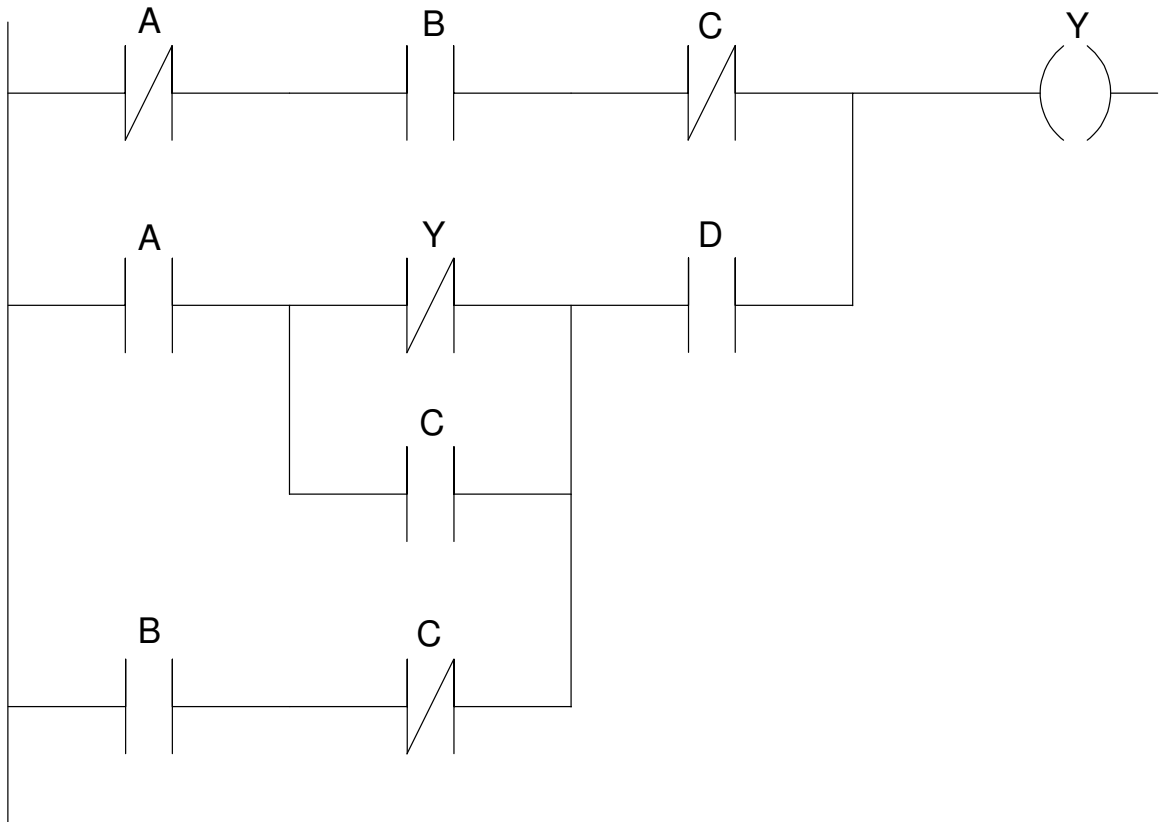
ECE 461/661 - Test #1: Name _____

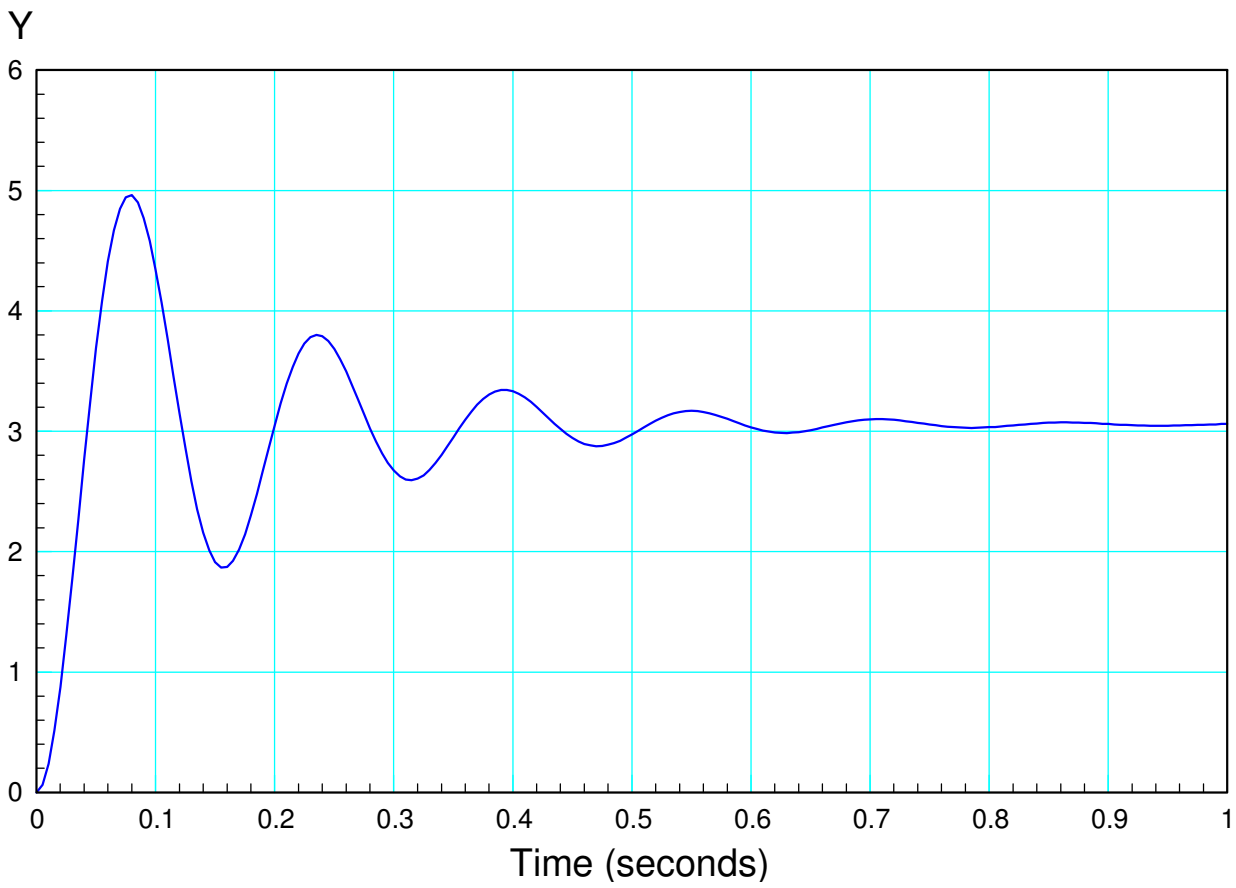
Fall 2022

1) Determine the functions for Y according to the following ladder diagram. (you don't need to simplify)



$$Y = \overline{A}B\overline{C} + \left(A\left(\overline{Y} + C\right) + B\overline{C}\right)D$$

2) Give the transfer function for a system with the following response to a unit step input:



DC gain = 3.1

2% settling time = 0.7 seconds (approx)

$$\sigma = \frac{4}{0.7} = 5.71$$

3 cycles in 0.47 seconds (approx)

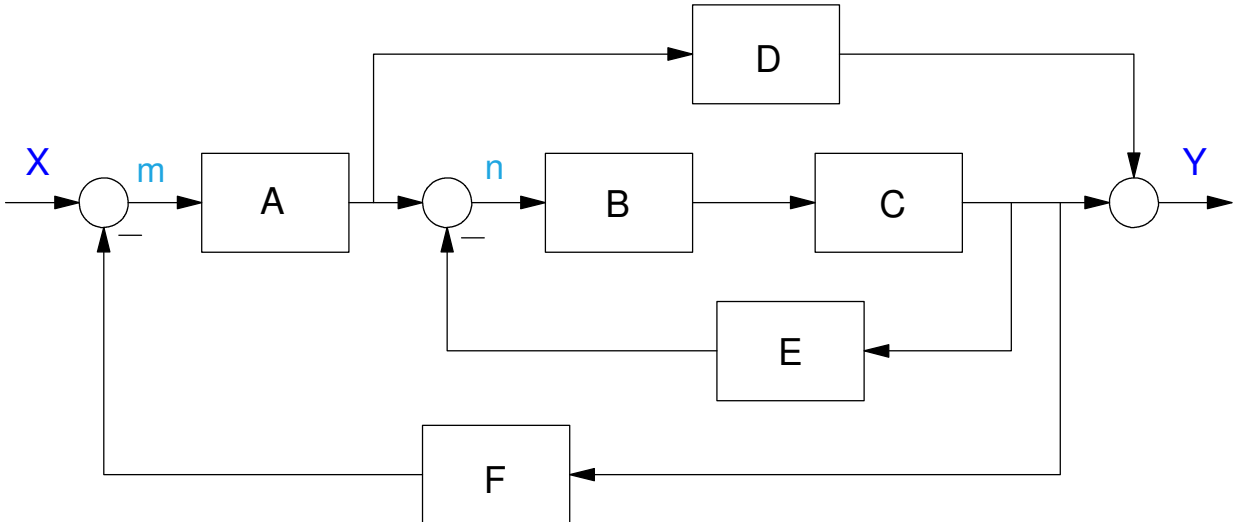
$$\omega_d = \left(\frac{3 \text{ cycles}}{0.47 \text{ sec}}\right) 2\pi = 40.1$$

so

$$G(s) \approx \left(\frac{5085.9}{(s+5.71+j40.1)(s+5.71-j40.1)}\right)$$

note: the numerator sets the DC gain to 3.1

3) Find the transfer funciton from X to Y



Shortcut

$$Y = \left(\frac{ABC+AD}{1+BCD+ABCF}\right)X$$

Long Way

$$m = X - FCBn$$

$$n = Am - ECBn$$

$$Y = DAm + CBn$$

Solving:

$$n = A(X - FCBn) - ECBn$$

$$(1 + AFCB + ECB)n = AX$$

$$n = \left(\frac{A}{1+AFCB+ECB}\right)X$$

$$Y = DA(X - FCBn) + CBn$$

$$Y = DAX - DAFCB\left(\frac{A}{1+AFCB+ECB}\right)X + CB\left(\frac{A}{1+AFCB+ECB}\right)X$$

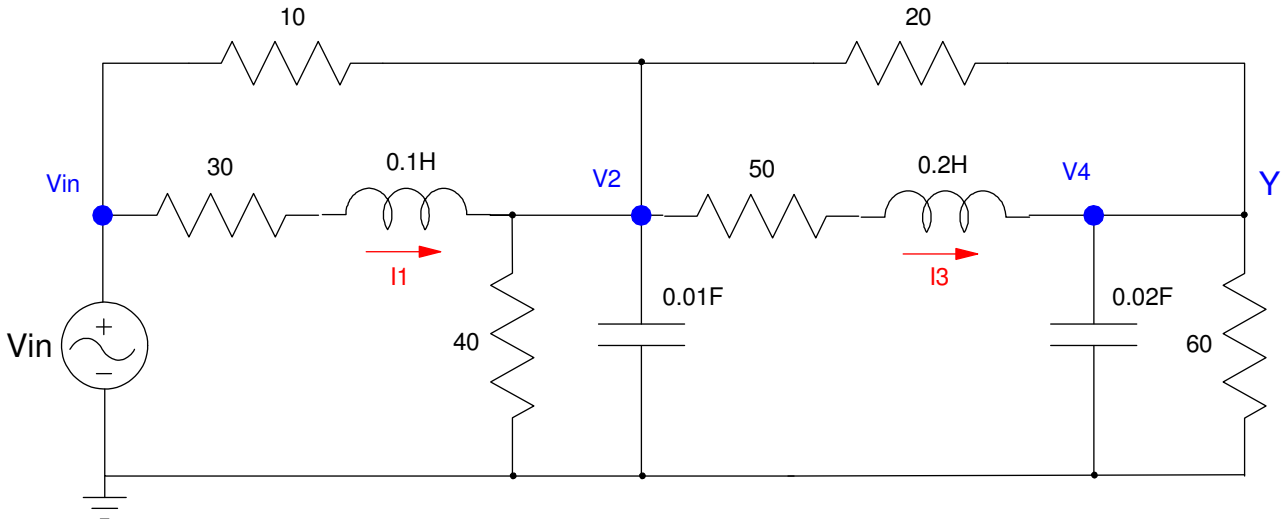
$$(1 + AFCB + ECB)Y = (1 + AFCB + ECB)DAX - DAFCB(A)X + CB(A)X$$

$$(1 + AFCB + ECB)Y = (1 + ECB)DAX + CB(A)X$$

$$Y = \left(\frac{DA+ECBDA+CBA}{1+AFCB+ECB}\right)X$$

4) For the following RLC circuit:

- Write the dynamics of this system as four compled differential equations in terms of {Vin, I1, V2, I3, V4}
- Express these dynamics in state-space form



$$0.1sI_1 = (V_{in} - 30I_1) - (V_2)$$

$$0.2sI_3 = (V_2 - 50I_3) - (V_4)$$

$$0.01sV_2 = I_1 + \left(\frac{V_{in}-V_2}{10}\right) - \left(\frac{V_2}{40}\right) - \left(\frac{V_2-V_4}{20}\right) - I_3$$

$$0.02sV_4 = I_3 - \left(\frac{V_4}{60}\right) - \left(\frac{V_4-V_2}{20}\right)$$

Group terms

$$sI_1 = -300I_1 - 10V_2 + 10V_{in}$$

$$sV_2 = 100I_1 - 17.5V_2 - 100I_3 + 5V_4 + 10V_{in}$$

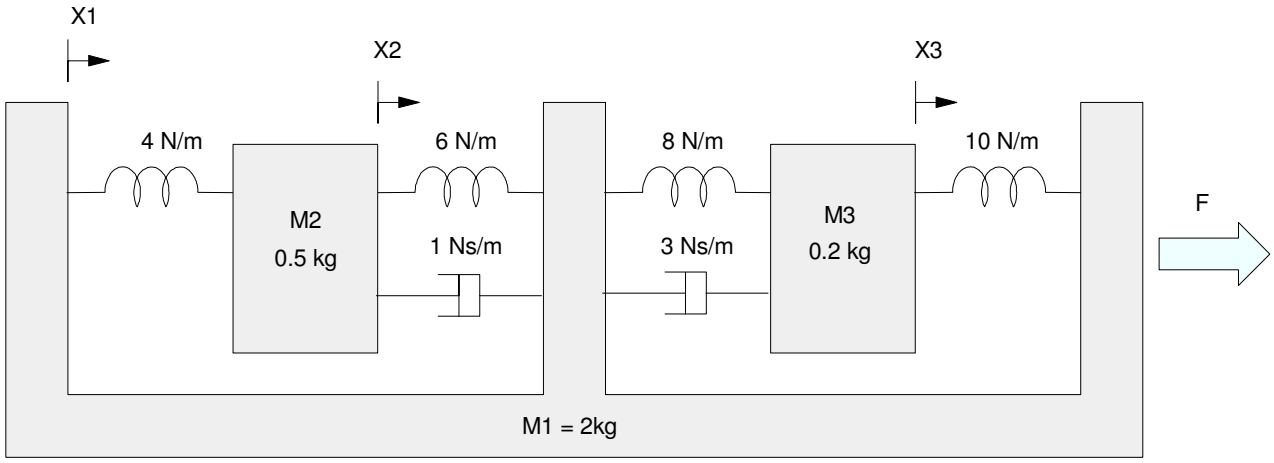
$$sI_3 = 5V_2 - 250I_3 - 5V_4$$

$$sV_4 = 50I_3 - 3.33V_4 + 2.5V_2$$

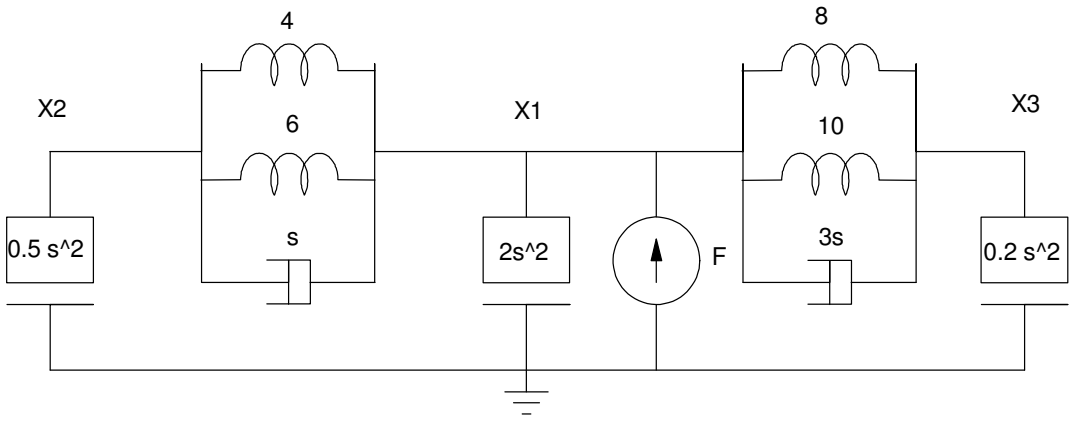
In matrix form

$$\begin{bmatrix} sI_1 \\ sV_2 \\ sI_3 \\ sV_4 \end{bmatrix} = \begin{bmatrix} -300 & -10 & 0 & 0 \\ 100 & -17.5 & -100 & 5 \\ 0 & 5 & -250 & -5 \\ 0 & 2.5 & 50 & -3.33 \end{bmatrix} \begin{bmatrix} sI_1 \\ sV_2 \\ sI_3 \\ sV_4 \end{bmatrix} + \begin{bmatrix} 10 \\ 10 \\ 0 \\ 0 \end{bmatrix} V_{in}$$

5) For the following mass-spring system



Draw the circuit equivalent for the following mass-spring system



Write the equations of motion (i.e. write the voltage node equations)

$$(0.5s^2 + s + 10)X_2 - (s + 10)X_1 = 0$$

$$(2s^2 + 4s + 28)X_1 - (s + 10)X_2 - (3s + 18)X_3 = F$$

$$(0.2s^2 + 3s + 18)X_3 - (3s + 18)X_1 = 0$$