ECE 461/661 - Test #3: Name ___________________

Digital Control & Frequemncy Domain techniques - Fall 2021

s to z conversion

1) Determine the discrete-time equivalent for G(s). Assume a sampling rate of T = 0.1 second

$$G(s) = \left(\frac{100}{(s+3)(s+8)(s+12)}\right)$$

Convert as $z = e^{sT}$

$$s = -3 \qquad z = e^{sT} = 0.7408$$

$$s = -8 \qquad z = 0.4493$$

$$s = -12 \qquad z = 0.3012$$

meaning

$$G(z) = \left(\frac{k}{(z-0.7408)(z-0.4493)(z-0.3012)}\right)$$

To find k, match the gain at DC

$$\left(\frac{100}{(s+3)(s+8)(s+12)}\right)_{s=0} = 0.3472$$

$$\left(\frac{k}{(z-0.7408)(z-0.4493)(z-0.3012)}\right)_{z=1} = 0.3472$$

$$k = 0.0346$$

(optional): Add zeros at the origin to match the phase at 1 rad/sec

$$\left(\frac{100}{(s+3)(s+8)(s+12)}\right)_{s=j} = 0.3257\angle -30.3236^0$$

$$\left(\frac{0.0346}{(z-0.7408)(z-0.4493)(z-0.3012)}\right)_{z=e^{j0.1}} = 0.3258\angle -39.9969^0$$

$$z = e^{j0.1} = 1\angle 5.7296^0$$

The number of zeros you need is

$$n = \left(\frac{39.9969^0 - 30.3236^0}{5.7296^0}\right) = 1.688$$

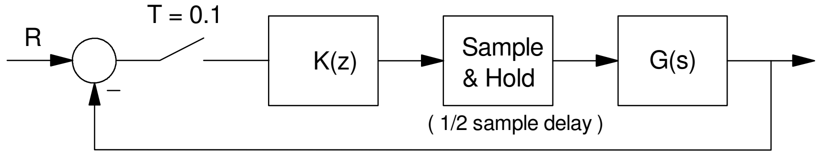
Let n=2 (slightly too large)

$$G(z) = \left(\frac{0.0346z^2}{(z-0.7408)(z-0.4493)(z-0.3012)}\right)$$

Digital Compensators: K(z)

2) Assume a unity feedback system with a sampling rate of T = 0.1 second

$$G(s) = \left(\frac{100}{(s+3)(s+8)(s+12)} \right)$$



Design a digital compensator, K(z), which results in

- No error for a step input
- 5% overshoot ($\zeta = 0.6901$), and
- A 2% settling time of 1.5 seconds

Translation:

- Make it a type-1 system
- Place the closed loop dominant pole at
  - s = -2.667 + j2.7966
  - z = 0.7362 + j0.2114

Let

$$K(z) = k\left(\frac{(z-0.7408)(z-0.4493)}{(z-1)(z-a)} \right)$$

Pick 'a' so that the phase is 180 degrees

$$\left(\frac{100}{(s+3)(s+8)(s+12)} \right) \cdot e^{-0.05s} \cdot k\left(\frac{(z-0.7408)(z-0.4493)}{(z-1)(z-a)} \right)_{s=-0.2667+j2.7966} = 1\angle 180^0$$

Evaluate what we know at the closed-loop dominant pole:

$$\left(\frac{100}{(s+3)(s+8)(s+12)} \right) \cdot e^{-0.05s} \cdot \left(\frac{(z-0.7408)(z-0.4493)}{(z-1)} \right) = 0.1541\angle -149.2144^0$$

$$\angle(z-a) = 30.7856^0$$

$$a = 0.7362 - \left(\frac{0.2114}{\tan\left(30.7856^0\right)} \right) = 0.3814$$

giving

$$\left(\frac{100}{(s+3)(s+8)(s+12)} \right) \cdot e^{-0.05s} \cdot k\left(\frac{(z-0.7408)(z-0.4493)}{(z-1)(z-0.3814)} \right)_{s=-0.2667+j2.7966} = 0.3732\angle 180^0$$

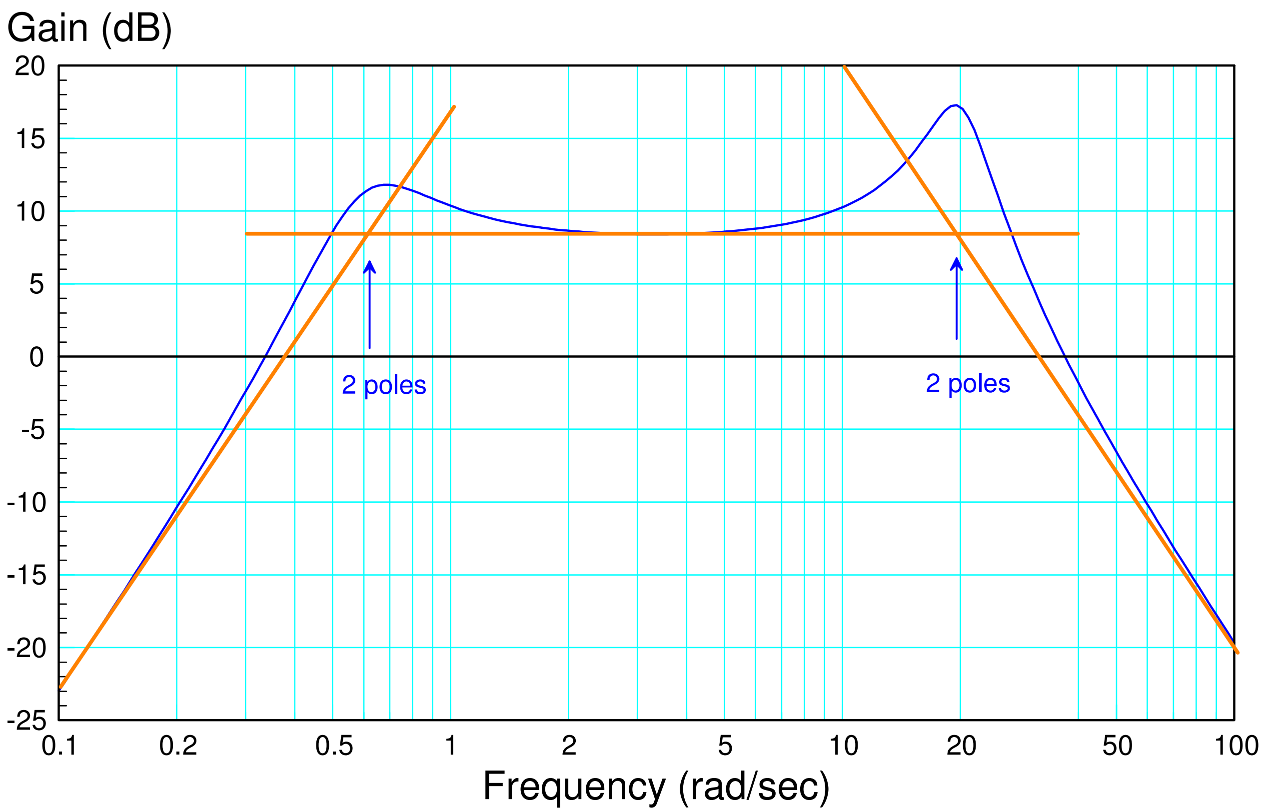
so

$$k = \frac{1}{0.3732} = 2.6796$$

$$K(z) = 2.6796\left(\frac{(z-0.7408)(z-0.4493)}{(z-1)(z-0.3814)} \right)$$

3) Bode Plots

Determine the system, G(s), which has the following gain vs. frequency



2 zeros left of 0.1

- assume zeros at s = 0

2 poles at 0.6 rad/sec

- gain at corner is +4dB (1.5849)
- $\zeta = \frac{1}{2 \cdot 1.5849} = 0.3155 \qquad (71.6^0)$

2 poles at 20 rad/sec

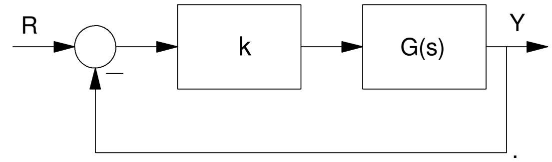
- gain at corner is +15dB (5.6234)
- $\zeta = \frac{1}{2 \cdot 5.6234} = 0.0889 \qquad (84.9^0)$

so

$$G(s) = \left(\frac{ks^2}{\left(s+0.6\angle\pm71.6^0\right)\left(s+20\angle\pm84.9^0\right)} \right) = \left(\frac{1004s^2}{\left(s+0.6\angle\pm71.6^0\right)\left(s+20\angle\pm84.9^0\right)} \right)$$

Pick 'k' to match the gain at some point, such as G(j3) = 8dB = 2.51 (results in k = 1004)

4) Nichols Charts

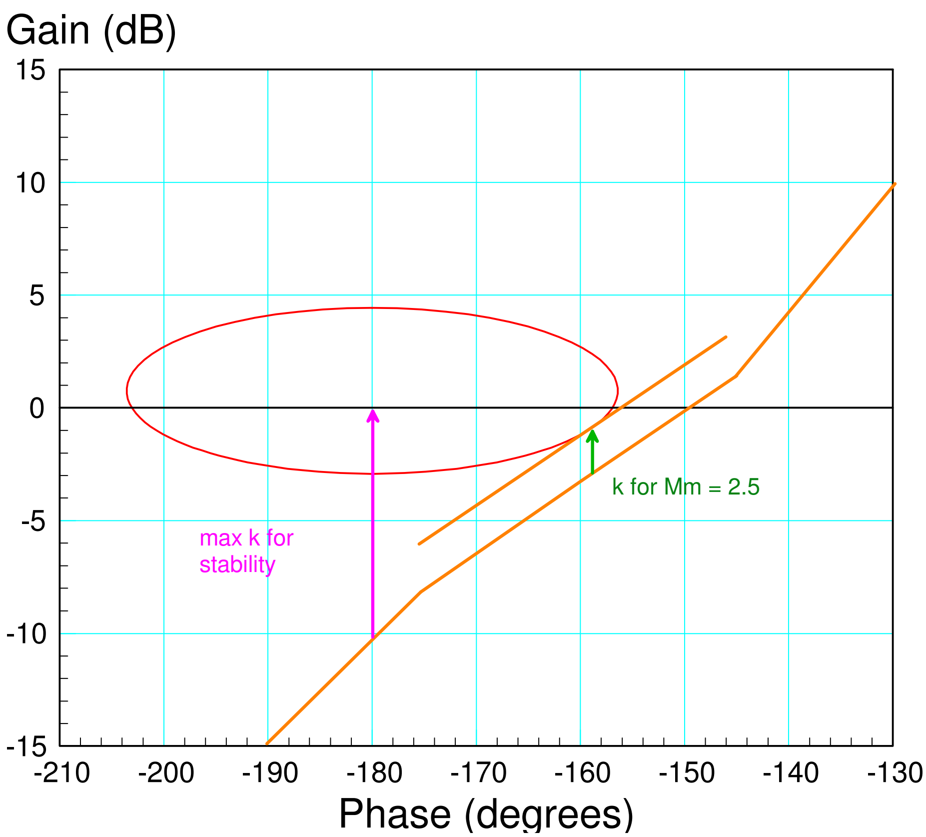


Assume a unity feedback system where the gain of G(s) is as follows:

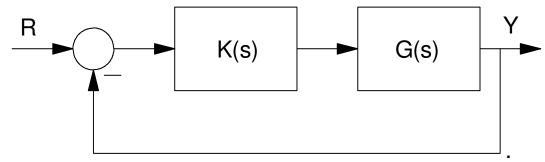
Determine

- The maximum gain, k, for stability **ans = +10dB**
- k that results in a resonance of Mm = 2.5 **ans = +2dB**

frequency (rad/sec)	7	8	9	10	12	15
Gain	10dB	2dB	-3dB	-8dB	-15dB	-22dB
Phase (degrees)	-130 deg	-145 deg	-160 deg	-175 deg	-190 deg	-205 deg



5) Analog Compensator (Bode Plots)



Assume a unity feedback system with

$$G(s) = \left(\frac{10}{(s+3)(s+8)(s+12)}\right)$$

Determine a compensator, K(s), which results in

- No error for a step input
- A phase margin of 60 degrees
- A 0dB gain frequency of 3 rad/sec

Let K(s) be of the form

$$K(s) = k\left(\frac{(s+3)(s+8)}{s(s+a)}\right)$$

Find 'a' so that the phase is -120 degrees (60 degree phase margin) at 3 rad/sec

$$GK = \left(\frac{10k}{s(s+a)(s+12)}\right)_{s=j3} = 1\angle-120^0$$

analyze what we know:

$$\left(\frac{10}{s(s+12)}\right)_{s=j3} = 0.2695\angle-104.0362^0$$

meaning

$$\angle(s+a) = 15.9638^0$$

$$a = \left(\frac{3}{\tan\left(15.9638^0\right)}\right) = 10.4873$$

Analyze what we now know

$$\left(\frac{10}{s(s+10.4873)(s+12)}\right)_{s=j3} = 0.0247\angle-120^0$$

$$k = \frac{1}{0.0247} = 40.4772$$

$$\boxed{K(s) = 40.4772\left(\frac{(s+3)(s+8)}{s(s+10.4873)}\right)}$$