ECE 461/661 - Test #2: Name ____________________

Feedback and Root Locus - Fall 2021

Root Locus

1) The root locus of G(s) is shown below.

$$G(s) = \left(\frac{10(s+1+j3)(s+1-j3)}{s(s+6)(s+5+j3)(s+5-j3)}\right)$$

Determine the following

Approach angle to the zero at s = -1 + j3	Departure angle from the pole at -5 + j3	real axis loci	Breakaway Point	Asymptotes
-74.291 degrees	173.088 degrees	(0, -6)	-1.51	2 asymptotes angle +/- 90 degrees intersect = -7.00

Approach Angle

$$\left(\frac{10(s+1+j3)}{s(s+6)(s+5+j3)(s+5-j3)}\right)_{s=-1+j3} = 0.113\angle -105.709^0$$

Add (zero) -74.291 degrees to get -180 degrees

Departure Angle:

$$\left(\frac{10(s+1+j3)(s+1-j3)}{s(s+6)(s+5+j3)}\right)_{s=-5+j3} = 2.607\angle -6.911^0$$

Subtract (pole) 173.089 degrees to get -180 degrees

Asymptote Intersect

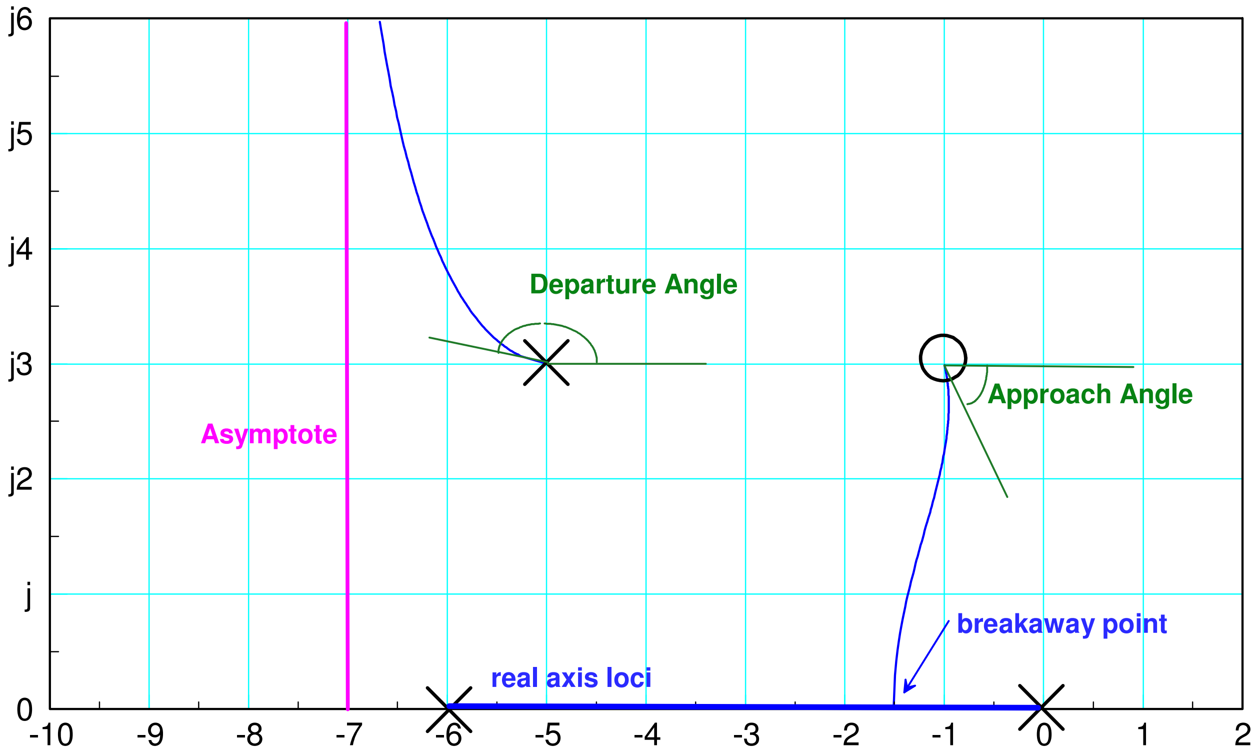
4 poles - 2 zeros = 2 asymptotes

$$2\theta = 180^0$$

$$\theta = \pm 90^0$$

$$\text{Intersect}=\left(\frac{\sum poles - \sum zeros}{\#poles - \#zeros}\right)$$

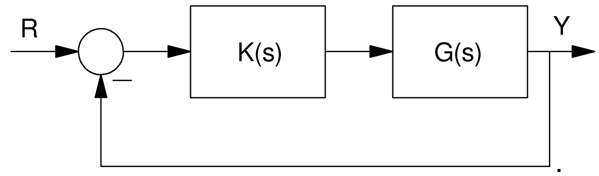
$$\text{Intersect}=\left(\frac{(0-6-5-5)-(-1-1)}{4-2}\right) = -7$$



j6
j5
j4
j3
j2
j
0
-10
-9
-8
-7
-6
-5
-4
-3
-2
-1
0
1
2
Departure Angle
Approach Angle
Asymptote
breakaway point
real axis loci

Gain Compensation

2) Design a gain compensator (K(s) = k) so that the feedback system has 50% overshoot for a step input.



Also determine

- The resulting error constant, Kp,
- The closed-loop dominant pole(s)

Assume

$$G(s) = \left(\frac{100}{(s-1)(s+5)(s+7)}\right)$$

Dampng Ratio	Design point	k	Kp	Closed-loop dominant pole
zeta = 0.215	**s = -0.678 + j3.076**	**1.307**	**-3.73**	**s = -0.678 + j3.076**

Step 1: Draw the damping line

- 50% overshoot means zeta = 0.215
- angle = 77.56 degrees

Step 2: Determine the design point (where the root locus intersects with the damping line)

- s = -0.678 + j3.076

Step 3: Find k

$$\left(\frac{100}{(s-1)(s+5)(s+7)}\right)_{s=-0.678+j3.076} = 0.765k\angle 180^0$$

$$k = \frac{1}{0.765} = 1.307$$

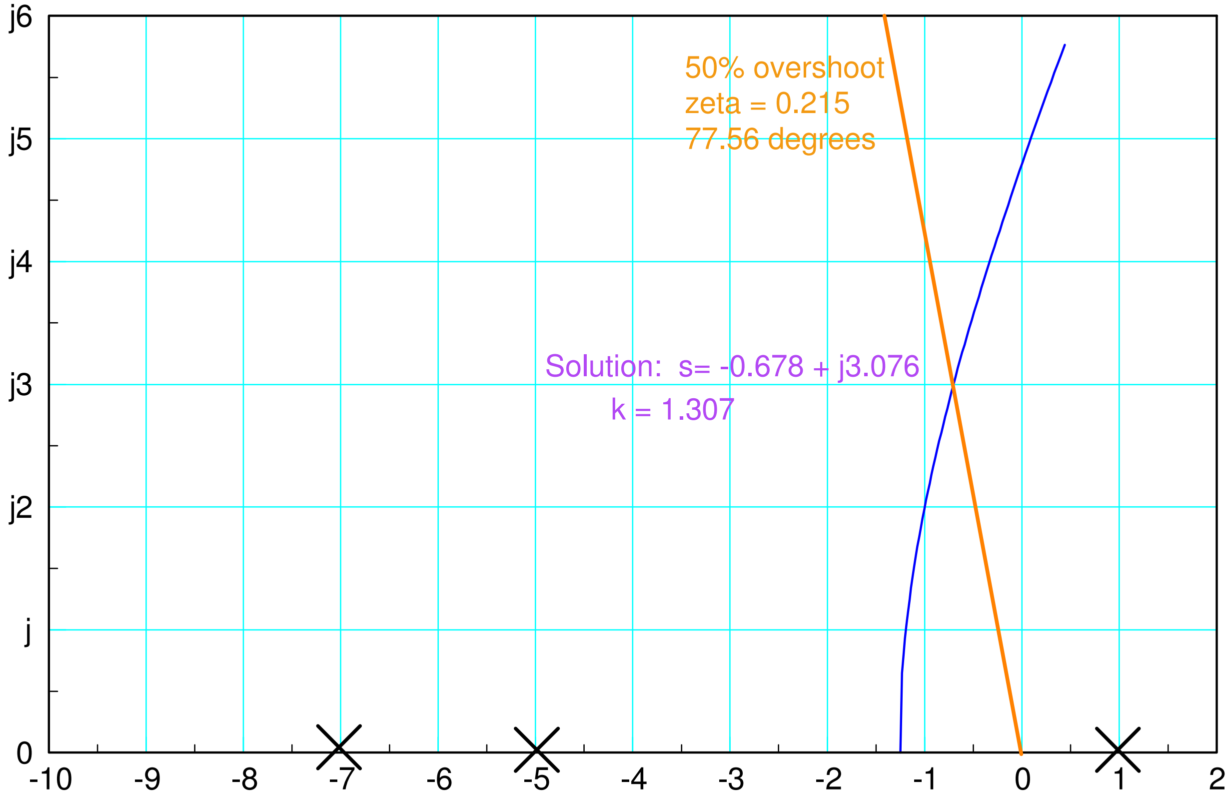
Kp:

$$K_p = (Gk)_{s\to 0}$$

$$K_p = \left(\frac{100}{(s-1)(s+5)(s+7)}\right)_{s\to 0} \cdot 1.307$$

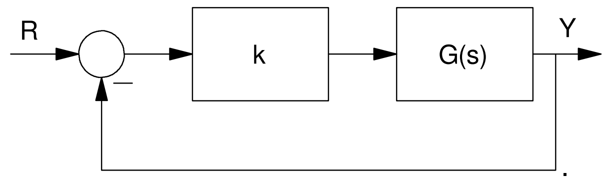
$$K_p = -3.733$$

Closed-Loop Dominant Pole(s) **s = -0.678 + j3.076**



50% overshoot
zeta = 0.215
77.56 degrees
Solution: s= -0.678 + j3.076
k = 1.307
j6
j5
j4
j3
j2
j
0
-10
-9
-8
-7
-6
-5
-4
-3
-2
-1
0
1
2

Lead/PI Compensation



3) Design a compensator, K(s), so that the closed-loop system has

- No error for a step input
- Closed-Loop dominant poles at s = -2 + j3, and
- Finite gain as s→ ∞ (i.e. have at least as many poles as zeros)

$$G(s) = \left(\frac{100}{(s+2)(s+5)(s+7)}\right)$$

Pick K(s)

- Add a pole at s = 0 to make it a type-1 system
- Cancel the two zeros at -2 and -5
- Add a pole at -a. Pick 'a' so that s = -2 + j3 is on the root locus (angles add up to 180 degrees)

$$K(s) = k\left(\frac{(s+2)(s+5)}{s(s+a)}\right)$$

and

$$GK = \left(\frac{100k}{s(s+7)(s+a)}\right)$$

Evaluate what we know at the design point

$$\left(\frac{100}{s(s+7)}\right)_{s=-2+j3} = 4.757\angle -154.654^0$$

So that the angle adds up to 180 degrees

$$\angle(s+a) = 25.346^0$$

$$a = \frac{3}{\tan(25.346^0)} + 2 = 8.333$$

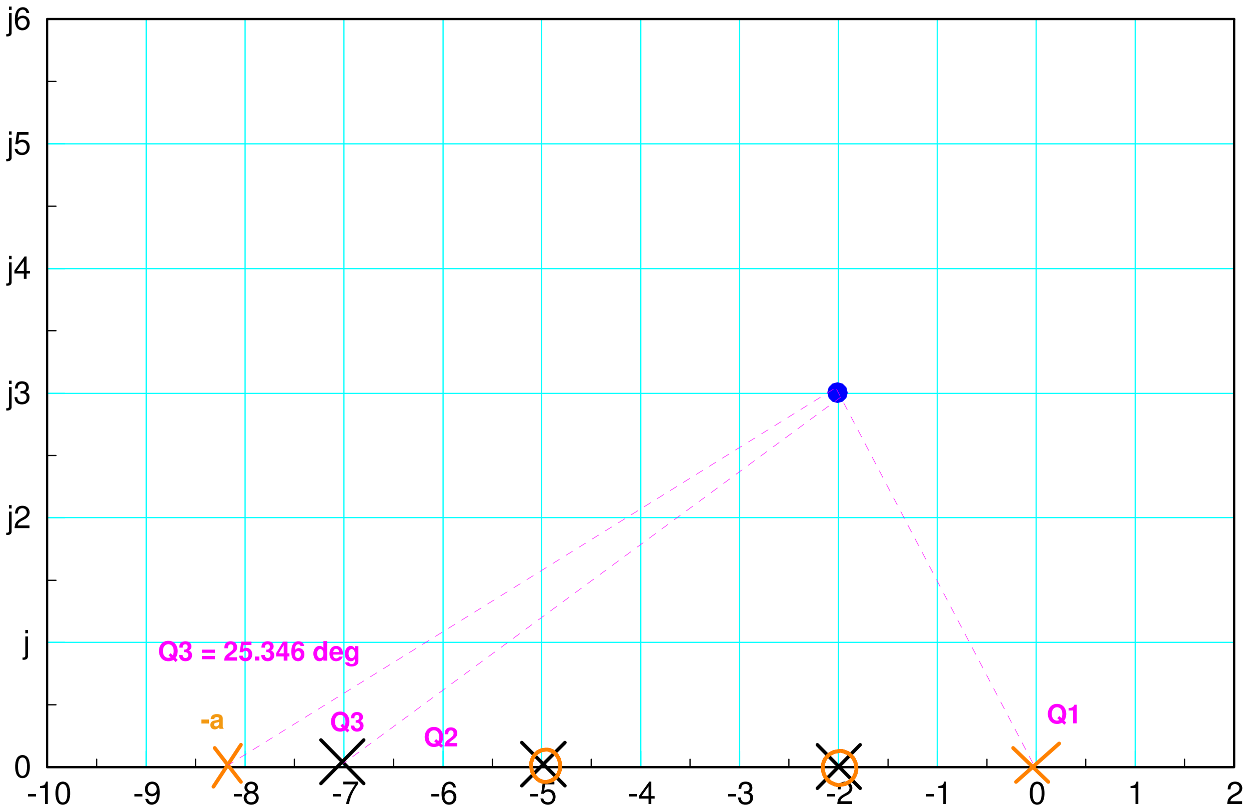
GK is now

$$GK = \left(\frac{100k}{s(s+7)(s+8.333)}\right)_{s=-2+j3} = 0.679k\angle 180^0$$

To make GK = -1

$$k = \frac{1}{0.679} = 1.473$$

$$K(s) = 1.473\left(\frac{(s+2)(s+5)}{s(s+8.333)}\right)$$



Compensator Design (hardware)

4) Design a circuit to implement K(s)

$$K(s) = \left(\frac{150(s+7)(s+11)}{s(s+22)} \right)$$

Rewrite as

$$K(s) = \left(15\left(\frac{s+11}{s+22} \right) \right) \left(10\left(\frac{s+7}{s} \right) \right)$$