ECE 461/661 - Test #2: Name ____________________

Feedback and Root Locus - Fall 2022

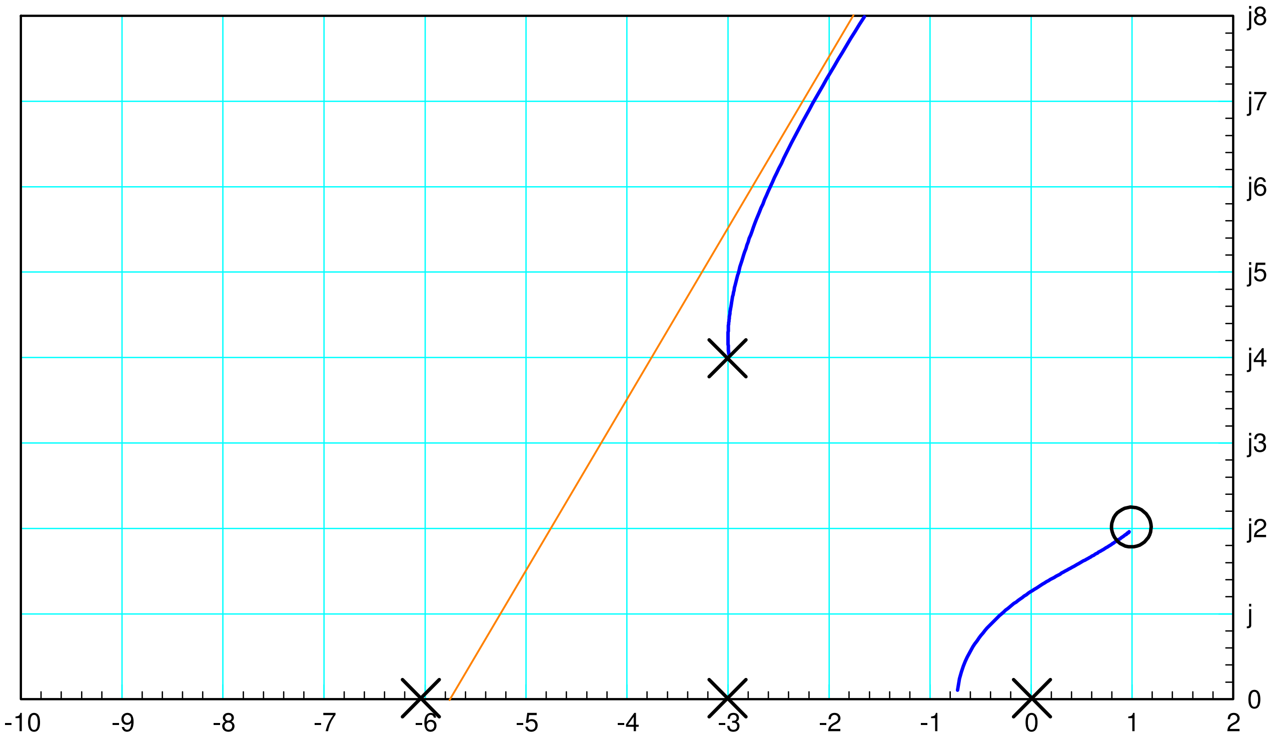
Root Locus

1) The root locus of G(s) is shown below.

$$G(s) = \left(\frac{10(s-1+j2)(s-1-j2)}{s(s+3)(s+6)(s+3+j4)(s+3-j4)}\right)$$

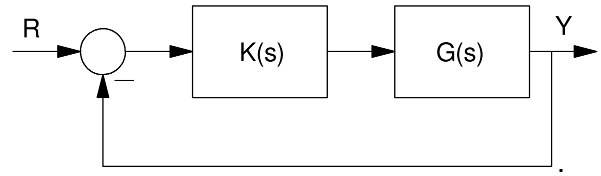
Determine the following

Approach Angle to the zero at +1+j2	Departure Angle from the pole at -3+j4	Real Axis Loci
-134.3097 deg	**97.125 deg**	**(0, -3), (-6, -inf)**
Breakaway Point (approx)	Asymptotes	jw Crossing(s)
s = -0.7406	+/- 60 degrees, 180 degrees intercept = -17/3	**j1.2723, j10.8935**



Gain Compensation

2) Determine the gain (K(s) = k) so that the feedback system has 60% overshoot for a step input. Also determine the closed-loop dominant pole(s) and error constant, Kp

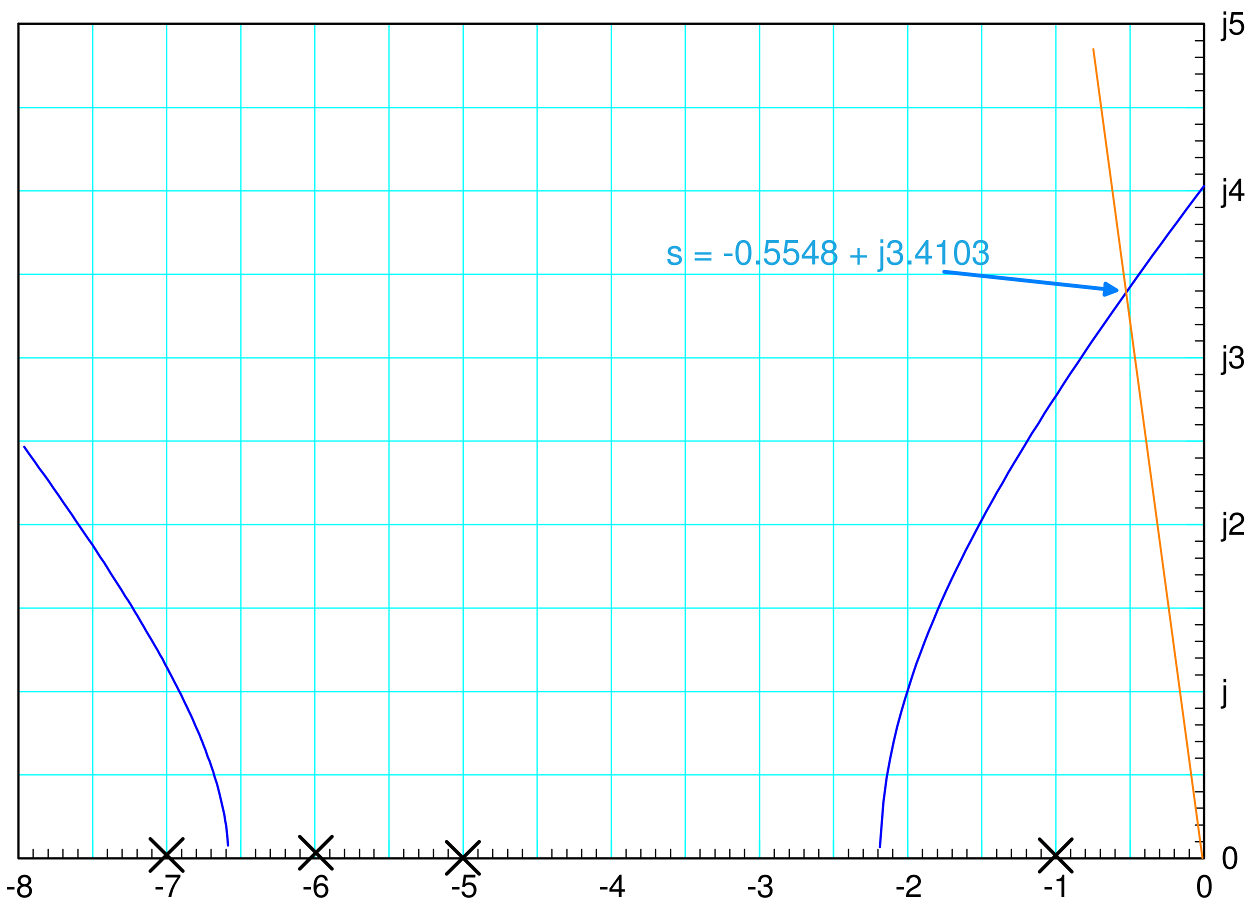


$$G(s) = \left(\frac{100}{(s+1)(s+5)(s+6)(s+7)} \right)$$

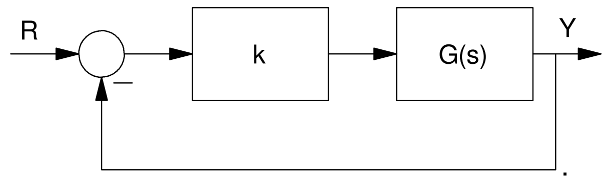
k 60% overshoot	Closed-Loop dominant pole(s)	Kp Error Constant
9.0275	**s = -0.5548 + j3.4103**	**4.3**

zeta = 0.1605, angle = 80.76 degrees, tan(angle) = 6.15

$$K_p = (GK)_{s=0} = \left(\frac{100}{(s+1)(s+5)(s+6)(s+7)} \right)_{s=0} \cdot 9.0275 = 4.2988$$



Lead/PI Compensation



3) Design a compensator, K(s), so that the closed-loop system has

- No error for a step input
- Closed-Loop dominant poles at s = -1 + j3, and
- Finite gain as s→ ∞ (i.e. have at least as many poles as zeros)

$$G(s) = \left(\frac{100}{(s+1)(s+5)(s+6)(s+7)}\right)$$

Let

$$K(s) = k\left(\frac{(s+1)(s+5)}{s(s+a)}\right)$$

Pick 'a' so that the angles add up to 180 degrees at s = -1 + j3

$$GK = \left(\frac{100k}{s(s+a)(s+6)(s+7)}\right)$$

Analyzing what we know

$$\left(\frac{100}{s(s+6)(s+7)}\right)_{s=-1+j3} = 0.8085\angle -165.9638^0$$

For the angles to add to 180 degrees

$$\angle(s+a) = 14.0362^0$$

$$a = \left(\frac{3}{\tan\left(14.0362^0\right)}\right) + 1 = 13.000($$

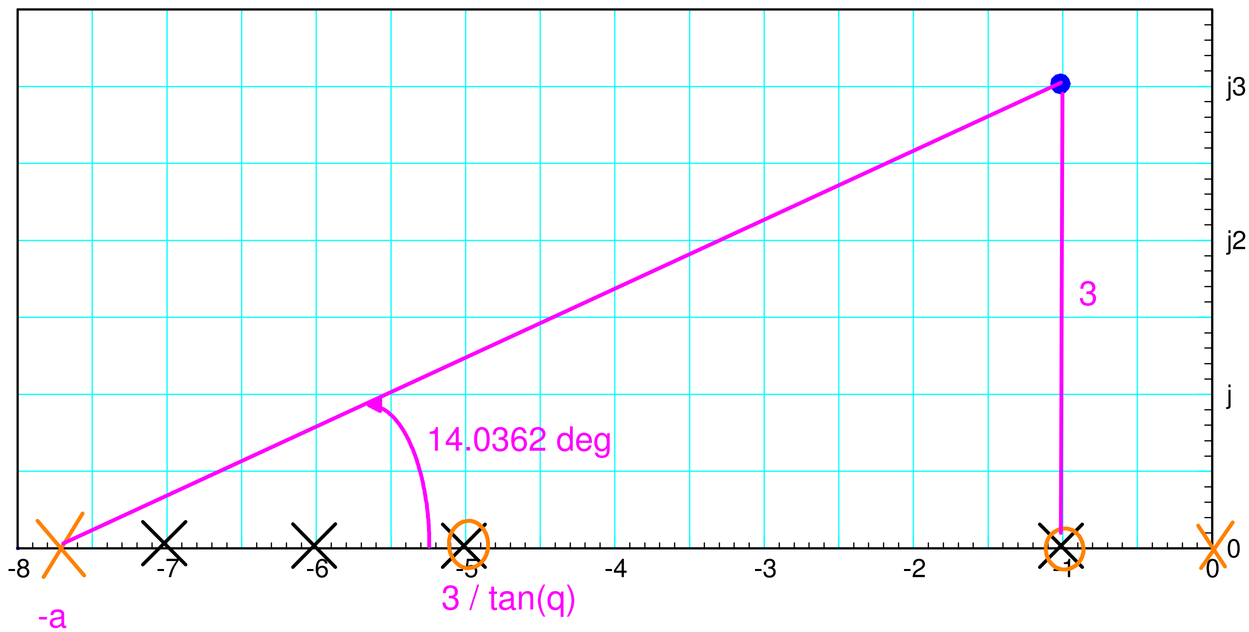
To find k

$$GK = \left(\frac{100k}{s(s+6)(s+7)(s+13)}\right)_{s=-1+j3} = 0.0654k\angle 180^0$$

$$k = \frac{1}{0.0645} = 15.300$$

and

$$K(s) = 15.30\left(\frac{(s+1)(s+5)}{s(s+13)}\right)$$



j3
j2
3
j
14.0362 deg
0
-8
-7
-6
-5
-4
-3
-2
-1
0
-a
3 / tan(q)

Compensator Design (hardware)

4) Design a circuit to implement K(s)

$$K(s) = \left(\frac{40(s+5)(s+6)}{s(s+17)}\right)$$

Rewrite as

$$K(s) = \left(\frac{8(s+6)}{s+17}\right)\left(\frac{5(s+5)}{s}\right)$$