Homework #8: ECE 461/661

Meeting Specs, Delays, Unstable Systems. Due Monday, October 25th
20 points per problem

Meeting Design Specs

1) Assume

$$G(s) = \left(\frac{170}{(s+0.47)(s+3.40)(s+9.00)(s+16.77)} \right)$$

Design a compensator, K(s), For the 4th-order model that results in

- No error for a step input
- A 2% settling time of 2 seconds, and
- 20% overshoot for the step response

Translation:

- Make it a type-1 system
  - *no error for a step input*
- Place the closed-loop dominant pole at s = -2 + j4

Let K(s) be of the form

$$K(s) = k\left(\frac{(s+0.47)(s+3.40}{s(s+a)} \right)$$

so that

$$GK = \left(\frac{170k}{s(s+a)(s+9.00)(s+16.77)} \right)$$

Pick 'a' so that s = -2 + j4 is on the root locus

- The angle of GK(s) = 180 degrees

Solving for what we know:

$$\left(\frac{170}{s(s+9.00)(s+16.77)} \right)_{s=-2+j4} = 0.3080\angle -161.463^0$$

For the angles to add up to 180 degrees

$$\angle(s+a) = 18.537^0$$

$$a = \frac{4}{\tan\left(18.537^0\right)} + 2$$

$$a = 13.929$$

meaning

$$K(s) = k\left(\frac{(s+0.47)(s+3.40}{s(s+13.929)} \right)$$

To find 'k', at any point on the root locus, GK = -1

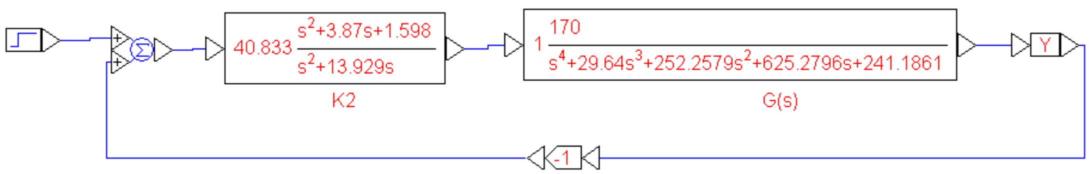
$$GK = \left(\frac{170k}{s(s+9.00)(s+13.929)(s+16.77)}\right)_{s=-2+j4} = 0.024k\angle 180^0$$

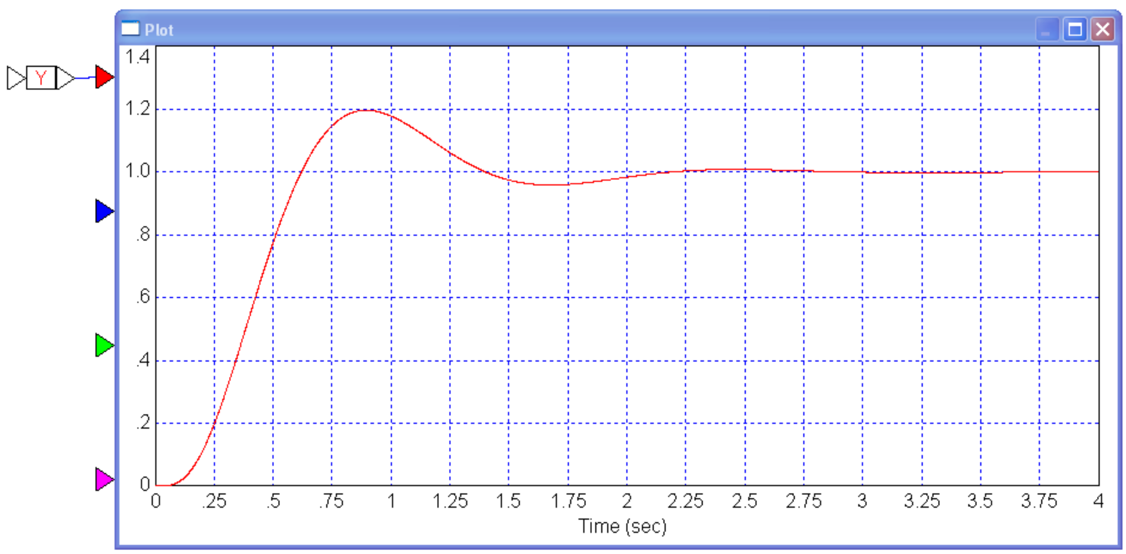
$$k = \frac{1}{0.024} = 40.833$$

meaning

$$\boxed{K(s) = 40.833\left(\frac{(s+0.47)(s+3.40}{s(s+13.929)}\right)}$$

Check your design in Matlab or Simulink or VisSim





Requirements:

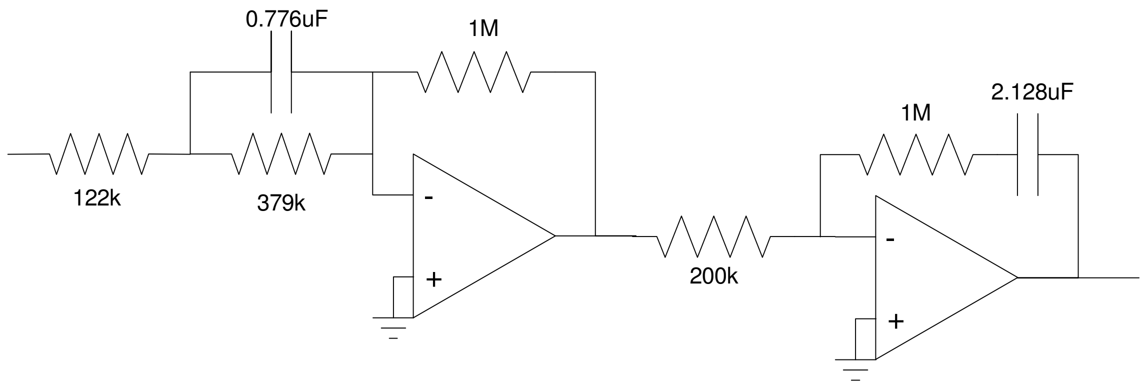
- No error for a step input
  - *check*
- 2% settling time of 2 seconds
  - *check*
- 20% overshoot
  - *check*

Give an op-amp circuit to implement K(s)

$$K(s) = 40.833\left(\frac{(s+0.47)(s+3.40}{s(s+13.929)}\right)$$

Rewrite as

$$K(s) = 8.167\left(\frac{s+3.40}{s+13.929}\right) \cdot 5\left(\frac{s+0.47}{s}\right)$$



Systems with Delays

2) Assume a 100ms delay is added to the system

$$G(s) = \left(\frac{170}{(s+0.47)(s+3.40)(s+9.00)(s+16.77)}\right)e^{-0.1s}$$

Design a compensator, K(s), For the 4th-order model that results in

- No error for a step input
- A 2% settling time of 2 seconds, and
- 20% overshoot for the step response

Same as before. Let K(s) be of the form

$$K(s) = k\left(\frac{(s+0.47)(s+3.40)}{s(s+a)}\right)$$

resulting in

$$GK = \left(\frac{170k}{s(s+a)(s+9.00)(s+16.77)}\right)e^{-0.1s}$$

Evaluate what we know:

$$\left(\left(\frac{170}{s(s+9.00)(s+16.77)}\right)e^{-0.1s}\right)_{s=-2+j4} = 0.3760\angle+175.618^0$$

There's too much phase shift.

Cancel another pole...

$$K(s) = k\left(\frac{(s+0.47)(s+3.40)(s+9)}{s(s+a)^2}\right)$$

$$GK = \left(\frac{170k}{s(s+a)^2(s+16.77)}\right)e^{-0.1s}$$

Evaluate what we know:

$$\left(\left(\frac{170}{s(s+16.77)}\right)e^{-0.1s}\right)_{s=-2+j4} = 3.043\angle-154.637^0$$

meaning

$$\angle(s+a)^2 = 25.363^0$$

$$\angle(s+a) = 12.682^0$$

$$a = \frac{4}{\tan\left(12.682^0\right)} + 2 = 19.776$$

and

$$K(s) = k\left(\frac{(s+0.47)(s+3.40)(s+9)}{s(s+19.776)^2}\right)$$

At any point on the root locus, GK = -1

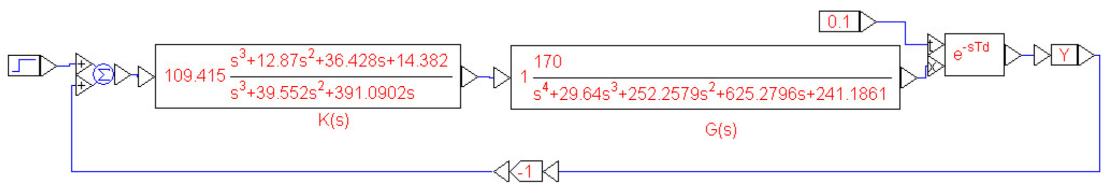
$$GK = \left(\left(\frac{170k}{s(s+19.776)^2(s+16.77)} \right) e^{-0.1s} \right)_{s=-2+j4} = 0.009k\angle 180^0$$

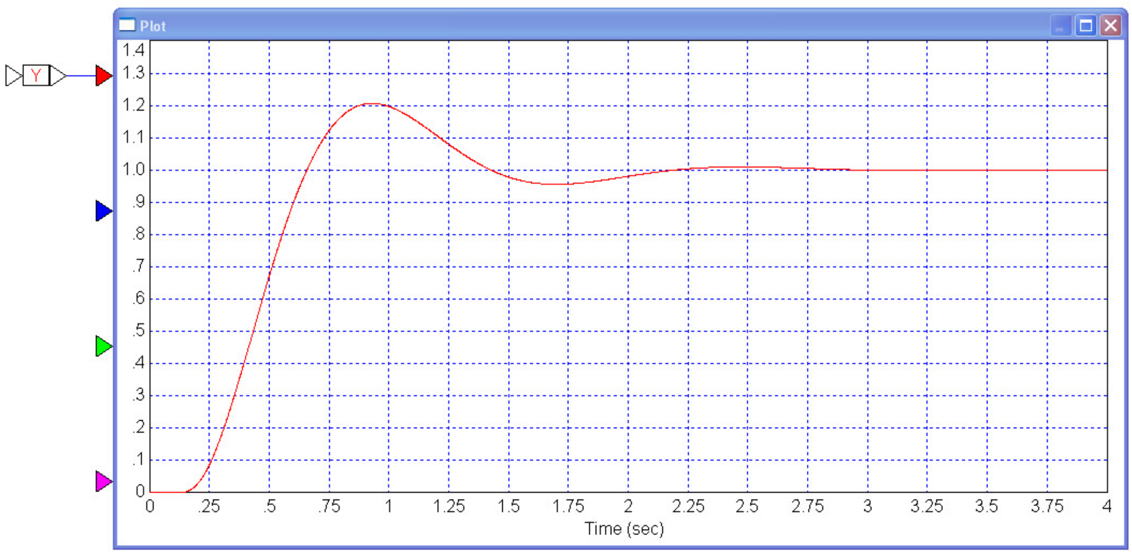
$$k = \frac{1}{0.009} = 109.415$$

so

$$\boxed{K(s) = 109.415 \left(\frac{(s+0.47)(s+3.40)(s+9)}{s(s+19.776)^2} \right)}$$

Check your design in Matlab or Simulink or VisSim



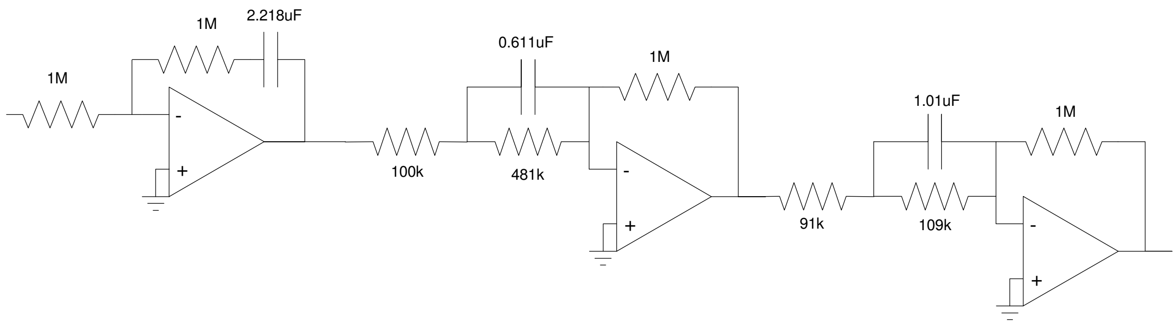


Give an op-amp circuit to implement K(s)

$$K(s) = 109.415\left(\frac{(s+0.47)(s+3.40)(s+9)}{s(s+19.776)^2}\right)$$

Rewrite as

$$K(s) = \left(\frac{s+0.47}{s}\right) \cdot 10\left(\frac{s+3.40}{s+19.776}\right) \cdot 10.94\left(\frac{s+9}{s+19.776}\right)$$



Unstable Systems

3) Assume the slow pole was unstable

$$G(s) = \left(\frac{170}{(s-0.47)(s+3.40)(s+9.00)(s+16.77)} \right)$$

Design a compensator, K(s), For the 4th-order model that results in

- No error for a step input
- A 2% settling time of 2 seconds, and
- 20% overshoot for the step response

Check your design in Matlab or Simulink or VisSim

Step 1: Just stabilize the system. Don't worry about the requirements

Let

$$K_1(s) = k\left(\frac{s+3.40}{s+10} \right)$$

Place the closed-loop pole at s = -1

```
>> G = zpk([],[0.47,-3.40,-9,-16.77],170)

              170
--------------------------------
(s-0.47) (s+3.4) (s+9) (s+16.77)

>> K1 = zpk(-3.40,-10,1)

(s+3.4)
-------
(s+10)

>> GK1 = minreal(G*K1)

              170
-------------------------------
(s-0.47) (s+9) (s+10) (s+16.77)

>> evalfr(GK1,-1)

ans =   -0.1019

>> k1 = 1/abs(ans)

k1 =    9.8182

>> K1 = zpk(-3.40,-10,9.8182)

9.8182 (s+3.4)
--------------
    (s+10)


>> G2 = minreal(G*K1 / (1 + G*K1))

               1669.094
--------------------------------------
(s+1) (s+4.112) (s^2 + 30.19s + 233.4)
```

Step 2: Now that the system is stable, meet the design specs

$$K_2(s) = k\left(\frac{(s+1)(s+4.112)}{s(s+a)}\right)$$

$$G_2K_2 = \left(\frac{1669.094k}{s(s+a)\left(s^2+30.19s+233.4\right)}\right)$$

Evaluate what we know

$$\left(\frac{1669.094}{s\left(s^2+30.19s+233.4\right)}\right)_{s=-2+j4} = 1.9429\angle -149.6132^0$$

meaning

$$\angle(s+a) = 30.3868^0$$

$$a = \frac{4}{\tan\left(30.3868^0\right)} + 2 = 8.821$$

and

$$K_2(s) = k\left(\frac{(s+1)(s+4.112)}{s(s+8.821)}\right)$$

At any point on the root locus, GK = -1

$$G_2K_2 = \left(\frac{1669.094k}{s(s+8.821)\left(s^2+30.19s+233.4\right)}\right)_{s=-2+j4} = 0.2457k\angle 180^0$$

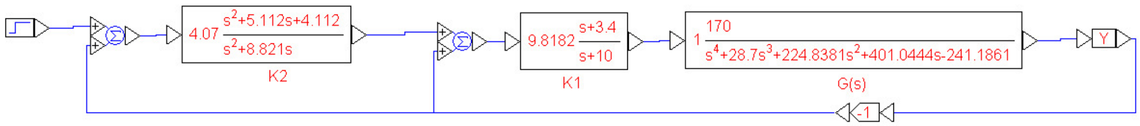
$$k = \frac{1}{0.2457} = 4.070$$

$$K_2(s) = 4.070\left(\frac{(s+1)(s+4.112)}{s(s+8.821)}\right)$$

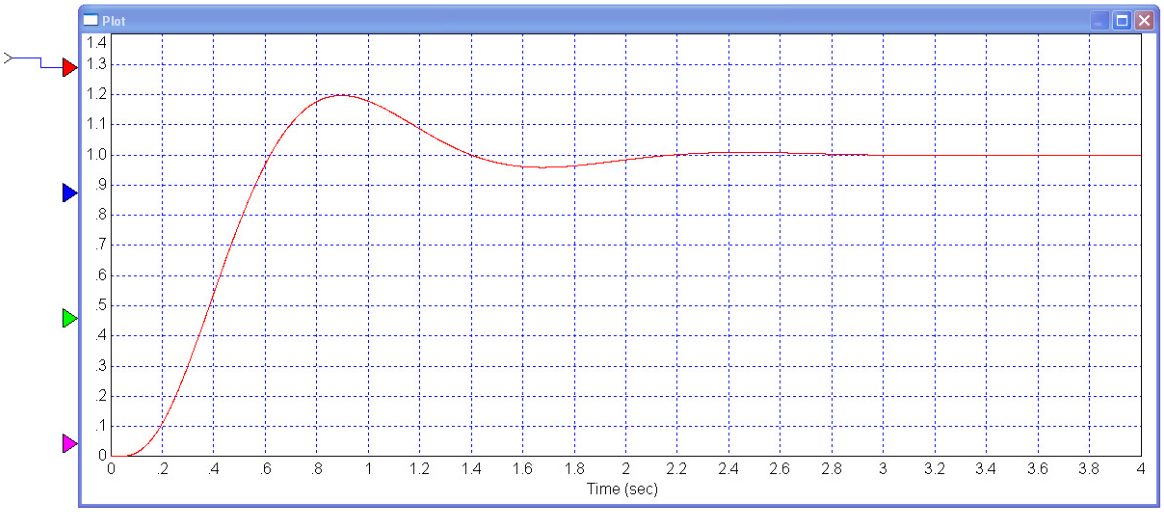
$$K_1(s) = 9.8182\left(\frac{s+3.4}{s+10}\right)$$

>>

Checking the resulting controller in VisSim



4.07 s^2+5.112s+4.112 / s^2+8.821s
K2
9.8182 s+3.4 / s+10
K1
1 170 / s^4+28.7s^3+224.8381s^2+401.0444s-241.1861
G(s)
Y
-1



Plot
1.4
1.3
1.2
1.1
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
0 .2 .4 .6 .8 1 1.2 1.4 1.6 1.8 2 2.2 2.4 2.6 2.8 3 3.2 3.4 3.6 3.8 4
Time (sec)