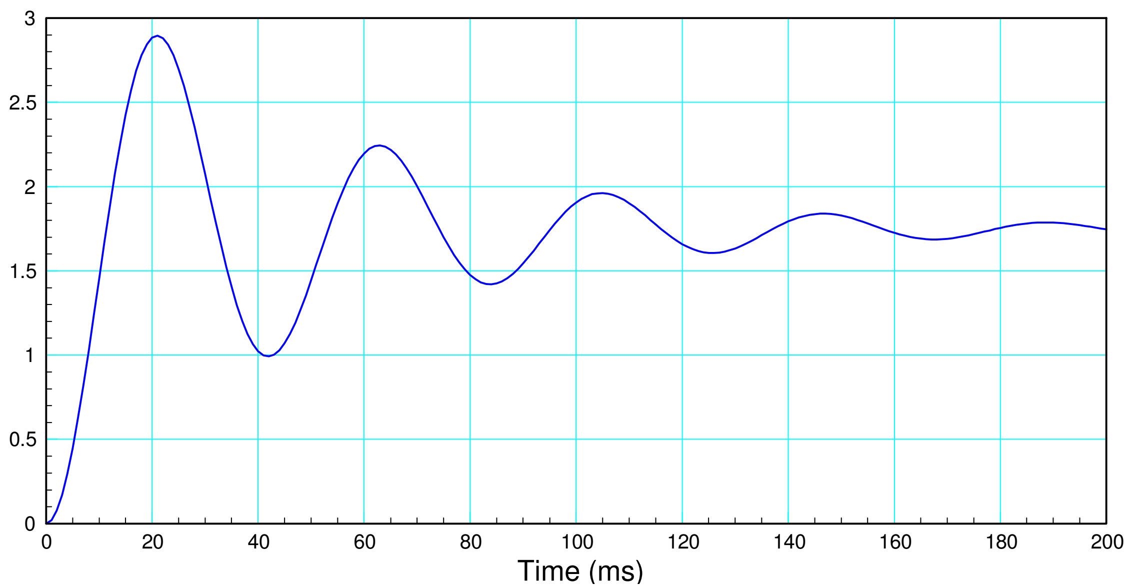
ECE 461 - Final: Name ____________________

Fall - 2022

1) Give the transfer function for a system with the following step response:



DC gain = 1.75

2% settling time = 200ms

$$\frac{4}{\sigma} = 200ms$$

$$\sigma = \frac{4}{200ms} = 20$$

Frequency of oscillation = 3 cycles in 125ms

$$\omega_d = \left(\frac{3 \text{ cycles}}{125\text{ms}}\right) 2\pi = 150.8 \frac{rad}{\text{sec}}$$

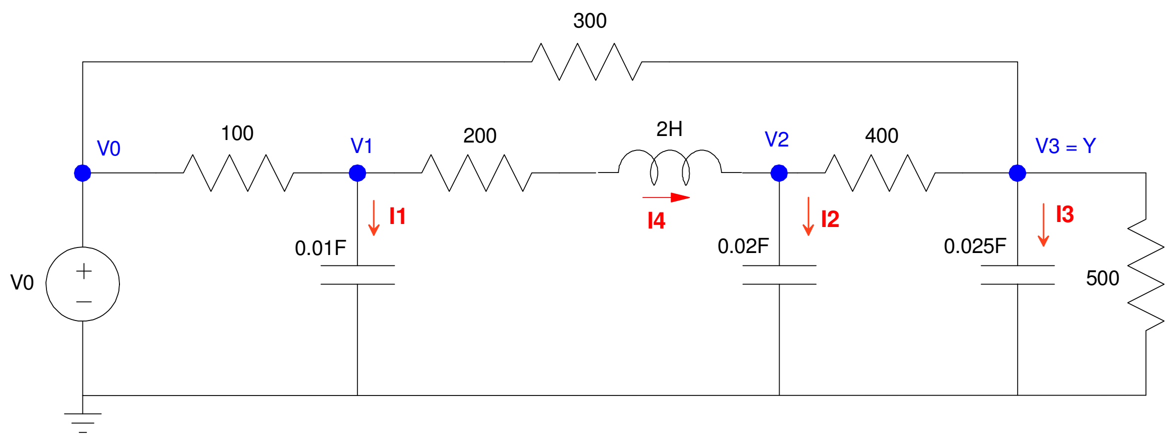
So

$$G(s) \approx \left(\frac{k}{(s+20+j150.8)(s+20-j150.8)}\right)$$

Pick 'k' to make the DC gain 1.75

$$\boxed{G(s) \approx \left(\frac{40{,}496}{(s+20+j150.8)(s+20-j150.8)}\right)}$$

2) Write the differential equations which describe the following circuit (i.e. write the N differential equations which correspond to the voltage node equations)



$$I_1 = 0.01sV_1 = \left(\frac{V_0 - V_1}{100}\right) - I_4$$

$$I_2 = 0.02sV_2 = I_4 - \left(\frac{V_2 - V_3}{400}\right)$$

$$I_3 = 0.025sV_3 = \left(\frac{V_2 - V_3}{400}\right) + \left(\frac{V_0 - V_3}{300}\right) - \left(\frac{V_3}{500}\right)$$

$$V_4 = 2sI_4 = V_1 - 200I_4 - V_2$$

2b) Express these dynamics in state-space form

Simplify and group terms

$$sV_1 = V_0 - V_1 - 100I_4$$

$$sV_2 = 50I_4 - 0.125V_2 + 0.125V_3$$

$$sV_3 = 0.133V_0 + 0.1V_2 - 0.3133V_3$$

$$sI_4 = 0.5V_1 - 100I_4 - 0.5V_2$$

Place in matrix form (state-space form)

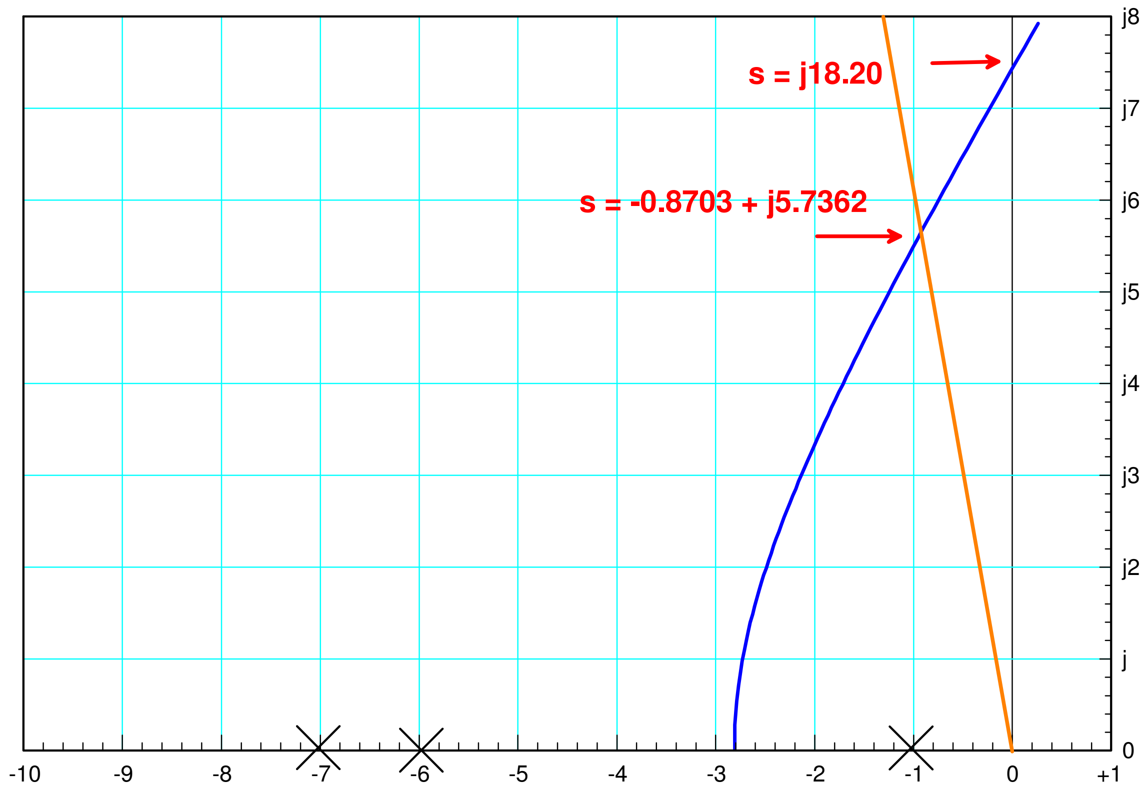
$$\begin{bmatrix} sV_1 \\ sV_2 \\ sV_3 \\ sI_4 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & -100 \\ 0 & -0.125 & 0.125 & 50 \\ 0 & 0.1 & -0.3133 & 0 \\ 0.5 & -0.5 & 0 & -100 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ I_4 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0.133 \\ 0 \end{bmatrix} V_0$$

3) Gain Compensation: The root locus for

$$G(s) = \left(\frac{40}{(s+1)(s+6)(s+7)} \right)$$

is shown below. Determine the following:

Maximum gain, k, for a stable closed-loop system	jw crossing = j7.4162 **k = 18.200**
k for a damping ratio of 0.15	s = -0.8703 + j5.7362 **k = 9.2669**
Closed-loop dominant pole(s) for a damping ratio of 0.15	**s = -0.8703 + j5.7362**
Closed-Loop DC gain for a damping ratio of 0.15	Kp = G(s=0) * k = 8.8256 **DC gain = Kp / (1+Kp) = 0.8982**



4) Given the following stable system

$$G(s) = \left(\frac{40}{(s+1)(s+6)(s+7)}\right)$$

Determine a compensator, K(s), which results in the closed-loop system having

- No error for a step input, and
- A closed-loop dominant pole at s = -3 + j2

Let

$$K(s) = k\left(\frac{(s+1)(s+6)}{s(s+a)}\right)$$

$$GK = \left(\frac{40k}{s(s+7)(s+a)}\right)$$

Analyze what we know

$$\left(\frac{40}{s(s+7)}\right)_{s=-3+j2} = 2.4807\angle -172.875^0$$

For the angle to add up to 180 degrees

$$\angle(s+a) = 7.125^0$$

$$a = 3 + \left(\frac{2}{\tan\left(7.125^0\right)}\right) = 19.0($$

Find k

$$GK = \left(\frac{40k}{s(s+7)(s+19)}\right)_{s=-3+j2} = 0.1538k \leq 180^0 = -1$$

$$k = 6.500$$

and

$$\boxed{K(s) = 6.500\left(\frac{(s+1)(s+6)}{s(s+19)}\right)}$$

Another valid solution is

$$\boxed{K(s) = 4.2937\left(\frac{(s+1)(s+6)(s+7)}{s(s+9.6056)^2}\right)}$$

5) Given the following stable system

$$G(z) = \left(\frac{0.04(z+1)}{(z-0.9)(z-0.4)(z-0.3)}\right)$$

Determine a digital compensator, K(z), which results in the closed-loop system having

- No error for a step input,
- A closed-loop dominant pole at z = 0.7 + j0.1, and
- A sampling rate of T = 0.01

Let

$$K(z) = k\left(\frac{(z-0.9)(z-0.4)}{z(z-a)}\right)$$

$$GK = \left(\frac{0.04k(z+1)}{z(z-0.3)(z-a)}\right)$$

Analyze what we know

$$\left(\frac{0.04(z+1)}{z(z-0.3)}\right)_{z=0.7+j0.1} = 0.5224\angle -172.2348^0$$

For the angles to add to 180 degrees

$$\angle(z-a) = 7.7652^0$$

$$a = 0.7 - \left(\frac{0.1}{\tan\left(7.7652^0\right)}\right) = -0.0333$$

To find k

$$GK = \left(\frac{0.04k(z+1)}{(z-1)(z-0.3)(z+0.0333)}\right)_{z=0.7+j0.1} = 0.7059k\angle 180^0 = -1$$

$$k = \frac{1}{0.7059} = 1.4166$$

and16

$$\boxed{K(z) = 1.4166\left(\frac{(z-0.9)(z-0.4)}{(z-1)(z+0.0333)}\right)}$$

Another solution

$$K(z) = 1.2935\left(\frac{(z-0.9)(z-0.4)(z-0.3)}{(z-1)(z-0.1817)^2}\right)$$

6) Given the following stable system

$$G(s) = \left(\frac{40}{(s+1)(s+6)(s+7)}\right)$$

Determine a compensator, K(s), which results in the closed-loop system having

- A closed-loop DC gain of 1.000 (i.e. no error for a step input),
- A 0dB gain frequency of 3 rad/sec, and
- A phase margin of 55 degrees

Translation: At 3 rad/sec, the open look gain is 1.000 and the phase shift is 55 degrees short of -180 degrees

$$GK(j3) = 1\angle -125^0$$

Let

$$K(s) = k\left(\frac{(s+1)(s+6)}{s(s+a)}\right)$$

$$GK = \left(\frac{40k}{s(s+7)(s+a)}\right)$$

Evaluate what we know

$$\left(\frac{40}{s(s+7)}\right)_{s=j3} = 1.7508\angle -113.1986^0$$

For the phase to be -125 degrees

$$\angle(s+a) = 11.8014^0$$

$$a = \left(\frac{3}{\tan\left(11.8014^0\right)}\right) = 14.3584$$

To find k

$$GK = \left(\frac{40k}{s(s+7)(s+14.3584)}\right)_{s=j3} = 0.1194k\angle -125^0$$

$$k = \frac{1}{0.1194} = 8.3784$$

and

$$\boxed{K(s) = 8.3784\left(\frac{(s+1)(s+6)}{s(s+14.3584)}\right)}$$

Another valid solution is

$$\boxed{K(s) = 7.4649\left(\frac{(s+1)(s+6)(s+7)}{s(s+9.5148)^2}\right)}$$