Homework #3: ECE 461 / 661

Structured Text, 1st and 2nd Order Approximations. Due Monday, September 13th
(will accept PLC code any time before December 1st so you can use the Micro810 PLC's)

LaPlace Transforms (Due September 13th)

5) Assume X and Y are related by the following transfer function

$$Y = \left(\frac{10(s+3)}{(s+2)(s+5)(s+10)}\right)X$$

a) What is the differential equation relating X and Y?

Multiply out and cross multiply

$$Y = \left(\frac{10s+30}{s^3+17s^2+80s+100}\right)X$$

$$(s^3 + 17s^2 + 80s + 100)Y = (10s + 30)X$$

sY means *the derivative of Y*

$$\frac{d^3y}{dt^3} + 17\frac{d^2y}{dt^2} + 80\frac{dy}{dt} + 100y = 10\frac{dx}{dt} + 30x$$

or

$$y''' + 17y'' + 80y' + 100y = 10x' + 30x$$

b) Determine y(t) assuming

$$x(t) = 4\cos(3t) + 5\sin(3t)$$

This is a phasor problem

$$Y = \left(\frac{10(s+3)}{(s+2)(s+5)(s+10)}\right)X$$

is true for all 's'. In this case

$$s = j3$$

$$X = 4 - j5$$

(real = cosine, imag = -sine)

$$Y = \left(\frac{10(s+3)}{(s+2)(s+5)(s+10)}\right)_{s=j3} \cdot (4 - j5)$$

$$Y = -0.430 - j1.161$$

meaning

$$y(t) = -0.430\cos(3t) + 1.161\sin(3t)$$

c) Determine y(t) assuming x(t) is a unit step input

$$x(t) = u(t)$$

This is a LaPlace problem. The LaPlace transfor for x(t) is

$$X(s) = \left(\frac{1}{s}\right)$$

$$Y = \left(\frac{10(s+3)}{(s+2)(s+5)(s+10)}\right)\left(\frac{1}{s}\right)$$

Use partial fractions

$$Y = \left(\frac{A}{s}\right) + \left(\frac{B}{s+2}\right) + \left(\frac{C}{s+5}\right) + \left(\frac{D}{s+10}\right)$$

$$Y = \left(\frac{0.30}{s}\right) + \left(\frac{-0.208}{s+2}\right) + \left(\frac{-0.267}{s+5}\right) + \left(\frac{0.175}{s+10}\right)$$

Convert back to time

$$y(t) = (0.3 - 0.208e^{-2t} - 0.267e^{-5t} + 0.175e^{-10t})u(t)$$

6) Assume X and Y are related by the following transfer function:

$$Y = \left(\frac{100}{(s+1+j5)(s+1-j5)(s+30)}\right)X$$

a) Use 2nd-order approximations to determine

- The 2% settling time
- The percent overshoot for a step input
- The steady-state output for a step input (x(t) = u(t))

The dominant pole is

$$s = -1 \pm j5$$

The 2% settling time is

$$T_s = \frac{4}{real(s)} = 4 \text{ sec}$$

The damping ratio comes from the angle of the dominant pole

$$\theta = \arctan\left(\frac{imag}{real}\right) = \arctan\left(\frac{5}{1}\right) = 78.69^0$$

$$\zeta = \cos(78.69^0) = 0.196$$

The overshoot is 53.3%

$$OS = \exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right) = 0.533$$

The steady-state output is the DC gain

$$G(s=0) = \left(\frac{100}{(s+1+j5)(s+1-j5)(s+30)}\right)_{s=0} = 0.128$$

b) Check your answers using the 3rd order model and Matlab, Simulink, of VisSim (your pick)

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>> G = zpk([],[-1+j*5,-1-j*5,-30],100)

Zero/pole/gain:
         100
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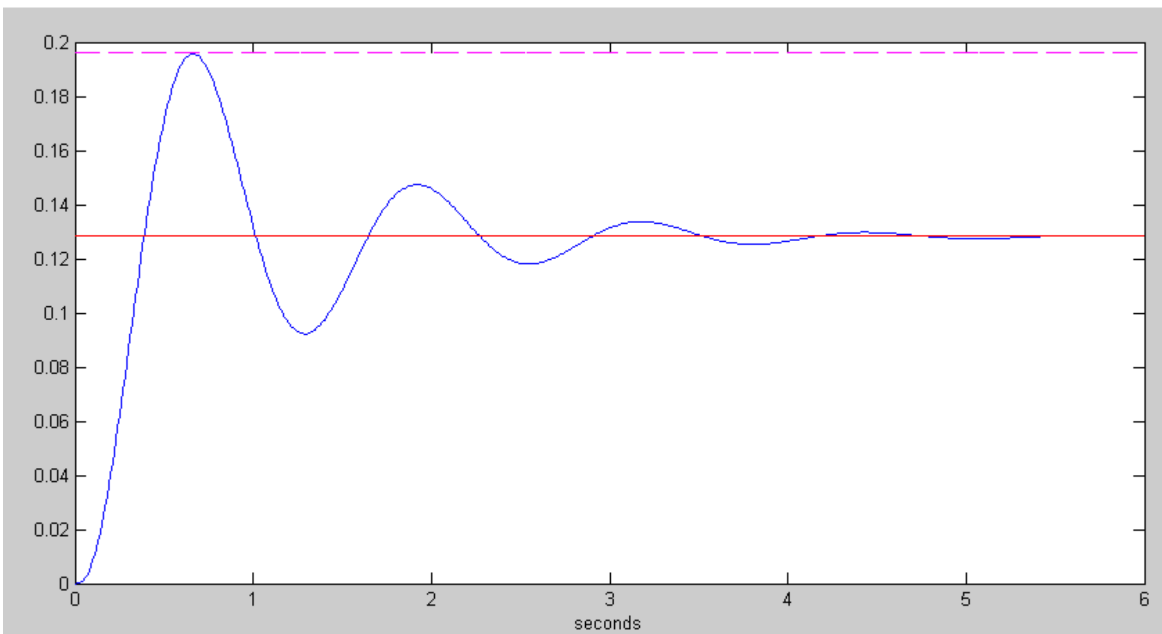
(s+30) (s^2 + 2s + 26)

>> t = [0:0.01:6]';
>> y = step(G,t);
>> plot(t,y);
>> xlabel('seconds');

>> DC = evalfr(G,0);
>> plot(t,y,'b',t,DC+t*0,'r',t,DC*1.53+t*0,'m--')
>> xlabel('seconds');
```



7) Determine the transfer function for a system with the following step response:

This is a 1st-order system (no oscillations) meaning

$$Y = \left(\frac{a}{s+b}\right)X$$

There are 2 unknowns. We need to take two measurements to find a and b

The DC gain is 2.95

$$\left(\frac{a}{s+b}\right)_{s=0} = \left(\frac{a}{b}\right) = 2.95$$

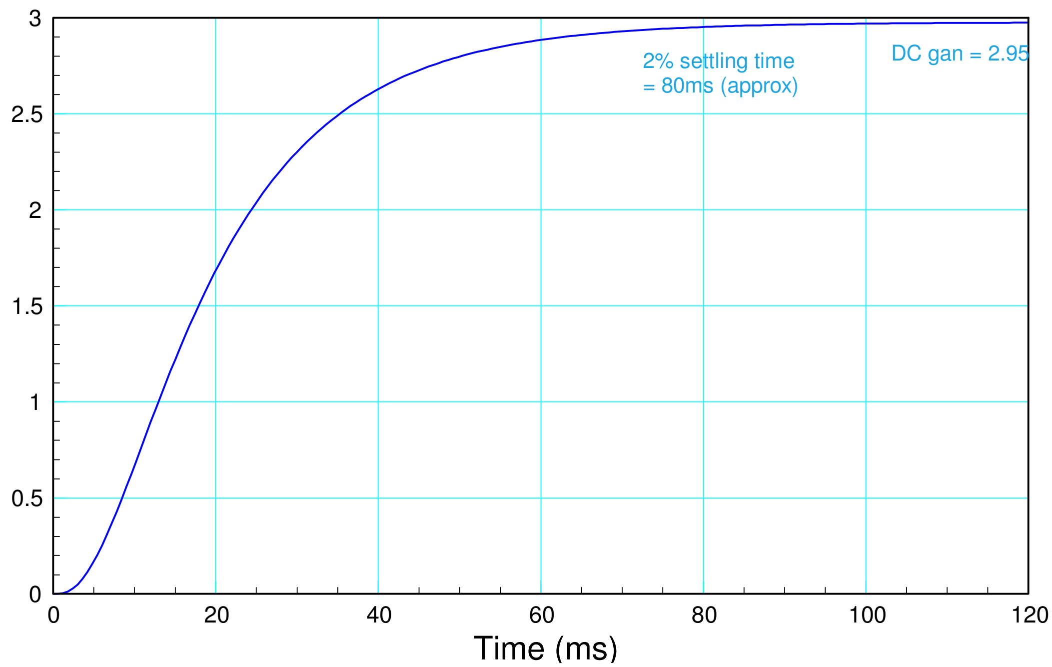
The 2% settling time is about 80ms

$$T_s = 80ms = \frac{4}{b}$$

$$b = 50$$

which makes a = 147.5 *(sets the DC gain to 2.95)*

$$Y \approx \left(\frac{147.5}{s+50}\right)$$



8) Determine the transfer function for a system with the following step response:

This is a 2nd-order system (it oscillates, meaning the poles are complex). In general

$$Y = \left(\frac{k}{(s+a+jb)(s+a-jb)}\right)X$$

There are 3 unknowns - we need to take 3 measurements

DC Gain = 3.4

$$\left(\frac{k}{(s+a+jb)(s+a-jb)}\right)_{s=0} = \left(\frac{k}{a^2+b^2}\right) = 3.4$$

Frequency of oscillation (imag(pole))

$$b = \left(\frac{3 \text{ cycles}}{82\text{ms}}\right) \cdot 2\pi = 229.9\frac{rad}{\text{sec}}$$

2% settlimg time = 110ms (approx)

$$T_s = \frac{4}{a} = 110ms$$

$$a = 36.4$$

so

$$G(s) \approx \left(\frac{182{,}648}{(s+36.4+j228.9)(s+36.4-j228.9)}\right)$$

the numerator sets the DC gain to 3.4