Homework #10: ECE 461/661

Digital PID Control. Due Monday, November 15th

PID Control

Assume T = 0.5 seconds:

$$G(s) = \left(\frac{170}{(s+0.47)(s+3.40)(s+9.00)(s+16.77)}\right)$$

1) Design a digital I controller

$$K(s) = k\left(\frac{z}{z-1}\right)$$

that results in 20% overshoot in the step response.

Simulate the step response of the closed-loop system (VisSim or Simulink preferred with K(z)*G(s))

Use the alternate method: Plant + Zero-Order Hold + K(z) = -1

$$\left(\left(\frac{170}{(s+0.47)(s+3.40)(s+9.00)(s+16.77)}\right) \cdot e^{-0.25s} \cdot k\left(\frac{z}{z-1}\right)\right)_{s=(-1+j2)a} = 1\angle 180^0$$

Search along the 20% overshoot damping line until the angle is 180 degrees

(time passes)

$$s = -0.1879 + j0.3758$$

$$z = 0.8943 + j0.1700$$

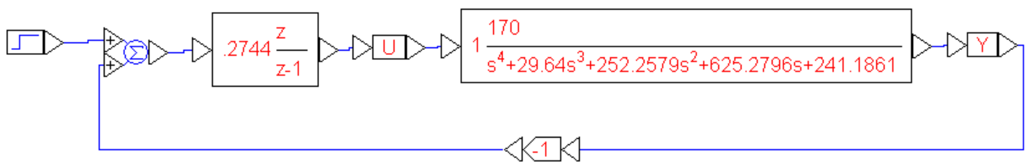
$$\left(\left(\frac{170}{(s+0.47)(s+3.40)(s+9.00)(s+16.77)}\right) \cdot e^{-0.25s} \cdot k\left(\frac{z}{z-1}\right)\right)_{s} = 3.6439k\angle 180^0$$

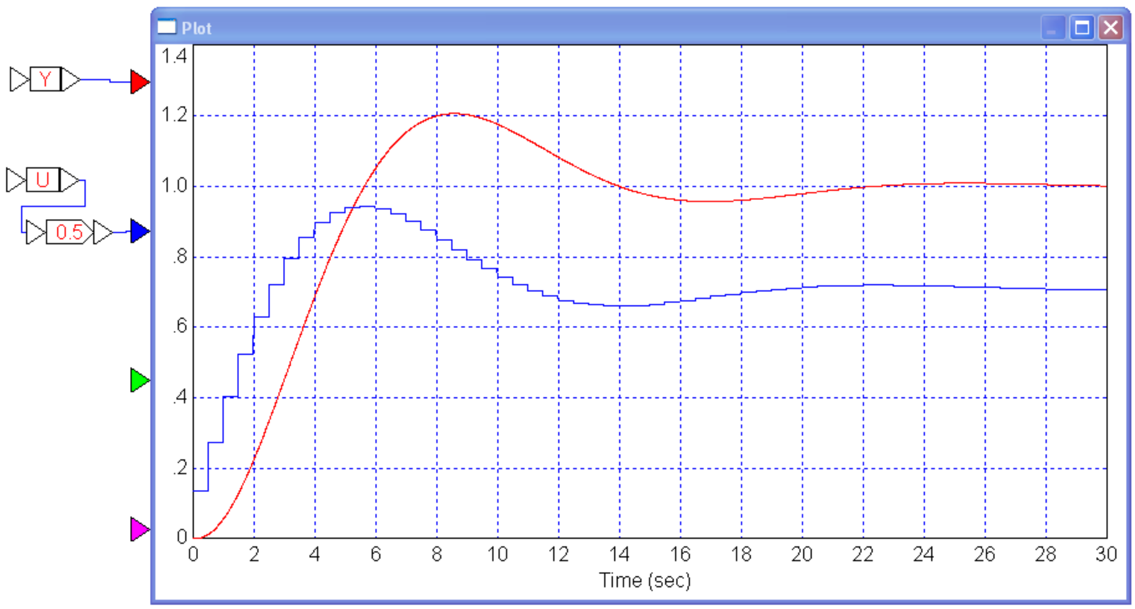
meaning

$$k = \frac{1}{3.6439} = 0.2744$$

and

$$K(z) = 0.2744\left(\frac{z}{z-1}\right)$$


+
+
.2744 z / z-1
U
1 170 / s4+29.64s3+252.2579s2+625.2796s+241.1861
Y
-1


Y
U
0.5
Plot
1.4
1.2
1.0
.8
.6
.4
.2
0
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30
Time (sec)

2) Assume T = 0.5 seconds and

$$G(s) = \left(\frac{170}{(s+0.47)(s+3.40)(s+9.00)(s+16.77)}\right)$$

Design a digital PI controller

$$K(s) = k\left(\frac{z-a}{z-1}\right)$$

that results in 20% overshoot in the step response.

Simulate the step response of the closed-loop system (VisSim or Simulink preferred with K(z)*G(s))

Pick 'a' to cancel the pole at s = -0.47

$$s = -0.47$$

$$z = e^{sT} = e^{(-0.47)(0.5)} = 0.7906$$

Again, use the alternate approach

$$\left(\left(\frac{170}{(s+0.47)(s+3.40)(s+9.00)(s+16.77)}\right) \cdot e^{-0.25s} \cdot k\left(\frac{z-0.7906}{z-1}\right)\right)_{s=(-1+j2)a} = 1\angle 180^0$$

Search along the damping line until the angle is 180 degrees

$$s = -0.7242 + j1.4483$$

$$z = e^{sT} = 0.5215 + j0.4613$$

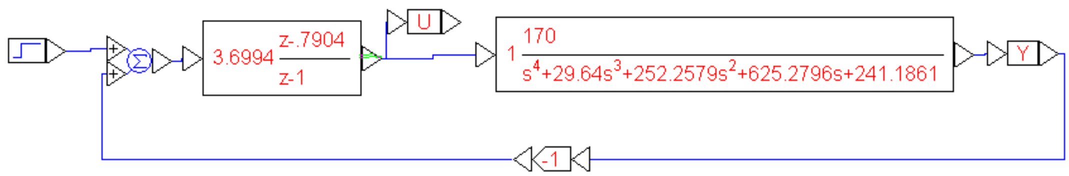
To find k, make the gain at this point equal to 1.0000

$$\left(\left(\frac{170}{(s+0.47)(s+3.40)(s+9.00)(s+16.77)}\right) \cdot e^{-0.25s} \cdot k\left(\frac{z-0.7906}{z-1}\right)\right)_{s} = 0.2703k\angle 180^0$$

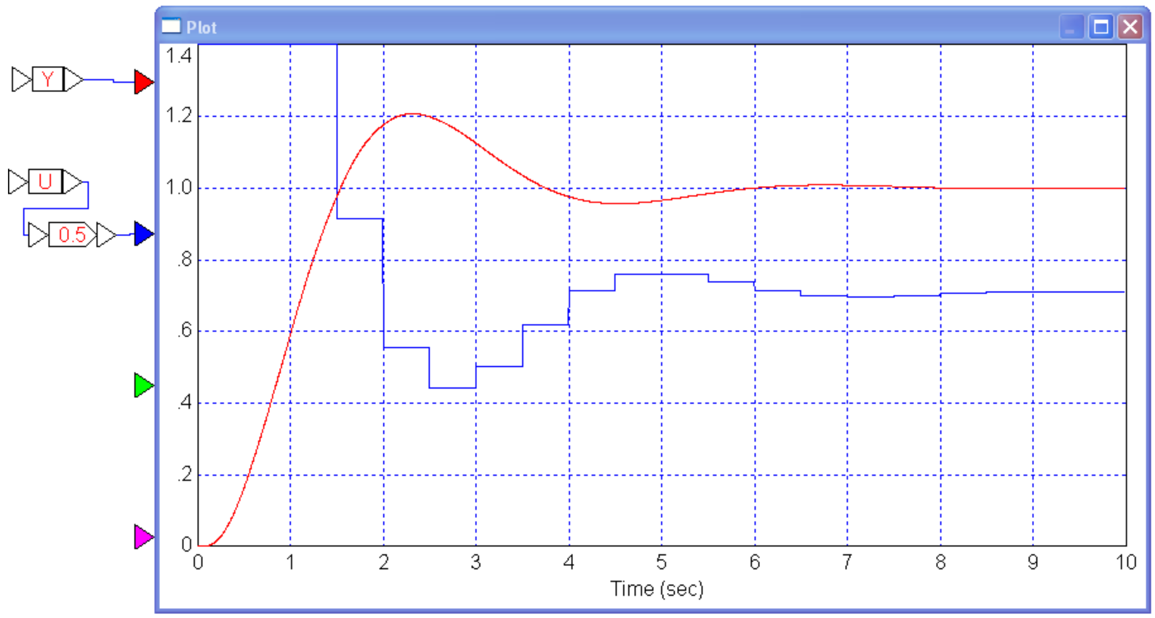
$$k = \frac{1}{0.2703} = 3.6994$$

and

$$K(z) = 3.6994\left(\frac{z-0.7906}{z-1}\right)$$



U
3.6994
z-.7904
z-1
1
170
s4+29.64s3+252.2579s2+625.2796s+241.1861
Y
-1



Plot
Y
U
0.5
1.4
1.2
1.0
.8
.6
.4
.2
0
0
1
2
3
4
5
6
7
8
9
10
Time (sec)

3) Assume T = 0.5 seconds and

$$G(s) = \left(\frac{170}{(s+0.47)(s+3.40)(s+9.00)(s+16.77)}\right)$$

Design a digital PID controller

$$K(s) = k\left(\frac{(z-a)(z-b)}{z(z-1)}\right)$$

that results in 20% overshoot in the step response.

Simulate the step response of the closed-loop system (VisSim or Simulink preferred with K(z)*G(s))

Pick the two zeros to cancel the two slowest poles

$$s = -0.47 \qquad z = e^{sT} = 0.7906$$

$$s = -3.40 \qquad z = e^{sT} = 0.1827$$

meaning K(z) is of the form

$$K(z) = k\left(\frac{(z-0.7906)(z-0.1827)}{z(z-1)}\right)$$

Again, use the approach where you search along the damping line until the angle is 180 degrees

$$\left(\left(\frac{170}{(s+0.47)(s+3.40)(s+9.00)(s+16.77)}\right) \cdot e^{-0.25s} \cdot k\left(\frac{(z-0.7906)(z-0.1827}{z(z-1)}\right)\right)_{s=(-1+j2)a} = 1\angle 180^0$$

Searching results in

$$s = -0.8892 + j1.7783$$

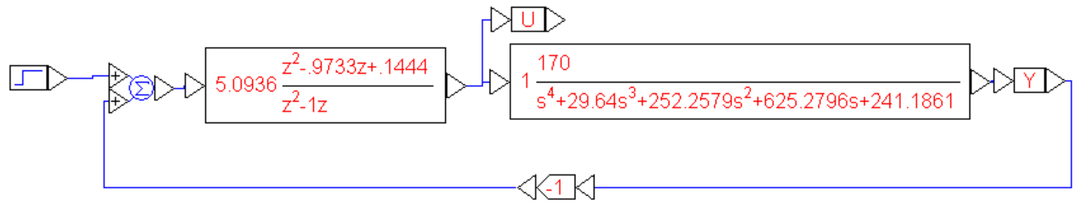
$$z = e^{sT} = 0.4039 + j0.4978$$

To make the gain equal to -1.0000 at this point

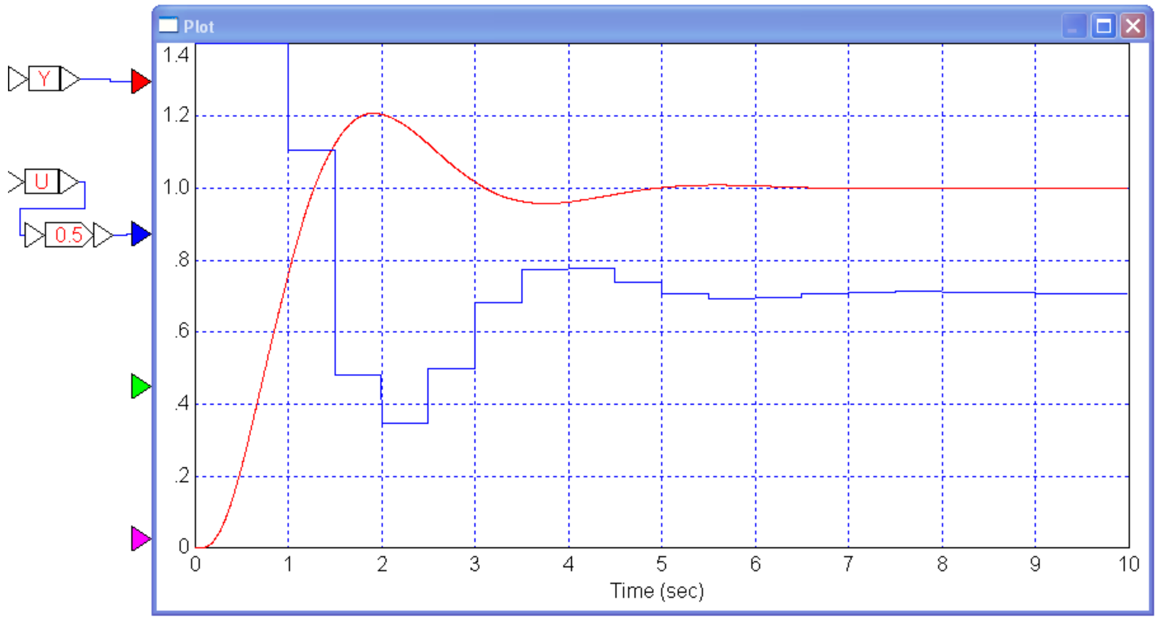
$$k = 5.0936$$

and

$$K(z) = 5.0936\left(\frac{(z-0.7906)(z-0.1827)}{z(z-1)}\right)$$



5.0936
z²-.9733z+.1444
z²-1z
U
1
170
s⁴+29.64s³+252.2579s²+625.2796s+241.1861
Y
-1



Plot
Y
U
0.5
1.4
1.2
1.0
.8
.6
.4
.2
0
0 1 2 3 4 5 6 7 8 9 10
Time (sec)

Meeting Design Specs

4) Assume a sampling rate of T = 0.5 seconds and

$$G(s) = \left(\frac{170}{(s+0.47)(s+3.40)(s+9.00)(s+16.77)}\right)$$

Design a digital controller that results in

- No error for a step input
- 20% overshoot for the step response, and
- A 2% settling time of 10 seconds

Simulate the step response of the closed-loop system (VisSim or Simulink preferred with K(z)*G(s))

Translation:

- Make it a type-1 system

Place the closed-loop dominant pole at

- s = -0.4 + j0.8
- z = 0.7541 + j0.3188

Assume K(z) is of the form

$$K(z) = k\left(\frac{(z-0.7906)(z-0.1827)}{(z-1)(z-a)}\right)$$

Pick 'a' so that s = -0.4 + j0.8 is on the root locus (the phase is 180 degrees)

$$\left(\left(\frac{170}{(s+0.47)(s+3.40)(s+9.00)(s+16.77)}\right) \cdot e^{-0.25s} \cdot k\left(\frac{(z-0.7906)(z-0.1827)}{(z-1)(z-a)}\right)\right)_{s=-0.4+j0.8} = 1\angle 180^0$$

Analyze what we know:

$$\left(\left(\frac{170}{(s+0.47)(s+3.40)(s+9.00)(s+16.77)}\right) \cdot e^{-0.25s} \cdot \left(\frac{(z-0.7906)(z-0.1827)}{(z-1)}\right)\right)_{s=-0.4+j0.8} = 0.2776\angle -121.45^0$$

meaing, to make the angle 180 degrees

$$\angle(z-a) = 58.5475^0$$

$$a = 0.7541 - \left(\frac{0.3188}{\tan\left(58.5475^0\right)}\right) = 0.5591$$

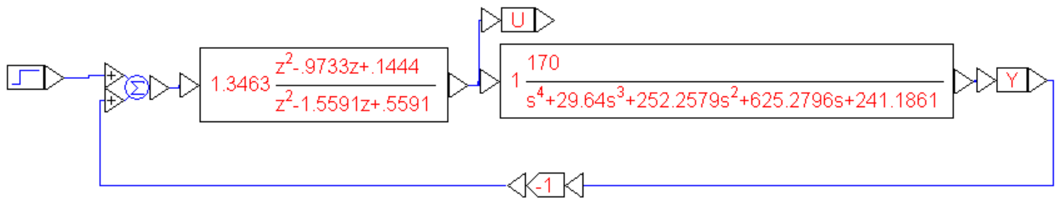
Find k so that the gain is one

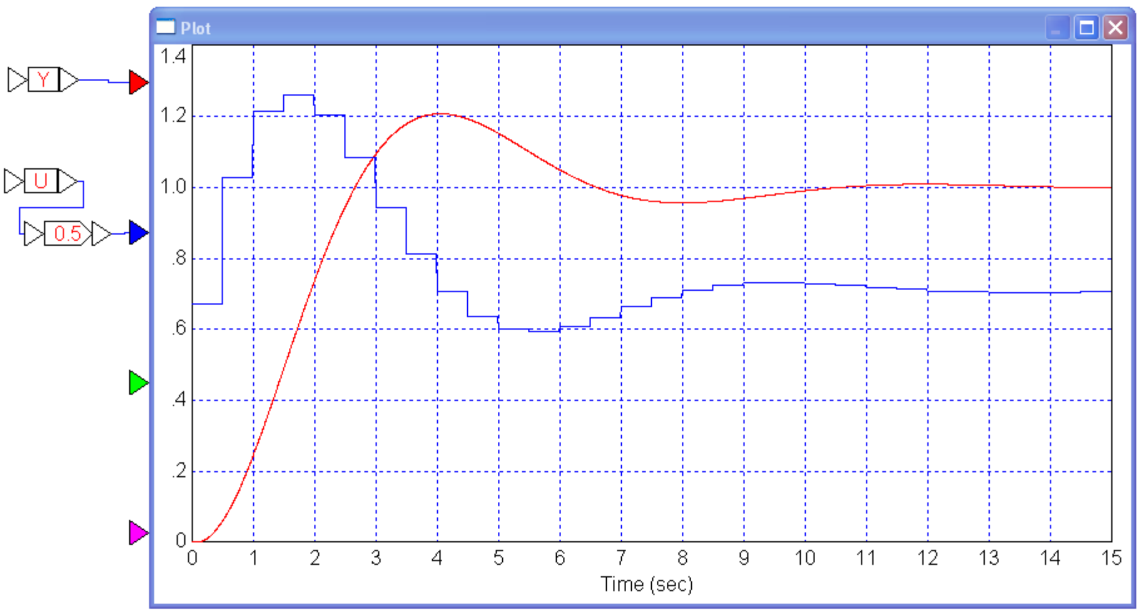
$$\left(\left(\frac{170}{(s+0.47)(s+3.40)(s+9.00)(s+16.77)}\right) \cdot e^{-0.25s} \cdot \left(\frac{(z-0.7906)(z-0.1827)}{(z-1)(z-0.5591)}\right)\right)_{s=-0.4+j0.8} = 0.7828\angle 180^0$$

$$k = \frac{1}{0.7828} = 1.3463$$

and

$$K(z) = 1.3463\left(\frac{(z-0.7906)(z-0.1827}{(z-1)(z-0.5591)}\right)$$





5) Assume

$$G(s) = \left(\frac{170}{(s+0.47)(s+3.40)(s+9.00)(s+16.77)} \right)$$

Design a digital controller with T = 0.1 second that results in

- No error for a step input
- 20% overshoot for the step response, and
- A 2% settling time of 10 seconds

Simulate the step response of the closed-loop system (VisSim or Simulink preferred with K(z)*G(s))

Note: Changing the sampling rate is a big deal: it means a complete redesign of K(z)

With T = 0.1

- s = -0.4 + j0.8
- z = 0.9577+j0.0768

Same procedure but with T = 0.1

$$K(z) = k\left(\frac{(z-0.9541)(z-0.7113}{(z-1)(z-a)} \right)$$

Analyze what we know:

$$\left(\left(\frac{170}{(s+0.47)(s+3.40)(s+9.00)(s+16.77)} \right) \cdot e^{-0.05s} \cdot \left(\frac{(z-0.9541)(z-0.7113}{(z-1)} \right) \right)_{s=-0.4+j0.8} = 0.1112\angle -124.566^0$$

meaning

$$\angle(z-a) = 55.4332^0$$

$$a = 0.9577 - \left(\frac{0.0768}{\tan\left(55.4332^0\right)} \right) = 0.9048$$

Analyzing what we now know

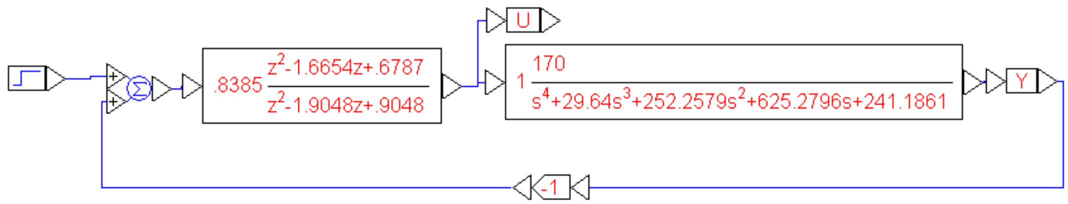
$$\left(\left(\frac{170}{(s+0.47)(s+3.40)(s+9.00)(s+16.77)} \right) \cdot e^{-0.05s} \cdot \left(\frac{(z-0.9541)(z-0.7113}{(z-1)(z-0.9048)} \right) \right)_{s=-0.4+j0.8} = 1.1927\angle 180^0$$

so

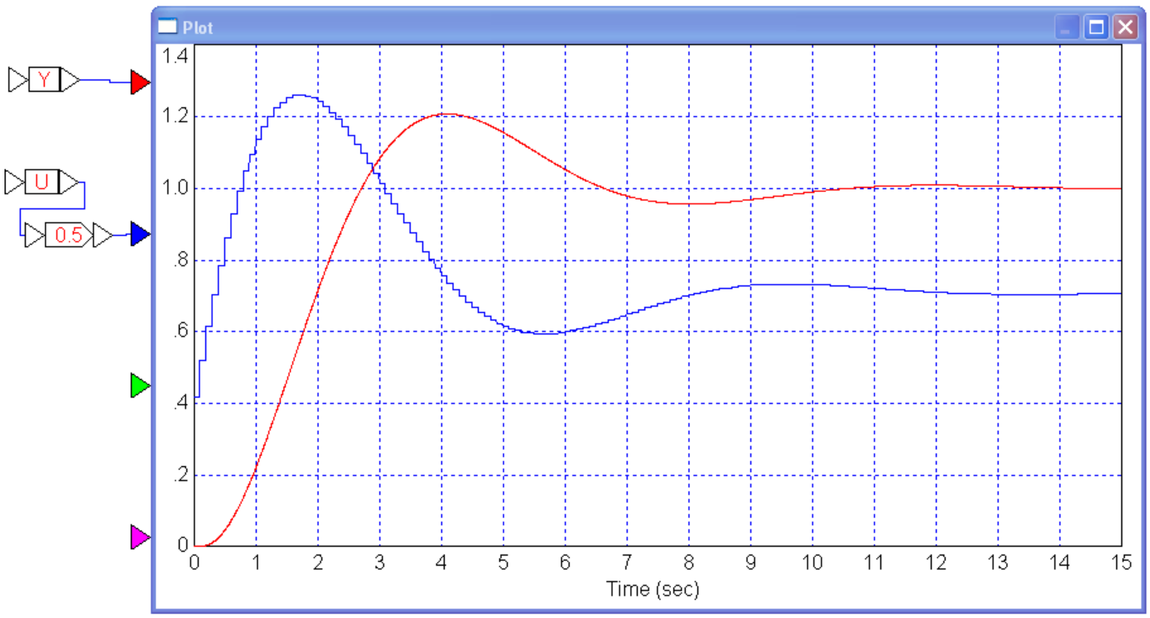
$$k = \frac{1}{1.1927} = 0.8385$$

and

$$K(z) = 0.8385\left(\frac{(z-0.9541)(z-0.7113)}{(z-1)(z-0.9048)} \right)$$



U
.8385
z^2-1.6654z+.6787
z^2-1.9048z+.9048
1
170
s^4+29.64s^3+252.2579s^2+625.2796s+241.1861
Y
-1



Plot
Y
U
0.5
1.4
1.2
1.0
.8
.6
.4
.2
0
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
Time (sec)