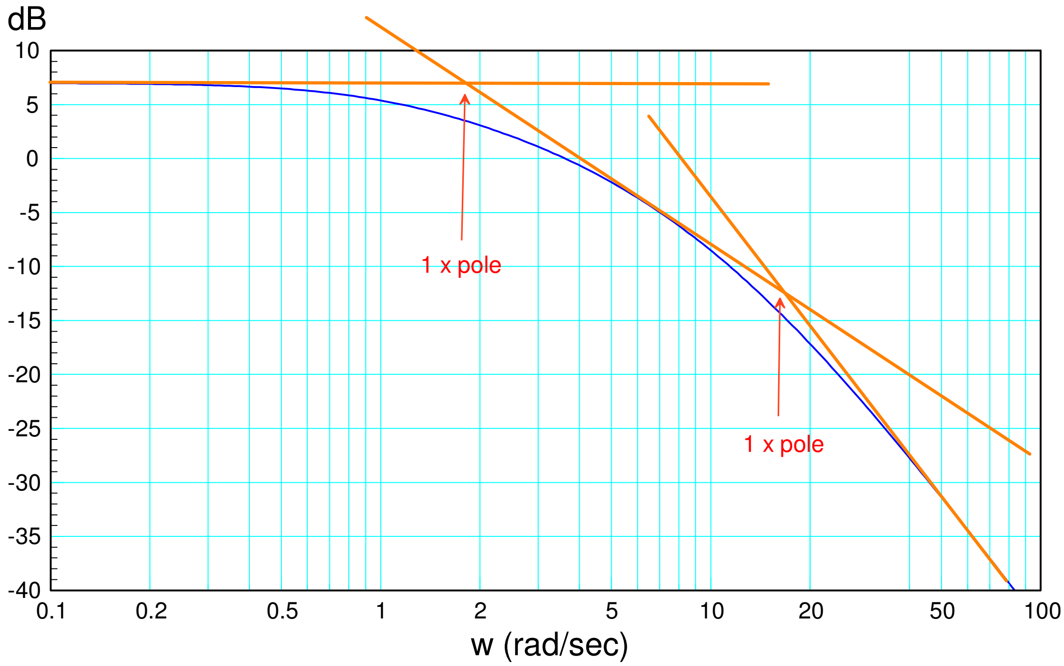
Homework #11: ECE 461/661

Bode Plots. Nichols charts and gain & lead compensation. Due Monday, November 23rd

Bode Plots

1) Determine the system, G(s), with the following gain vs. frequency



Step 1: Draw in the asymptotes at multiples of 20dB / decade (shown in orange)

Step 2: Each corner is a pole (gain drops) or a zero (gain increases). The number of poles is the number of 20dB/decade changes

- pole at 1.8 rad/sec
- pole at 17 rad/sec

$$G(s) \approx \left(\frac{k}{(s+1.8)(s+17)}\right)$$

Step 3: Determine k by matching the gain at a frequency

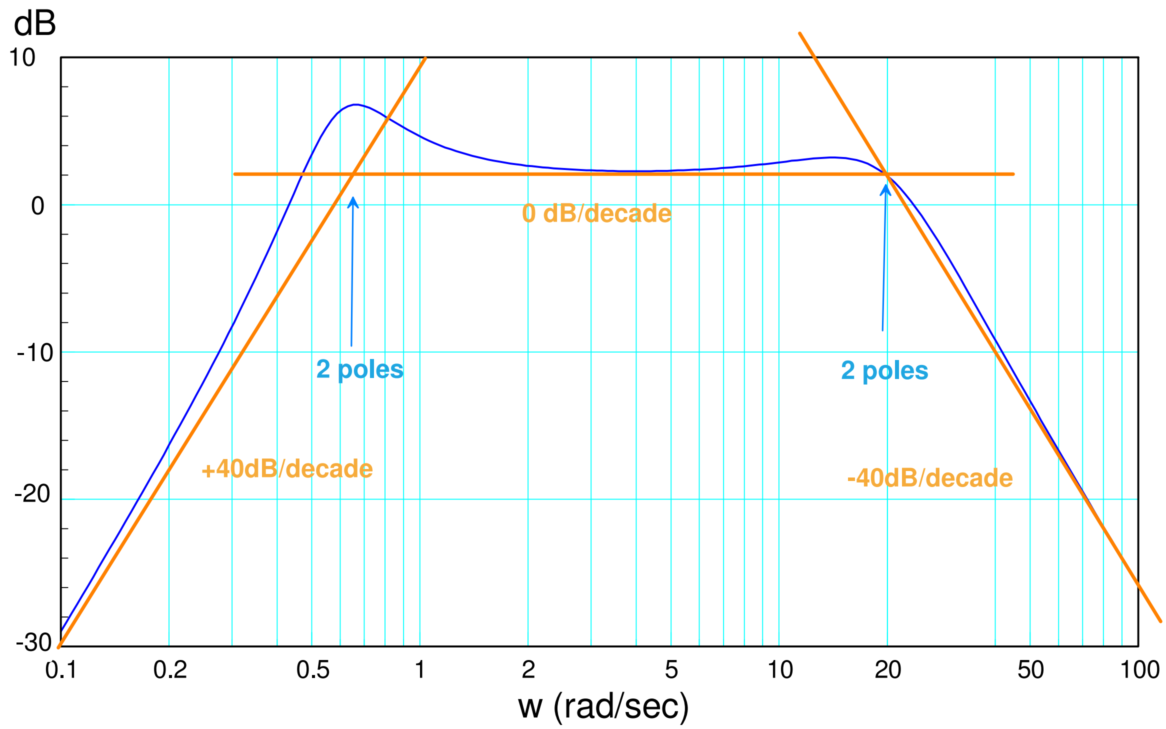
At 0.1 rad/sec, the gain should be +7dB = 2.239

$$\left(\frac{k}{(s+1.8)(s+17)}\right)_{s=j0.1} = 2.239$$

$$k = 68.50$$

$$G(s) \approx \left(\frac{68.50}{(s+1.8)(s+17)}\right)$$

2) Determine the system, G(s), with the following gain vs. frequency



Step 1: Draw in the asymptotes (shown in orange)

Step 2: Determine the poles and zeros (magnitude = corner frequency)

- 2 zeros left of 0.1 (assume at s = 0)
- 2 poles at 0.63 rad/sec
- 2 poles at 20 rad/sec

Step 3: Determine the angle from the gain at the corner

pole at 0.63 rad/sec

gain at corner = +4dB above the corner = 1.585

$$\frac{1}{2\zeta} = 1.585$$

$$\zeta = 0.315 \qquad (71.6^0)$$

pole at 20 rad/sec

gain at corner = +0dB = 1

$$\frac{1}{2\zeta} = 1.00$$

$$\zeta = 0.50 \qquad (60^0)$$

Add a gain, k, so that at 5 rad/sec, the gain is +2dB

$$G(s) = \left(\frac{503 \cdot s^2}{\left(s + 0.63\angle \pm 71.6^0\right)\left(s + 20\angle \pm 60^0\right)} \right)$$

Nichols Charts

3) The gain vs. frequency of a system is measured

w (rad/sec)	2	3	4	5	6	10
Gain (dB)	3.29	-0.97	-4.36	-7.25	-9.81	-17.56
Phase (deg)	-117.51	-129.49	-139.97	-149.04	-156.89	-180

Using this data

- Transfer it to a Nichols chart
- Determine the maximum gain that results in a stable system **ans = k = +17.56 dB**
- Determine the gain, k, that results in a maximum closed-loop gain of Mm = 1.5 **ans: k = 1.70**

In Matlab:

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>> GdB = [3.29,-0.97,-4.36,-7.25,-9.81,-17.56]'
>> Pdeg = [-117.51,-129.49,-139.97,-149.04,-156.89,-180]'

>>G = 10.^(GdB/20) .* exp(j*Pdeg*pi/180)

>> Nichols2(G,1.5);
>> Nichols2(G*[1,1.5],1.5);
>> Nichols2(G*[1,1.7],1.5);
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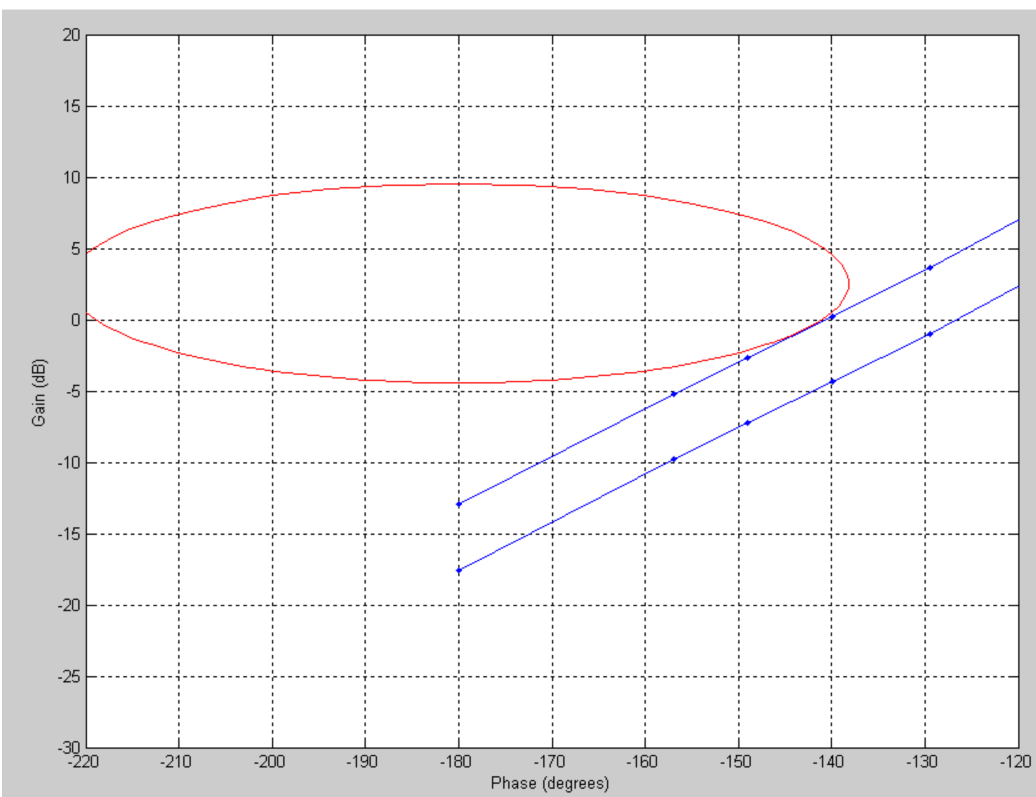


Gain and Lead Compensation

Problem 4 & 5) Assume

$$G(s) = \left(\frac{170}{(s+0.47)(s+3.40)(s+9.00)(s+16.77)}\right)$$

4) Design a gain compensator that results in a 50 degree phase margin.

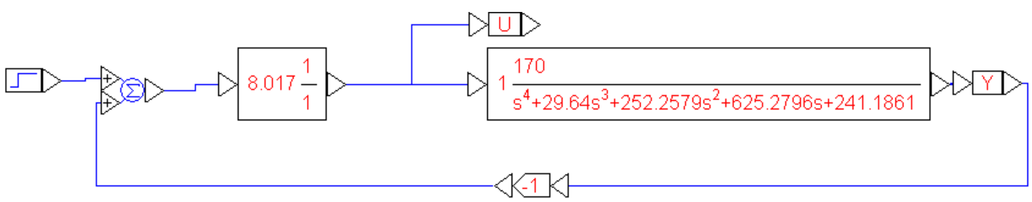
- Check the resulting step response in Matlab

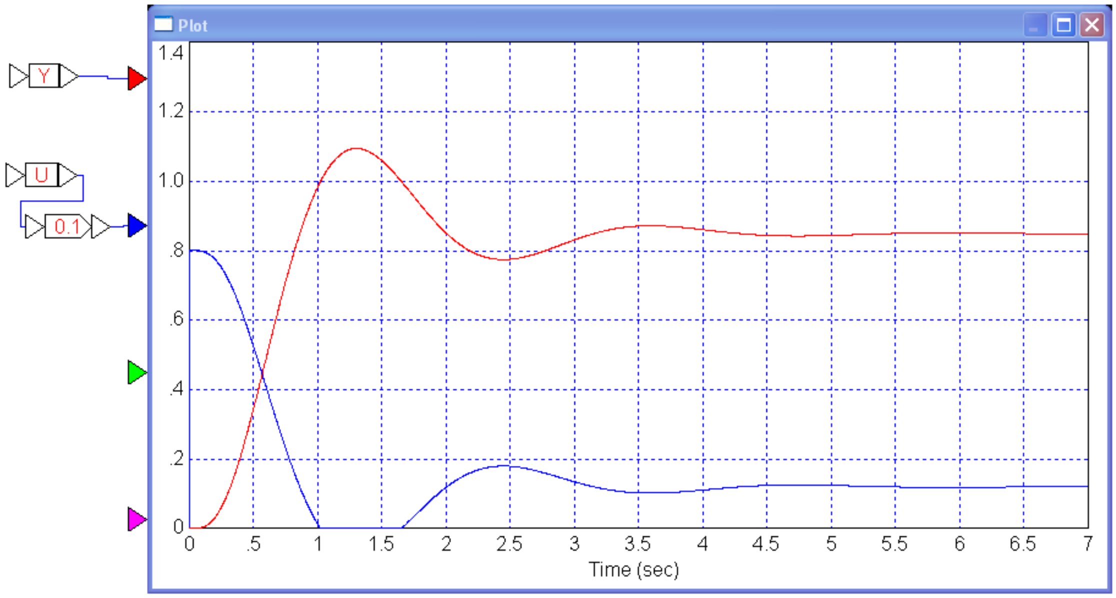
Determine the frequency which has a 50 degree phase margin (-130 degrees of 50 degrees from unstable)

$$G(j2.124) = 0.125\angle -130^0$$

Pick 'k' to make the gain equal to 1.00 at this frequency

$$k = \frac{1}{0.125} = 8.017$$





5) Design a lead compensator that results in a 50 degree phase margin.

- Check the resulting step response in Matlab

Let

$$K(s) = k\left(\frac{s+3.40}{s+34}\right)$$

$$GK = \left(\frac{170k}{(s+0.47)(s+9.00)(s+16.77)(s+34)}\right)$$

Determine the frequency where the angle of GK is -130 degrees (for a 50 degree phase margin)

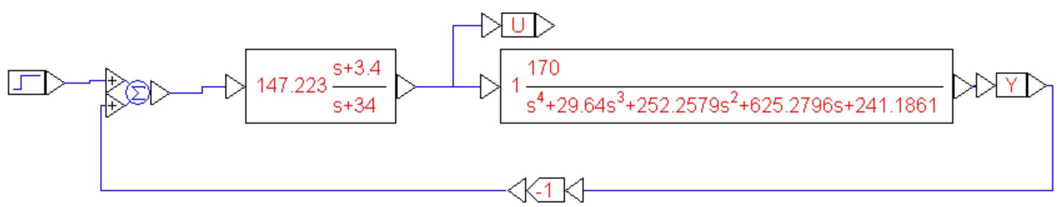
$$GK(j4.223) = 0.00679\angle -130^0$$

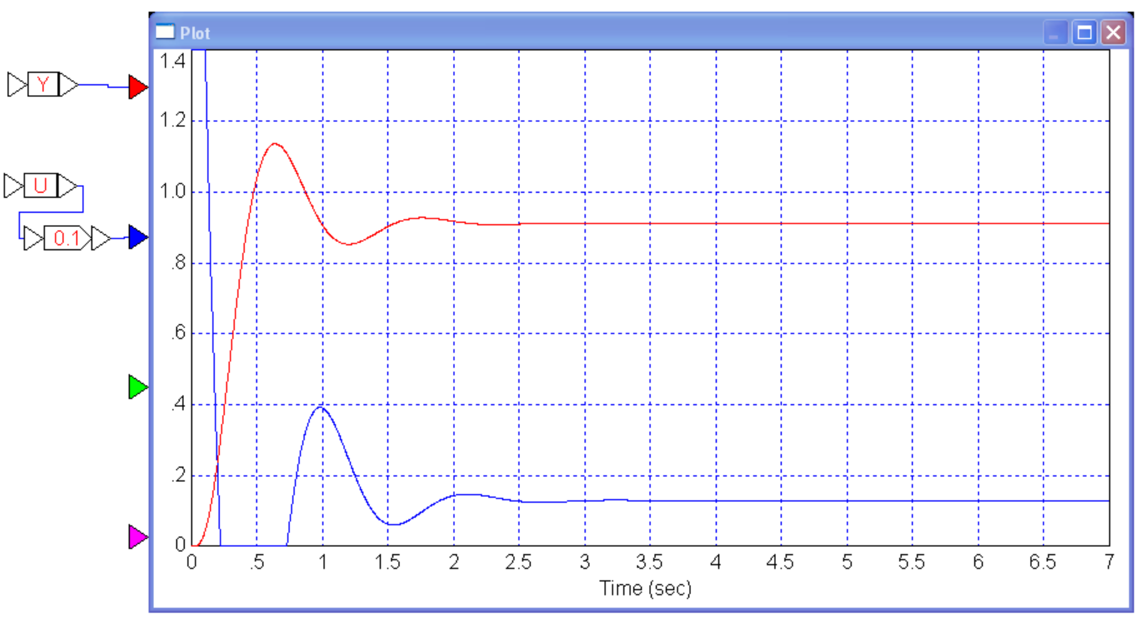
Pick 'k' to make the gain one at this frequency

$$k = \frac{1}{0.00679} = 147.223$$

and

$$K(s) = 147.223\left(\frac{s+3.40}{s+34}\right)$$





Problem 6 & 7) Assume a 500ms delay is added

$$G(s) = \left(\frac{170}{(s+0.47)(s+3.40)(s+9.00)(s+16.77)} \right) e^{-0.5s}$$

6) Design a gain compensator that results in a 50 degree phase margin.

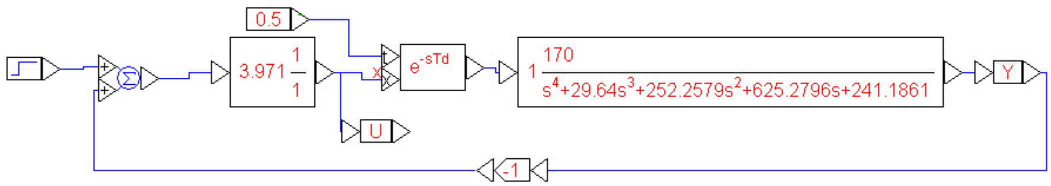
- Check the resulting step response in Matlab

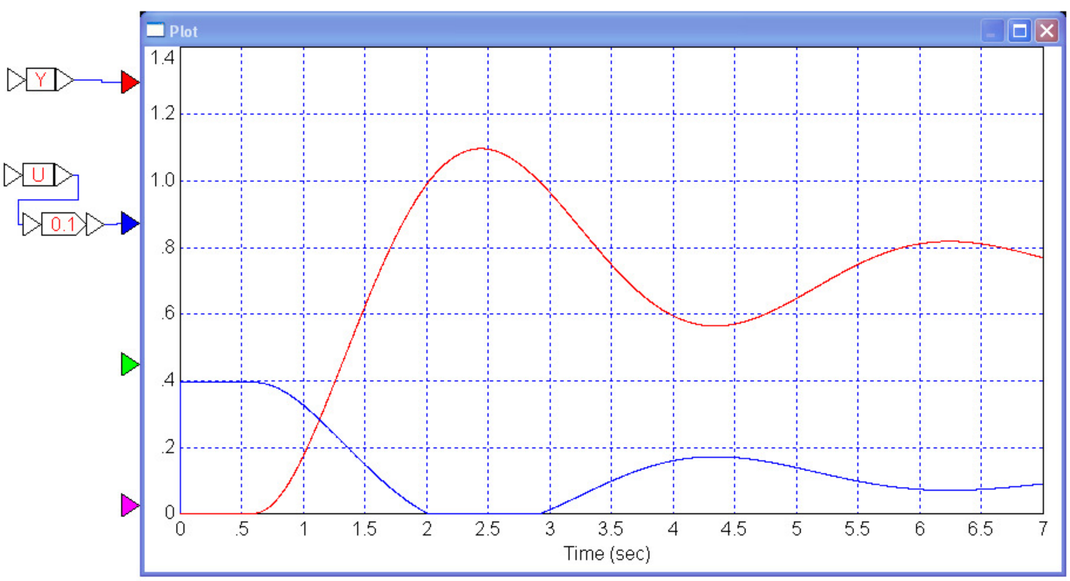
Find the frequency where the phase of G(s) is -130 degrees

$$G(j1.141) = 0.252\angle -130^0$$

Pick 'k' to make the gain 1.00 at this frequency

$$k = \frac{1}{0.252} = 3.9710$$





7) Design a lead compensator that results in a 50 degree phase margin.

- Check the resulting step response in Matlab

Let

$$K(s) = k\left(\frac{s+3.4}{s+34}\right)$$

$$GK = \left(\frac{170}{(s+0.47)(s+9.00)(s+16.77)(s+34)}\right) e^{-0.5s}$$

Find the frequency where the phase shift is -130 degrees

$$GK(j1.448) = 0.0214\angle -130^0$$

Pick k to make the gain 1.00 at this frequency

$$k = \frac{1}{0.0214} = 46.753$$

$$K(s) = 46.753\left(\frac{s+3.4}{s+34}\right)$$