ECE 461/661 - Test #3: Name ____________________

Digital Control & Frequemncy Domain techniques - Fall 2022

s to z conversion

1) Determine the discrete-time equivalent for G(s). Assume a sampling rate of T = 0.1 second

$$G(s) = \left(\frac{100(s+2)}{(s+5)(s+2+j6)(s+2-j6)}\right)$$

Convert as $z = e^{sT}$

- s = -2 z = 0.8187
- s = -5 z = 0.6065
- s = -2 + j6 z = 0.6757 + 0.4623i
- s = -2 - j6z = 0.6757 - 0.4623i

so

$$G(z) = k\left(\frac{z-0.8187}{(z-0.6065)(z-0.6757+j0.4623)(z-0.6757-j0.4623)}\right)$$

To find k, match the gain at some frequency, like s = 0

$$\left(\frac{100(s+2)}{(s+5)(s+2+j6)(s+2-j6)}\right)_{s=0} = 1.000$$

$$k\left(\frac{z-0.8187}{(z-0.6065)(z-0.6757+j0.4623)(z-0.6757-j0.4623)}\right)_{z=1} = 1.000$$

$$k = 0.6921$$

$$G(z) = 0.6921\left(\frac{z-0.8187}{(z-0.6065)(z-0.6757+j0.4623)(z-0.6757-j0.4623)}\right)$$

Digital Compensators: K(z)

2) Assume a unity feedback system with a sampling rate of T = 0.1 second

$$G(z) = \left(\frac{100z^2}{(z-0.95)(z-0.9)(z-0.8)} \right)$$

Design a digital compensator, K(z), which results in

- No error for a step input,
- Closed-Loop Dominant poles at z = 0.8 + j0.2, and
- Is causal (the number of poles in K(z) is equal to or greater than the number of zeros)

Let

$$K(z) = k\left(\frac{(z-0.95)(z-0.9)}{(z-1)(z-a)} \right)$$

so

$$GK = \left(\frac{100k \cdot z^2}{(z-1)(z-0.8)(z-a)} \right)$$

Evaluate what we know at z = 0.8 + j0.2

$$\left(\frac{100 \cdot z^2}{(z-1)(z-0.8)} \right)_{z=0.8+j0.2} = 1.202\angle+163^0$$

The phase is past -180 degrees, so we need another zero. Let

$$K(z) = k\left(\frac{(z-0.95)(z-0.9)(z-0.8)}{(z-1)(z-a)^2} \right)$$

$$GK = \left(\frac{100k \cdot z^2}{(z-1)(z-a)^2} \right)$$

Evaluate what we know

$$\left(\frac{100 \cdot z^2}{(z-1)(z-0.8)} \right)_{z=0.8+j0.2} = 240.4163\angle-106.92^0$$

For the angle to add up to 180 degrees

$$\angle(z-a)^2 = 73.0725^0$$

$$\angle(z-a) = 36.5362^0$$

$$a = 0.8 - \left(\frac{0.2}{\tan\left(35.5362^0\right)} \right) = 0.5301$$

so

$$K(z) = k\left(\frac{(z-0.95)(z-0.9)(z-0.8)}{(z-1)(z-0.5301)^2}\right)$$

To find k

$$GK = \left(\frac{100k \cdot z^2}{(z-1)(z-0.5301)^2}\right)_{z=0.8+j0.2} = 2130.2038k\angle 180^0$$

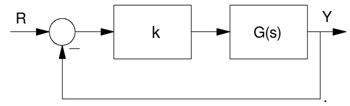
$$k = \frac{1}{2130.2038} = 0.0004694$$

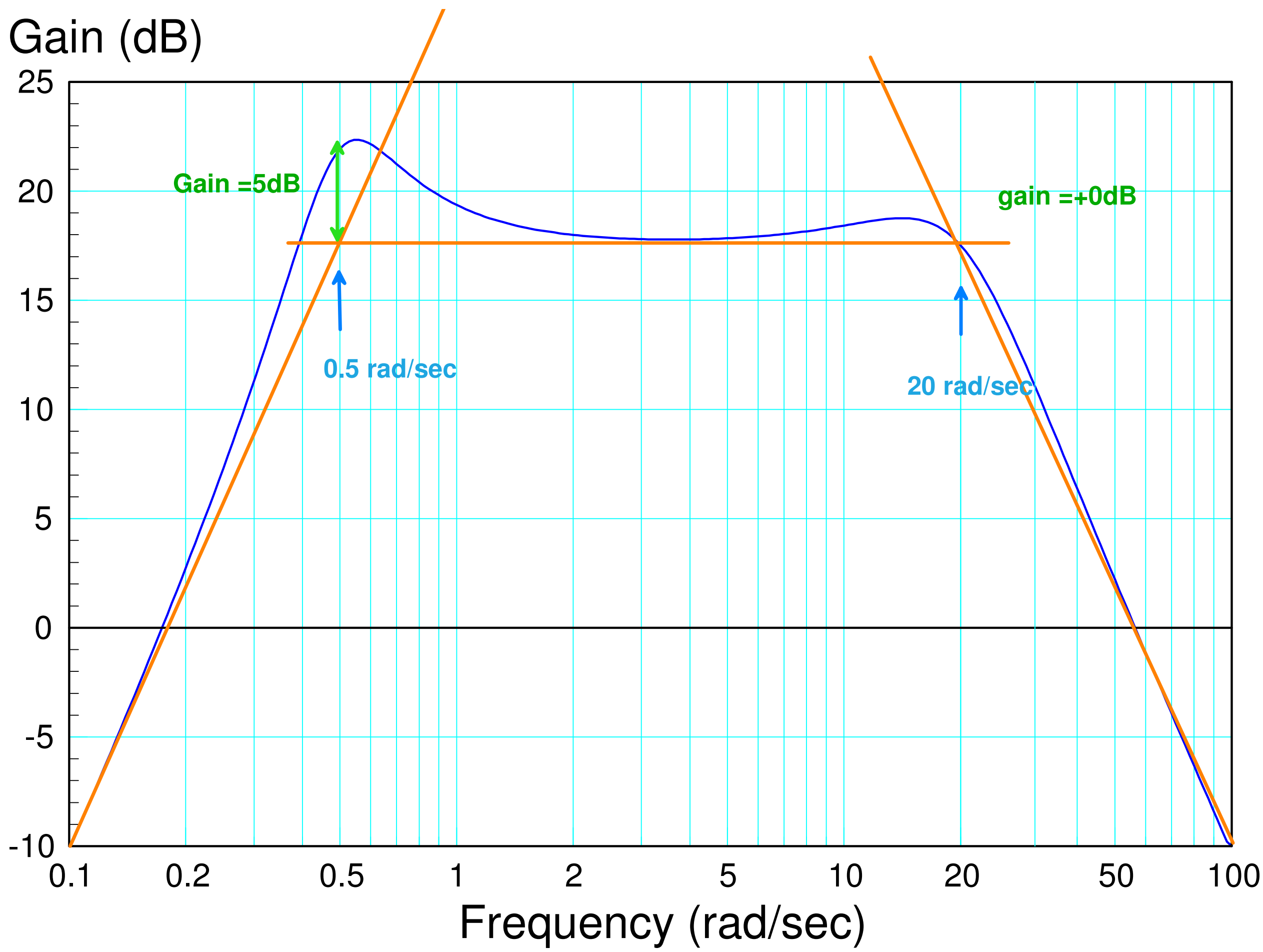
and

$$\boxed{K(z) = 0.0004694\left(\frac{(z-0.95)(z-0.9)(z-0.8)}{(z-1)(z-0.5301)^2}\right)}$$

3) Bode Plots

Determine the system, G(s), which has the following gain vs. frequency





There are two zeros left of 0.1 (assume s = 0)

There are two poles at 0.5 rad/sec

$$\frac{1}{2\zeta} = 5dB = 1.7783$$

$$\zeta = 0.2812$$

$$\theta = 73.67^0$$

There are two poles at 20 rad/sec

$$\frac{1}{2\zeta} = 0dB = 1$$

$$\zeta = 0.5$$

$$\theta = 60^0$$

So

$$G(s) = k\left(\frac{s^2}{\left(s+0.5\angle\pm 73.67^0\right)\left(s+20\angle\pm 60^0\right)}\right)$$

To find k, match the gain at some frequency, such as 3 rad/sec

$$G(j3) = 18dB = 7.94$$

$$k\left(\frac{s^2}{\left(s+0.5\angle\pm73.67^0\right)\left(s+20\angle\pm60^0\right)}\right)_{s=j3} = 7.94$$

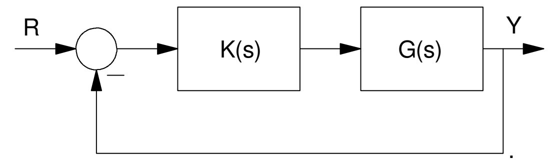
$$k = 3067.8$$

and

$$G(s) \approx \left(\frac{3067.7\cdot s^2}{\left(s+0.5\angle\pm73.67^0\right)\left(s+20\angle\pm60^0\right)}\right)$$

There are two poles at 20 rad/sec

4) Nichols Charts

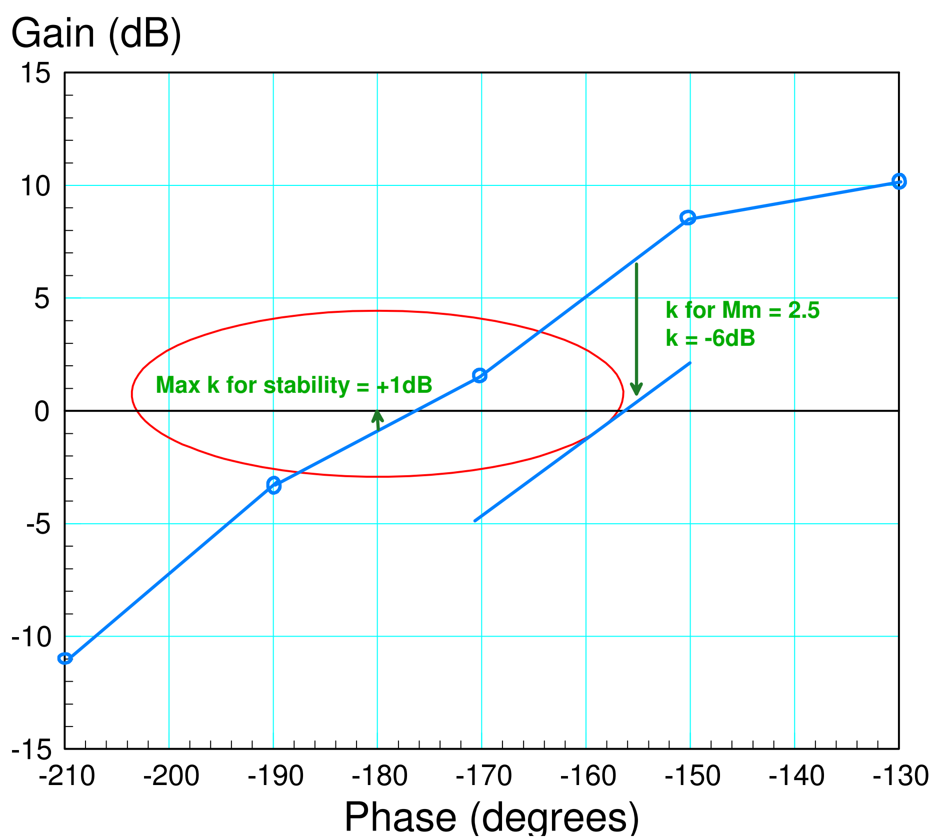


Assume a unity feedback system where the gain of G(s) is as follows:

Determine

- The maximum gain, k, for stability
- k that results in a resonance of Mm = 2.5

frequency (rad/sec)	7	8	9	10	12
Gain	10dB	8dB	2dB	-3dB	-11dB
Phase (degrees)	-130 deg	-150 deg	-170 deg	-190 deg	-210 deg



5) Analog Compensator (Bode Plots)

Assume a unity feedback system with

$$G(s) = \left(\frac{10}{(s+2)(s+10)(s+12)}\right)$$

Determine a compensator, K(s), which results in

- No error for a step input
- A phase margin of 40 degrees
- A 0dB gain frequency of 4 rad/sec

Let

$$K(s) = k\left(\frac{(s+2)(s+10)}{s(s+a)}\right)$$

$$GK = \left(\frac{10k}{s(s+12)(s+a)}\right)$$

Analyze what we know at 4 rad/sec

$$GK(j4) = 1\angle -140^0$$ *definition of a 40 degree phase margin with a 0dB gain frequency of 4 rad/sec*

$$\left(\frac{10}{s(s+12)}\right)_{s=j4} = 0.1976\angle -108.4349^0$$

For the angle to be -140 degrees

$$\angle(s+a) = 31.5651^0$$

$$a = \left(\frac{4}{\tan\left(31.5651^0\right)}\right) = 6.5108$$

and

$$GK = \left(\frac{10k}{s(s+6.5108)(s+12)}\right)_{s=j4} = 0.0259k\angle -140^0$$

meaning

$$k = \frac{1}{0.0259} = 38.6626$$

and

$$K(s) = 38.6626\left(\frac{(s+2)(s+10)}{s(s+6.5108))}\right)$$

note: this also works (cancel all three poles)

$$K(s) = 35.8330\left(\frac{(s+2)(s+10)(s+12)}{s(s+8.5780)^2}\right)$$