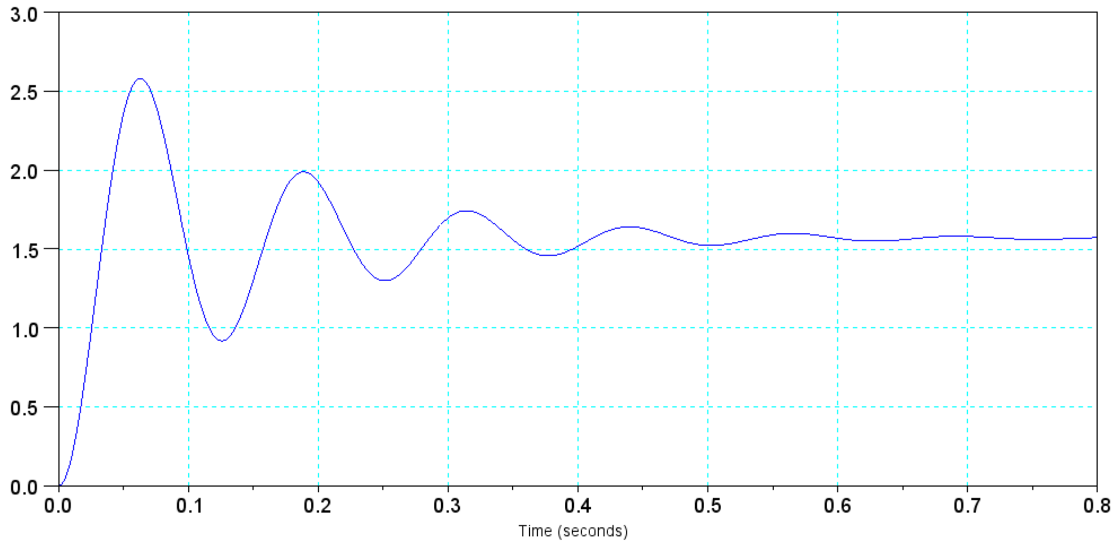
ECE 461 - Final: Name ____________________

Fall - 2021

1a) Give the transfer function for a system with the following step response:



DC gain = 1.6

2% settling time = 0.6 seconds

$$\sigma = \frac{4}{0.6} = -6.67$$

Frequency of oscillation

$$\omega_d = \left(\frac{4 \text{ cycles}}{0.5 \text{ seconds}}\right) 2\pi = 50.27$$

meaning

$$G(s) \approx \left(\frac{4277}{(s+6.67+j50.27)(s+6.67-j50.27)}\right)$$

The numberator is whatever it takes to make the DC gain 1.6

1b) What is the step response for the following system:

$$Y = \left(\frac{20{,}000}{(s+12+j30)(s+12-j30)(s+140)} \right) X$$

DC Gain	2% Settling Time	% Overshoot
0.137	**4/12 sec**	**28.5%**

domiant pole is -12 + j30

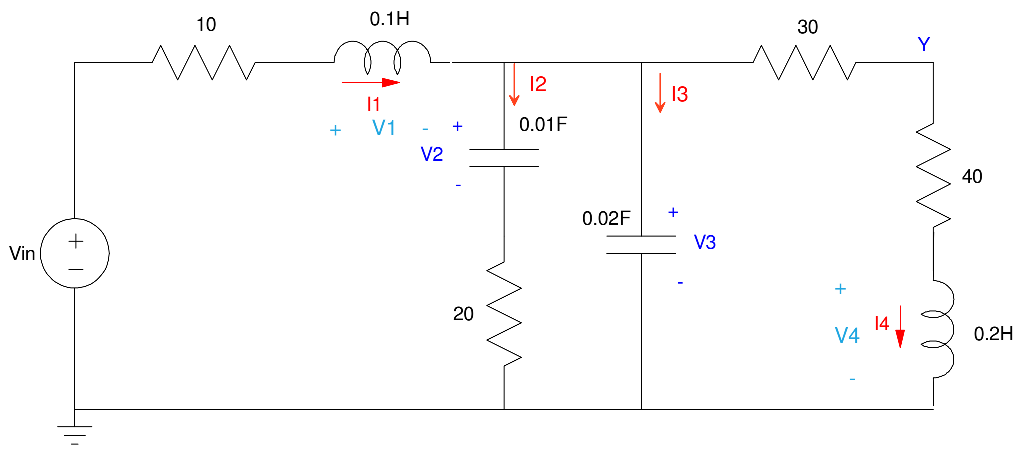
$$\theta = \arctan\left(\frac{30}{12}\right) = 68.199^0$$

$$\zeta = \cos(\theta) = 0.371$$

$$OS = \exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right) = 0.285$$

28.5% overshoot

2a) Write the differential equations which describe the following circuit (i.e. write the N differential equations which correspond to the voltage node equations)



$$V_1 = 0.1sI_1 = V_{in} - 10I_1 - V_3$$

$$I_2 = 0.01sV_2 = \left(\frac{V_3 - V_2}{20}\right)$$

$$I_3 = 0.02sV_3 = I_1 - \left(\frac{V_3 - V_2}{20}\right) - I_4$$

$$V_4 = 0.2sI_4 = V_3 - 70I_4$$

2b) Express these dynamics in state-space form

$$sI_1 = 10V_{in} - 100I_1 - 10V_3$$

$$sV_2 = 5V_3 - 5V_2$$

$$sV_3 = 50I_1 - 2.5V_3 + 2.5V_2 - 50I_4$$

$$sI_4 = 5V_3 - 350I_4$$

$$Y = V_3 - 30I_4$$

$$s\begin{bmatrix} I_1 \\ V_2 \\ V_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} -100 & 0 & -10 & 0 \\ 0 & -5 & 5 & 0 \\ 50 & 2.5 & -2.5 & -50 \\ 0 & 0 & 5 & -350 \end{bmatrix}\begin{bmatrix} I_1 \\ V_2 \\ V_3 \\ I_4 \end{bmatrix} + \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \end{bmatrix} V_{in}$$

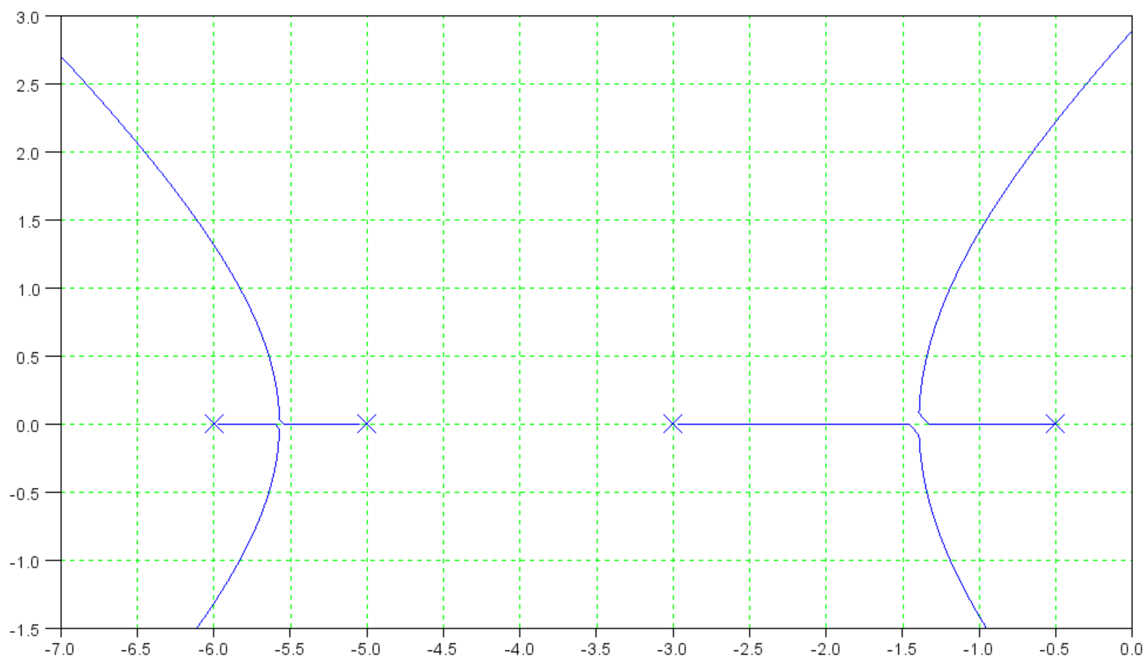
$$Y = \begin{bmatrix} 0 & 0 & 1 & -30 \end{bmatrix}\begin{bmatrix} I_1 \\ V_2 \\ V_3 \\ I_4 \end{bmatrix} + [0]V_{in}$$

3) Gain Compensation: The root locus for

$$G(s) = \left(\frac{40}{(s+0.5)(s+3)(s+5)(s+6)}\right)$$

is shown below. Determine the following:

Maximum gain, k, for a stable closed-loop system	s = j2.895 **k = 11.783**
k for a damping ratio of 0.4	s = -0.783 + j1.795 **k = 3.275**
Closed-loop dominant pole(s) for a damping ratio of 0.4	**-0.783 + j1.795**
Closed-Loop DC gain for a damping ratio of 0.4	**0.74**



DC gain of the open-loop system

$$K_p = GK = \left(\frac{40}{(s+0.5)(s+3)(s+5)(s+6)}\right)_{s=0} \cdot 3.275 = 2.911$$

DC gain of the closed-loop system

$$DC = \left(\frac{GK}{1+GK}\right) = \left(\frac{2.911}{1+2.911}\right) = 0.744$$

4) Given the following stable system

$$G(s) = \left(\frac{40}{(s+0.5)(s+3)(s+5)(s+6)}\right)$$

Determine a compensator, K(s), which results in the closed-loop system having

- No error for a step input, and
- A closed-loop dominant pole at s = -1.5 + j4

Let

$$K(s) = k\left(\frac{(s+0.5)(s+3)}{s(s+a)}\right)$$

$$GK = \left(\frac{40k}{s(s+5)(s+6)(s+a)}\right)$$

Evaluate what we know at the design point

$$\left(\frac{40}{s(s+5)(s+6)}\right)_{s=-1.5+j4} = 0.293\angle -201^0$$

Let

$$K(s) = k\left(\frac{(s+0.5)(s+3)(s+5)}{s(s+a)^2}\right)$$

$$GK = \left(\frac{40k}{s(s+6)(s+a)^2}\right)$$

$$\left(\frac{40}{s(s+6)}\right)_{s=-1.5+j4} = 1.555\angle -152.19^0$$

$$\angle(s+a)^2 = 27.810^0$$

$$\angle(s+a) = 13.905^0$$

$$a = \left(\frac{4}{\tan\left(13.905^0\right)}\right) + 1.5 = 17.657$$

so

$$K(s) = k\left(\frac{(s+0.5)(s+3)(s+5)}{s(s+17.657)^2}\right)$$

evaluate what we know

$$\left(\frac{40}{s(s+6)(s+17.657)^2}\right)_{s=-1.5+j4} = 0.006\angle 180^0$$

$$k = \frac{1}{0.006} = 178.148$$

$$K(s) = 178.148\left(\frac{(s+0.5)(s+3)(s+5)}{s(s+17.657)^2}\right)$$

It also works to cancel all four poles:

$$K(s) = 112.517\left(\frac{(s+0.5)(s+3)(s+5)(s+6)}{s(s+10.8561)^3}\right)$$

5) Given the following stable system

$$G(s) = \left(\frac{40}{(s+0.5)(s+3)(s+5)(s+6)}\right)$$

Determine a digital compensator, K(z), which results in the closed-loop system having

- No error for a step input,
- A closed-loop dominant pole at s = -1.5 + j7, and
- A sampling rate of T = 0.1

$$s = -1.5 + j7$$

$$z = e^{sT} = 0.6583 + j0.5545$$

Let

$$K(z) = k\left(\frac{(z-0.9512)(z-0.7408)(z-0.6065)(z-0.5488)}{(z-1)(z-a)^3}\right)$$

Evaluate what we know

G(s) * 50ms delay * K(z)

$$\left(\left(\frac{40}{(s+0.5)(s+3)(s+5)(s+6)}\right) \cdot e^{-0.05s} \cdot \left(\frac{(z-0.9512)(z-0.7408)(z-0.6065)(z-0.5488)}{(z-1)}\right)\right)_{s=-1.5+j7} = 0.0022\angle -58.6^0$$

$$\angle(z-a)^3 = 121.36^0$$

$$\angle(z-a) = 40.45^0$$

$$a = 0.6583 - \left(\frac{0.5545}{\tan\left(40.45^0\right)}\right) = 0.0080$$

Evaluate what we know

$$\left(\left(\frac{40}{(s+0.5)(s+3)(s+5)(s+6)}\right) \cdot e^{-0.05s} \cdot \left(\frac{(z-0.9512)(z-0.7408)(z-0.6065)(z-0.5488)}{(z-1)(z-0.0080)^3}\right)\right)_{s=-1.5+j7} = 0.0036\angle 180^0$$

$$k = \frac{1}{00046} = 280.9$$

$$K(z) = 280.9\left(\frac{(z-0.9512)(z-0.7408)(z-0.6065)(z-0.5488)}{(z-1)(z-0.0080)^3}\right)$$

6) Given the following stable system

$$G(s) = \left(\frac{40}{(s+0.5)(s+3)(s+5)(s+6)}\right)$$

Determine a compensator, K(s), which results in the closed-loop system having

- A closed-loop DC gain of 1.000 (i.e. no error for a step input),
- A 0dB gain frequency of 7 rad/sec, and
- A phase margin of 24 degrees

Let

$$K(s) = k\left(\frac{(s+0.5)(s+3)(s+5)}{s(s+a)^2}\right)$$

$$GK = \left(\frac{40k}{s(s+6)(s+a)^2}\right)$$

Evaluate what we know

$$\left(\frac{40}{s(s+6)}\right)_{s=j7} = 0.6198\angle -139.39^0$$

We want the phase to be -156 degrees for a 24 degre phase margin. Meaning

$$\angle(s+a)^2 = 16.6013^0$$

$$\angle(s+a) = 8.3006^0$$

$$a = \left(\frac{7}{\tan\left(8.3006^0\right)}\right) = 47.9795$$

Evaluate what we now know

$$GK = \left(\frac{40}{s(s+6)(s+47.9795)^2}\right)_{s=j7} = 0.0003\angle -156^0$$

$$k = \frac{1}{0.0003} = 3793$$

meaning

$$K(s) = 3793\left(\frac{(s+0.5)(s+3)(s+5)}{s(s+47.9795)^2}\right)$$

It also works to cancel all four poles

$$K(s) = 1141.8\left(\frac{(s+0.5)(s+3)(s+5)(s+6)}{s(s+17.3256)^3}\right)$$