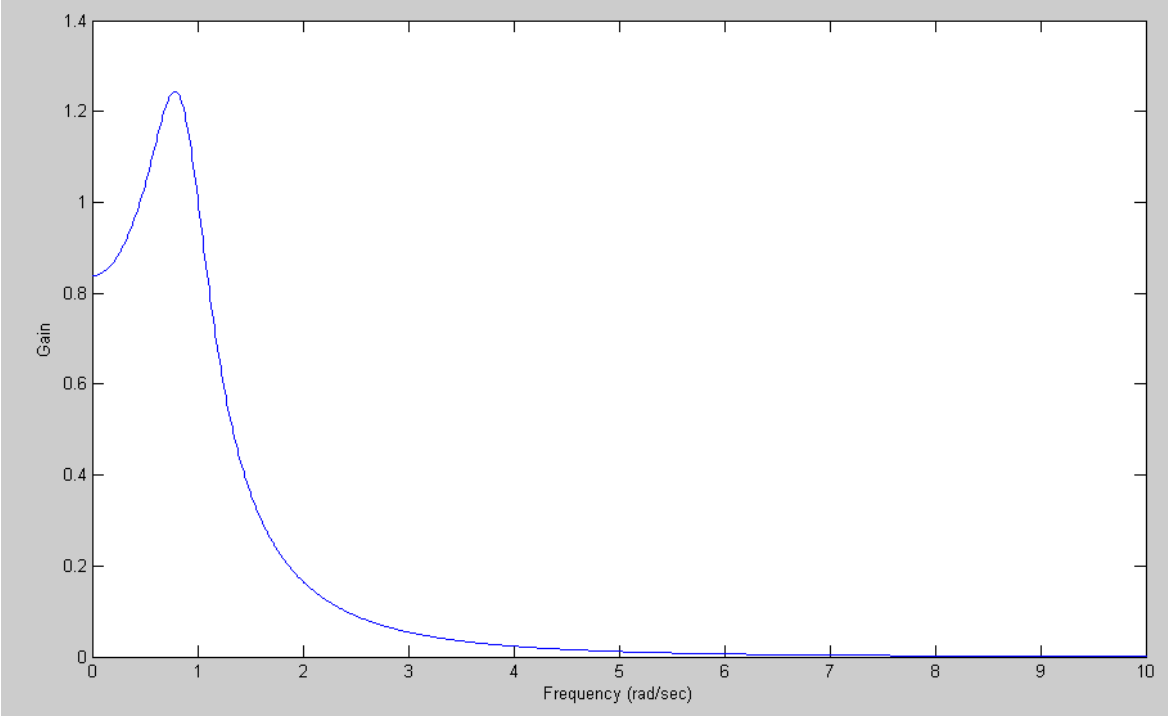
$$G(j0.6631) = 0.1202 \angle -130^0$$

Pick k to make the gain one at this frequency

$$k = \frac{1}{0.1202} = 8.3174$$

Resulting 0dB gain frequency = 0.6631 rad/sec

Resulting gain vs. frequency



2) Design a lead compensator of the form

$$K(s) = k\left(\frac{s+a}{s+10a}\right)$$

that results in

- A 50 degree phase margin

Also determine

- The resulting 0dB gain frequency, **w = 1.3202 rad/sec**
- The closed-loop frequency response:

Pick the zero:

- Pick the zero to be 1..3 times 0.6631 rad/sec (problem #1)
- Pick the zero to cancel the pole at s = -1.04

$$K(s) = k\left(\frac{s+1}{s+10}\right)$$

$$GK = \left(\frac{1.4427}{(s+0.1617)(s+1.04)(s+2.719)(s+5.05)}\right)\left(\frac{s+1}{s+10}\right)k$$

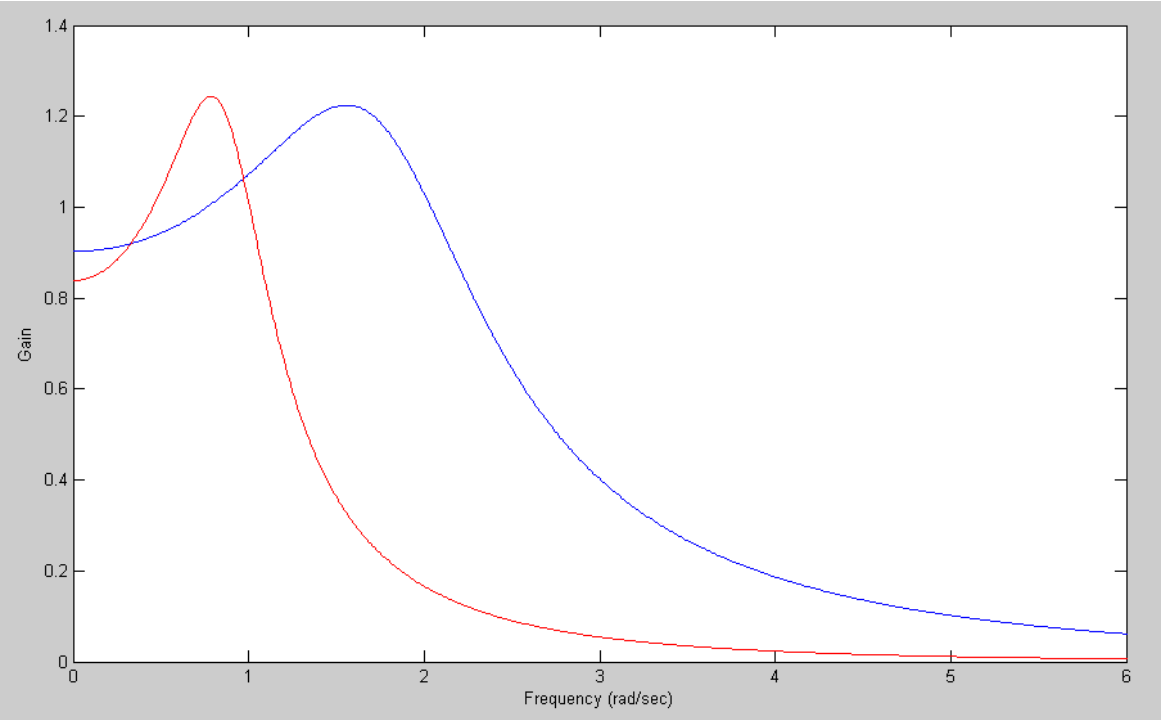
Search w until the phase is -130 degrees

$$GK(j1.3202) = 0.0067\angle -130^0$$

$$k = \frac{1}{0.0067} = 148.88$$

$$K(s) = 148.88\left(\frac{s+1}{s+10}\right)$$

```
Kw = 148.88 * (s+1) ./ (s+10);
GclwLead = (Gw.*Kw) ./ (1 + Gw.*Kw);
plot(w,abs(GclwLead),'b',w,abs(Gclw),'r')
```



Problem 3&4: Assume a 500ms delay is added

$$G(s) = \left(\frac{1.4427}{(s+0.1617)(s+1.04)(s+2.719)(s+5.05)} \right) e^{-0.5s}$$

3) Design a gain compensator, K(s) = k, that results in

- A 50 degree phase margin

Also determine

- The resulting 0dB gain frequency, **w = 0.5134 rad/sec**
- The closed-loop frequency response:

Search along w until the pase is -130 degrees

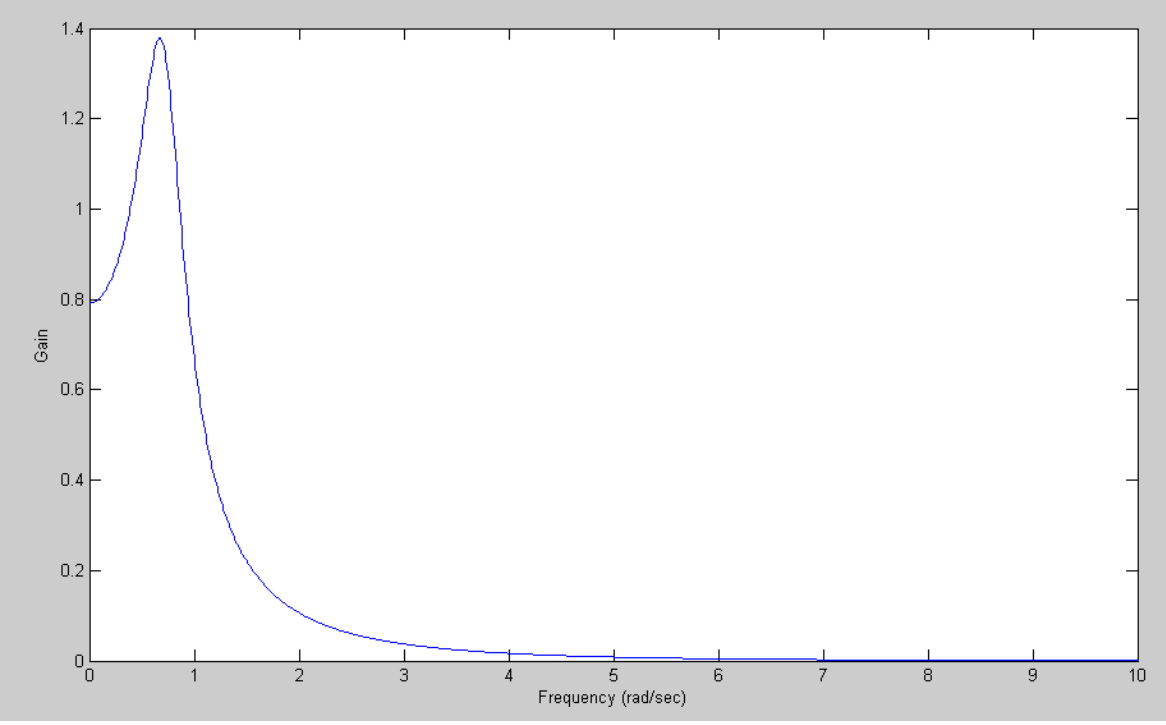
$$G(j0.5134) = 0.1645 \angle -130^0$$

Pick k to make the gain one at this frequency

$$k = \frac{1}{0.1645} = 6.078$$

```
Gw = 1.4427 ./ ( (s+0.1617).*(s+1.04).*(s+2.719).*(s+5.05) ) .*
exp(-0.5*s);

k = 6.078;
Gclw = (Gw*k) ./ (1 + Gw*k);
plot(w,abs(Gclw))
xlim([0,10]);
xlabel('Frequency (rad/sec)');
ylabel('Gain');
```



4) Design a lead compensator of the form

$$K(s) = k\left(\frac{s+a}{s+10a}\right)$$

that results in

- A 50 degree phase margin

Also determine

- The resulting 0dB gain frequency,
- The closed-loop frequency response:

Place the zero

- 1 to 3 times 0.5134 rad/sec, or
- To cancel the pole at s = -1

Let

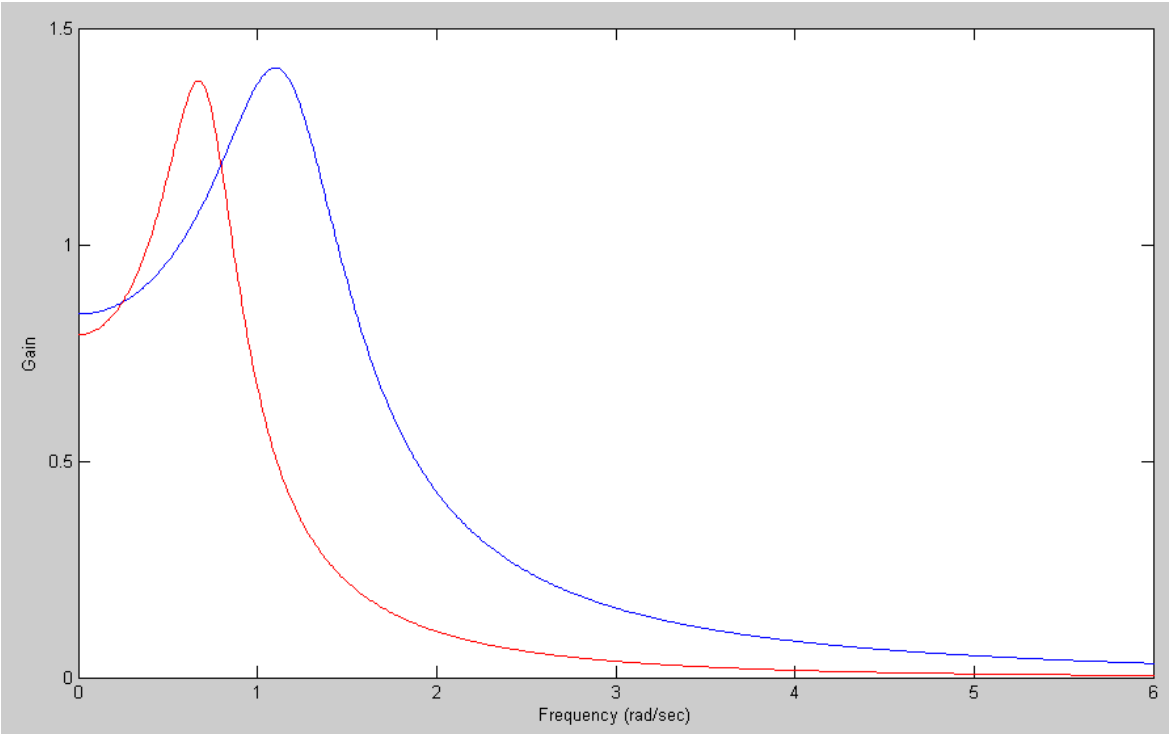
$$K(s) = k\left(\frac{s+1}{s+10}\right)$$

Search w until the phase of GK is -130 degrees

$$GK(j0.7953) = 0.01190\angle -130^0$$

$$k = \frac{1}{0.01190} = 83.74$$

```
>> Kw = 83.74 * (s+1) ./ (s+10);
>> Gclw = (Gw.*Kw) ./ (1 + Gw.*Kw);
>> plot(w,abs(Gclw))
>> xlabel('Frequency (rad/sec)');
>> ylabel('Gain');
```



Gain (red) & Gain + Lead (blue)