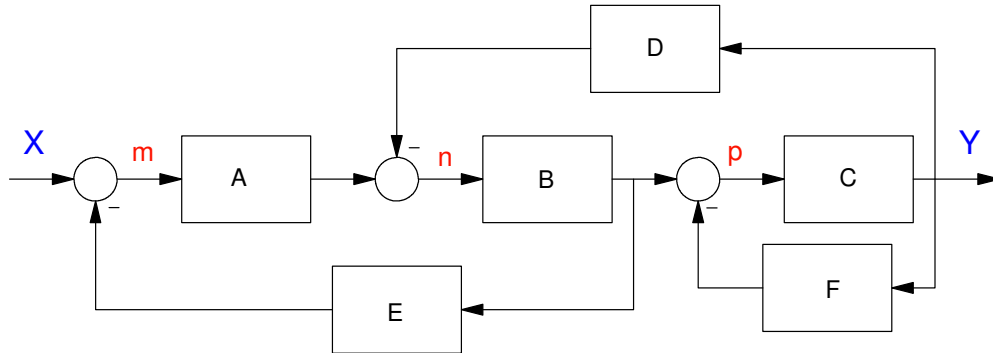


Homework #4: ECE 461/661

Block Diagrams, Canonical Forms, Electrical Circuits. Due Monday, September 20th

Block Diagrams

1) Determine the transfer function from X to Y



Option 1:

$$G = \left(\frac{\sum (\text{gains from X to Y})}{1 + \sum (\text{loop gains})} \right) = \left(\frac{ABC}{1 + ABE + BCD + CF} \right)$$

Option 2: Write out equations

$$m = X - EBn$$

$$n = Am - DY$$

$$p = Bn - FY$$

$$Y = Cp$$

Simplify (get m, n, p to drop out)

$$m = X - EB(Am - DY)$$

$$(1 + EBA)m = X + EBDY$$

$$m = \left(\frac{1}{1 + EBA} \right) X + \left(\frac{EBD}{1 + EBA} \right) Y$$

-

$$p = Bn - FY$$

$$p = B(Am - DY) - FY$$

$$p = B \left(A \left(\left(\frac{1}{1 + EBA} \right) X + \left(\frac{EBD}{1 + EBA} \right) Y \right) - DY \right) - FY$$

$$Y = Cp = C\left(B\left(A\left(\left(\frac{1}{1+EBA}\right)X + \left(\frac{EBD}{1+EBA}\right)Y\right) - DY\right) - FY\right)$$

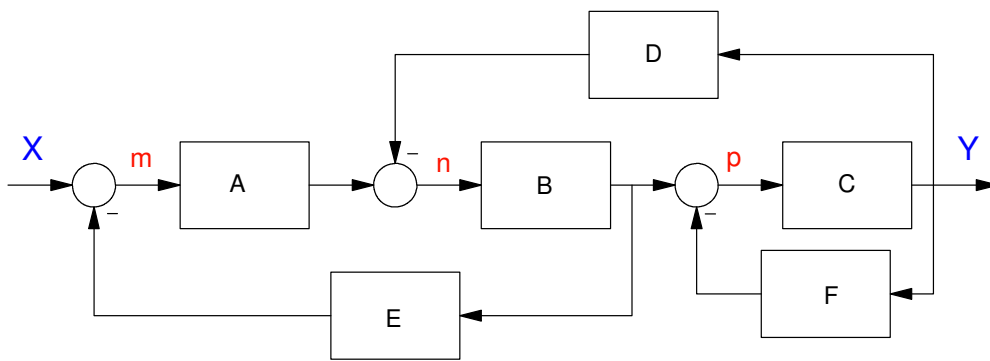
$$Y = \left(\frac{CBA}{1+EBA}\right)X + \left(\frac{CBAEBD}{1+EBA}\right)Y - CBDY - CFY$$

$$(1 + EBA)Y = (CBA)X + (CBAEBD)Y - (1 + EBA)CBDY - (1 + EBA)CFY$$

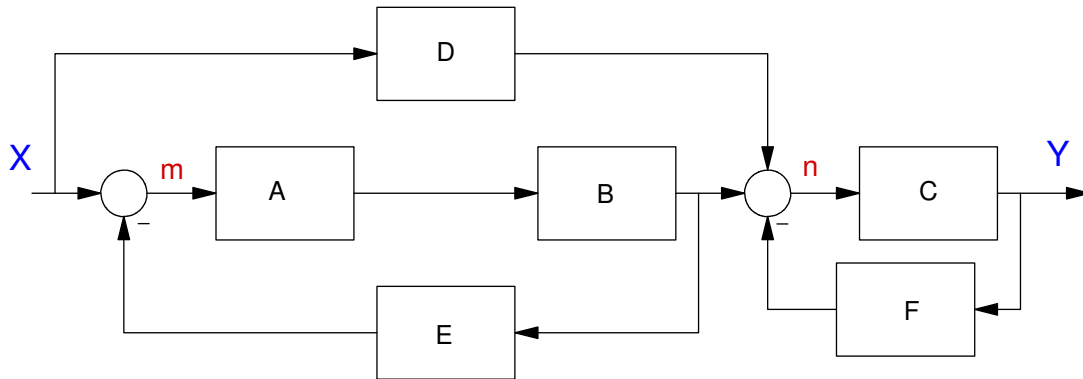
$$(1 + EBA + CBD + CF + EBACF)Y = (CBA)X$$

$$Y = \left(\frac{CBA}{1+EBA+CBD+CF+EBACF}\right)X$$

The shortcut missed the EBACF term



2) Determine the transfer function from X to Y



Shortcut - note that this is two systems cascaded

$$Y = \left(\frac{AB}{1+ABE} \right) \left(\frac{C}{1+CF} \right) + \left(\frac{DC}{1+CF} \right)$$

Long Way

$$m = X - EBAm$$

$$n = DX + BAm - FY$$

$$Y = Cn$$

simplify

$$m = \left(\frac{1}{1+EBA} \right) X$$

$$n = DX + BAm - FY$$

$$n = DX + BA \left(\left(\frac{1}{1+EBA} \right) X \right) - FY$$

$$Y = Cn$$

$$Y = C \left(DX + BA \left(\left(\frac{1}{1+EBA} \right) X \right) - FY \right)$$

$$(1 + EBA)Y = (1 + EBA)CDX + CBAX - (1 + EBA)CFY$$

$$(1 + EBA + CF + EBACF)Y = (CD + EBACD + CBA)X$$

$$Y = \left(\frac{CD+EBACD+CBA}{(1+EBA+CF+EBACF)} \right) X$$

which is the same as the shortcut

Canonical Forms

3) Give two different state-space models that produce the following transfer function

$$Y = \left(\frac{30}{(s+2)(s+3)(s+4)} \right) U$$

Cascade Form

$$sX = \begin{bmatrix} -2 & 0 & 0 \\ 1 & -3 & 0 \\ 0 & 1 & -4 \end{bmatrix} X + \begin{bmatrix} 30 \\ 0 \\ 0 \end{bmatrix} U$$

$$Y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} X$$

Controller Form

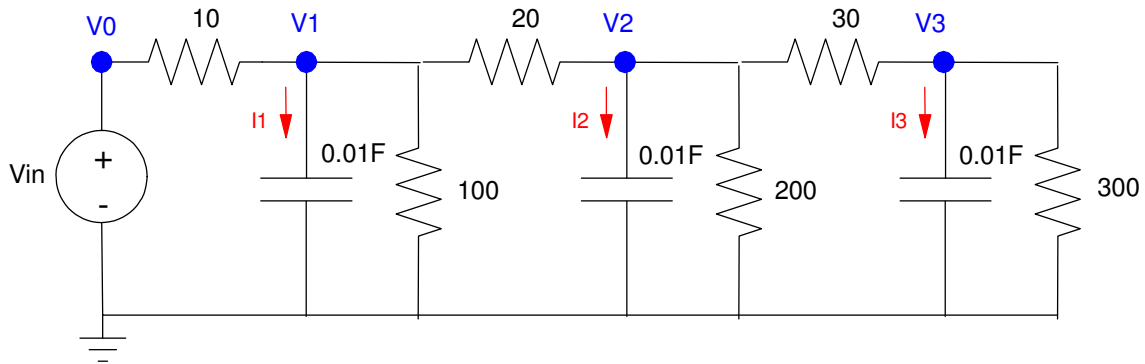
$$Y = \left(\frac{30}{(s+2)(s+3)(s+4)} \right) U = \left(\frac{30}{s^3 + 14s^2 + 26s + 24} \right) U$$

$$sX = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -26 & -14 \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} U$$

$$Y = \begin{bmatrix} 30 & 0 & 0 \end{bmatrix} X$$

Electrical Circuits

4) Using state-space methods, find the transfer function from V_{in} to V_3



$$I_1 = CsV_1 = \left(\frac{V_0 - V_1}{10} \right) + \left(\frac{V_2 - V_1}{20} \right) - \left(\frac{V_1}{100} \right)$$

$$I_2 = CsV_2 = \left(\frac{V_1 - V_2}{20} \right) + \left(\frac{V_3 - V_2}{30} \right) - \left(\frac{V_2}{200} \right)$$

$$I_3 = CsV_3 = \left(\frac{V_2 - V_3}{30} \right) - \left(\frac{V_3}{300} \right)$$

Simplifying and solving for sV_x

$$sV_1 = 10V_0 - 16V_1 + 5V_2$$

$$sV_2 = 5V_1 - 8.833V_2 + 3.333V_3$$

$$sV_3 = 3.333V_2 - 3.667V_3$$

In state-space

$$\begin{bmatrix} sV_1 \\ sV_2 \\ sV_3 \end{bmatrix} = \begin{bmatrix} -16 & 5 & 0 \\ 5 & -8.833 & 3.333 \\ 0 & 3.333 & -3.667 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} + \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix} V_0$$

$$y = V_3 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

Use Matlab to find the transfer function

```
>> A = [-16, 5, 0 ; 5, -8.833, 3.333 ; 0, 3.333, -3.667]
```

```
A =
```

```
   -16.0000    5.0000         0
     5.0000   -8.8330    3.3330
         0     3.3330   -3.6670
```

```
>> B = [10; 0; 0]
```

```
B =
```

```
   10
     0
     0
```

```
>> C = [0, 0, 1]
```

```
C =
```

```
     0     0     1
```

```
>> G = ss(A, B, C, 0);
```

```
>> zpk(G)
```

```
          166.65
-----
(s+18.73) (s+8.138) (s+1.633)
```

5) Using state-space methods, find the transfer function from V_{in} to V_2

All that changes is the C matrix

```
>> C = [0, 1, 0];
```

```
>> G = ss(A, B, C, 0);
```

```
>> zpk(G)
```

```
          50 (s+3.667)
-----
(s+18.73) (s+8.138) (s+1.633)
```

Note

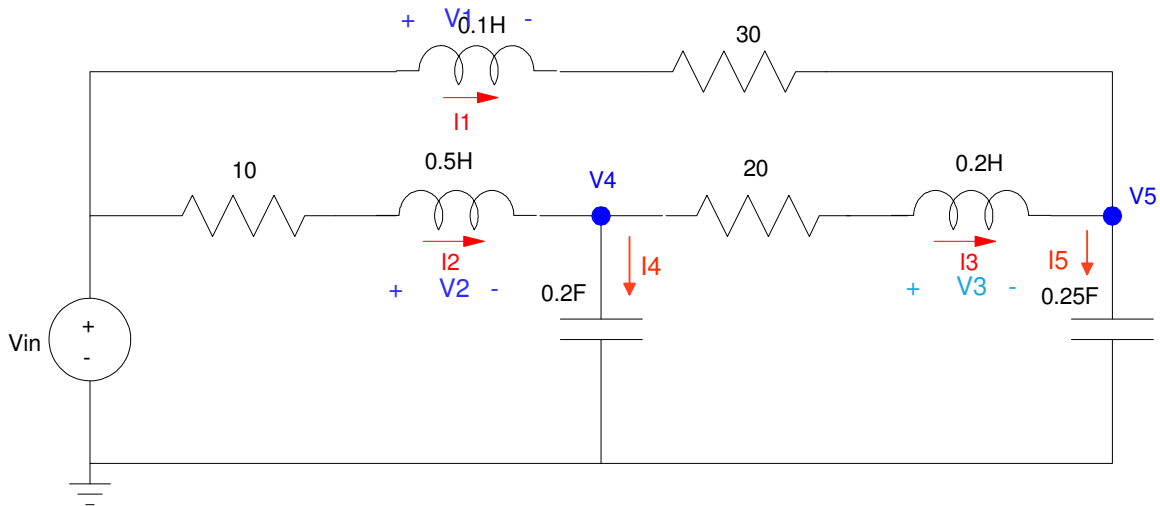
- The poles don't change when you change the output
- The zeros do change

Poles tell you how the energy in the system dissipates. Changing the output doesn't change this.

Zeros tell you how the energy states contribute to the output. Changing the output does change the zeros.

6) Express the dynamics for the following RLC circuit in state-space form.

- Find the transfer function from V_{in} to V_5



The energy in the system is defined by the current in the inductors and the voltage across the capacitors. Let the states be $\{ I_1, I_2, I_3, V_4, V_5 \}$

$$V_1 = 0.1sI_1 = V_{in} - 30I_1 - V_5$$

$$V_2 = 0.5sI_2 = V_{in} - 10I_2 - V_4$$

$$V_3 = 0.2sI_3 = V_4 - 20I_3 - V_5$$

$$I_4 = 0.2sV_4 = I_2 - I_3$$

$$I_5 = 0.25sV_5 = I_1 + I_3$$

Simplifying

$$sI_1 = 10V_{in} - 300I_1 - 10V_5$$

$$sI_2 = 2V_{in} - 20I_2 - 2V_4$$

$$sI_3 = 5V_4 - 100I_3 - 5V_5$$

$$sV_4 = 5I_2 - 5I_3$$

$$sV_5 = 4I_1 + 4I_3$$

Place in matrix (state-space) form

$$\begin{bmatrix} sI_1 \\ sI_2 \\ sI_3 \\ sV_4 \\ sV_5 \end{bmatrix} = \begin{bmatrix} -300 & 0 & 0 & 0 & -10 \\ 0 & -20 & 0 & -2 & 0 \\ 0 & 0 & -100 & 5 & -5 \\ 0 & 5 & -5 & 0 & 0 \\ 4 & 0 & 4 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ V_4 \\ V_5 \end{bmatrix} + \begin{bmatrix} 10 \\ 2 \\ 0 \\ 0 \\ 0 \end{bmatrix} V_{in}$$

Use Matlab to find the transfer function to V5

```
>> A = [-300,0,0,0,-10 ; 0,-20,0,-2,0 ; 0,0,-100,5,-5;0,5,-5,0,0;4,0,4,0,0]

-300    0    0    0   -10
    0   -20    0   -2    0
    0    0  -100    5   -5
    0    5   -5    0    0
    4    0    4    0    0

>> B = [10;2;0;0;0]

10
 2
 0
 0
 0

>> C = [0,0,0,0,1];
>> D = 0;
>> G = ss(A,B,C,D);
>> zpk(G)
```

```
-----
          40 (s+99.88) (s+18.5) (s+1.624)
-----
(s+299.9) (s+99.55) (s+19.48) (s+0.8701) (s+0.2372)
```

7) Assume $V_{in} = 0$. Specify the initial conditions so that the total energy at $t = 0$ is 1.0 Joules and

- The transients decay as slow as possible
- The transients decay as fast as possible

This is an eigenvalue / eigenvector problem

```
>> [M,V] = eig(A)

M (eigenvector)

 0.9999    0.0020   -0.0300    0.0117    0.0001
-0.0000    0.0013   -0.0439   -0.0971   -0.9675
-0.0003    0.9979   -0.0233    0.0646    0.0159
-0.0000    0.0501    0.4338    0.9291    0.2524
-0.0133   -0.0402    0.8991   -0.3506   -0.0033

V (eigenvalues)

-299.8667    0         0         0         0
 0   -99.5479    0         0         0
 0         0   -0.2372    0         0
 0         0    0   -0.8701    0
 0         0    0         0  -19.4782
```

The red eigenvector decays the fastest. Its energy is

$$E = \sum \left(\frac{1}{2}LI^2 + \frac{1}{2}CV^2 \right) =$$


```

>> X0 = M(:,1);
>> Energy = 0.5 * [0.1, 0.5, 0.2, 0.2, 0.25] * X0.^2

Energy =

    0.0500

>> X0 / Energy

I1    19.9929A
I2    -0.0000A
I3    -0.0067A
V4    -0.0001V
V5    -0.2666V

```

The 3rd eigenvector (blue) is the slow mode. For its initial energy to be 1.00 Joule

```

>> X0 = M(:,3);
>> Energy = 0.5 * [0.1, 0.5, 0.2, 0.2, 0.25] * X0.^2

Energy =

    0.1205

>> X0 / Energy

I1    -0.2490A
I2    -0.3645A
I3    -0.1936A
V4    3.6014V
V5    7.4645V

```