Homework #4: ECE 461 / 661

1st and 2nd Order Approximations. Due Monday, September 12th

LaPlace Transforms (Due September 12th)

1) Assume X and Y are related by the following transfer function

$$Y = \left(\frac{5(s+2)}{(s+3)(s+4)(s+5)}\right)X$$

a) What is the differential equation relating X and Y?

Multiply out and cross multiply

$$(s^3 + 12s^2 + 47s + 60)Y = (5s + 10)X$$

'sY' means 'the derivative of y(t)'

$$y''' + 12y'' + 47y' + 60y = 5x' + 10x$$

b) Determine y(t) assuming

$$x(t) = 4\cos(2t) + 3\sin(2t)$$

Use phasor analysis

$$s = j2$$

$$X = 4 - j3$$

$$Y = \left(\frac{5(s+2)}{(s+3)(s+4)(s+5)}\right)X$$

$$Y = \left(\frac{5(s+2)}{(s+3)(s+4)(s+5)}\right)_{s=j2} \cdot (4 - j3)$$

$$Y = 0.2255 - j0.7825$$

convert back to time

$$y(t) = 0.2255\cos(2t) + 0.7825\sin(2t)$$

c) Determine y(t) assuming x(t) is a unit step input

$$x(t) = u(t)$$

$$Y = \left(\frac{5(s+2)}{(s+3)(s+4)(s+5)}\right)X$$

Replace x(t) with it's LaPlace transform, X

$$Y = \left(\frac{5(s+2)}{(s+3)(s+4)(s+5)}\right)\left(\frac{1}{s}\right)$$

Use partial fractions

$$Y = \left(\frac{0.1667}{s}\right) + \left(\frac{0.8333}{s+3}\right) + \left(\frac{-2.5}{s+4}\right) + \left(\frac{1.5}{s+5}\right)$$

Take the inverse LaPlace transform

$$y(t) = (0.1667 + 0.8333e^{-3t} - 2.5e^{-4t} + 1.5e^{-5t})u(t)$$

2) Assume X and Y are related by the following transfer function:

$$Y = \left(\frac{300}{(s+1+j4)(s+1-j4)(s+15)}\right)X$$

a) Use 2nd-order approximations to determine

- The 2% settling time
- The percent overshoot for a step input
- The steady-state output for a step input (x(t) = u(t))

The dominant poles are at

$$s = -1 \pm j4 = -4.123\angle 75.964^0$$

This gives

$$T_s = \left(\frac{4}{1}\right) = 4 \text{ seconds}$$

$$\zeta = \cos\theta = 0.2425$$

$$OS = \exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right) = 45.59\%$$

$$\text{DC Gain} = \left(\frac{300}{(s+1+j4)(s+1-j4)(s+15)}\right)_{s=0} = 1.1765$$

b) Check your answers using the 3rd order model and Matlab, Simulink, of VisSim (your pick)

```
>> G = zpk([],[-15,-1+j*4,-1-j*4],300)

         300
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(s+15) (s^2 + 2s + 17)

>> DC = evalfr(G,0)

DC = 1.1765

>> t = [0:0.01:6]';
>> y = step(G,t);
>> plot(t,y);
>> xlabel('Time (seconds');

>> max(y) / DC

ans =    1.4380
```
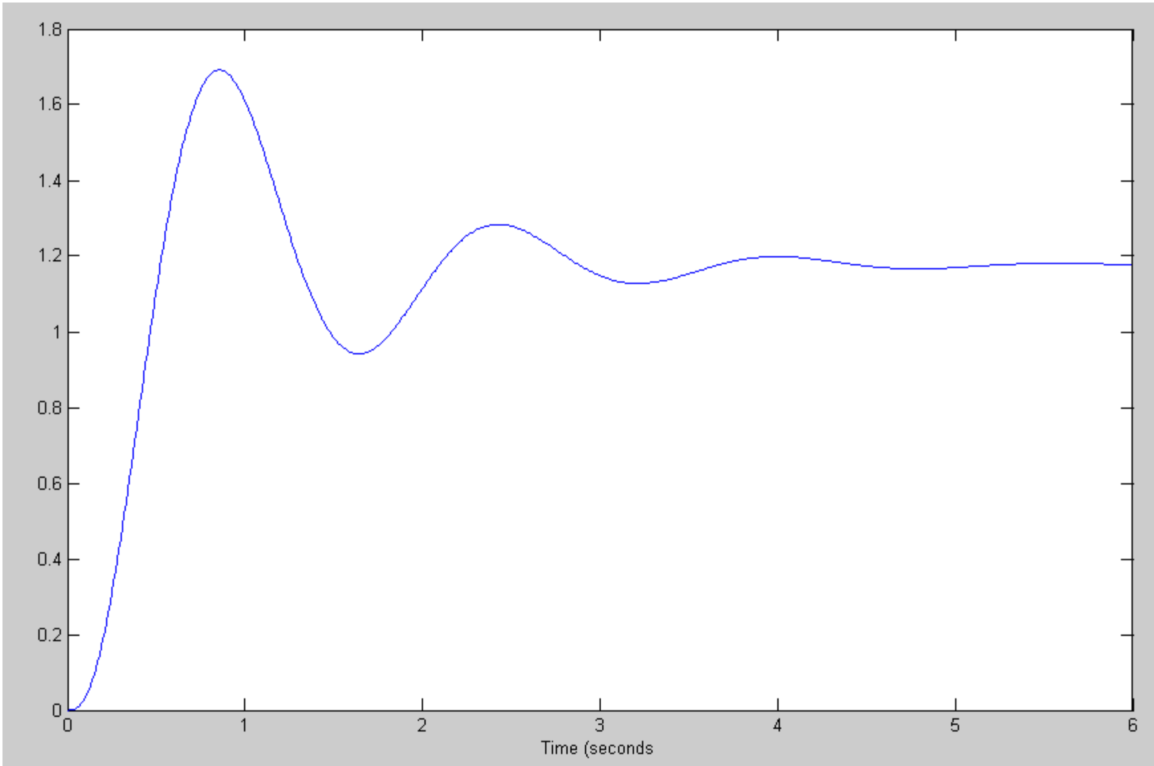
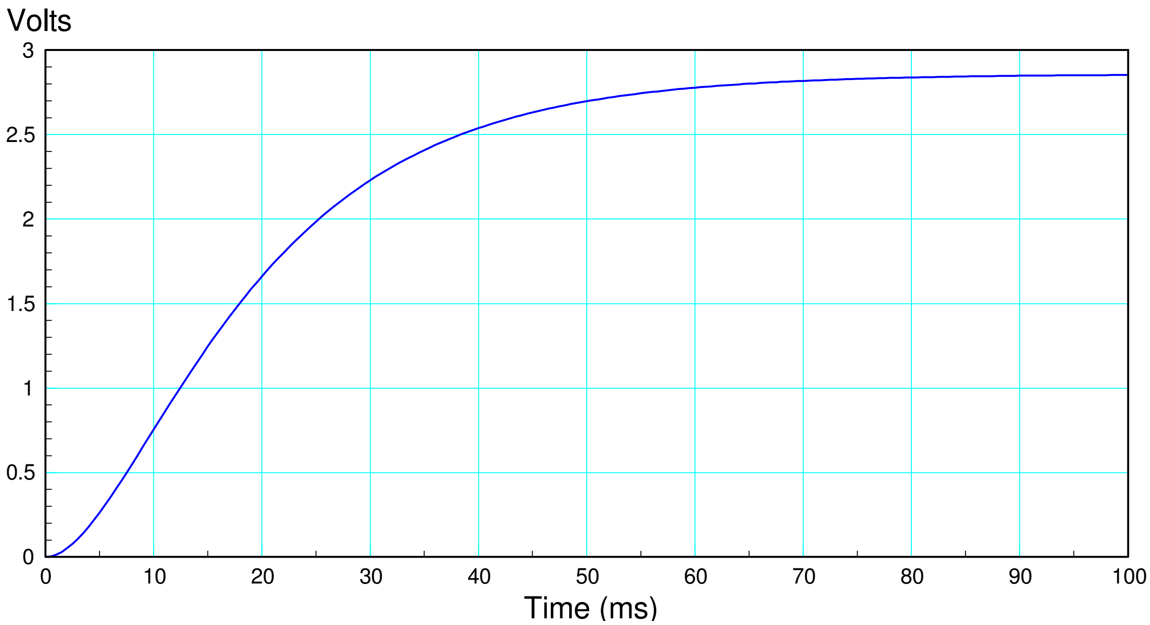

System actually has 43.8% overshoot

- 45.59% calculated



1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
1
2
3
4
5
6
Time (seconds

3) Determine the transfer function for a system with the following step response:



This is a 1st-order system (no oscillations), so G(s) is of the form

$$G(s) = \left(\frac{a}{s+b}\right)$$

Pull two pieces of information from the graph

DC gain = 2.8 = a/b

2% settling time = 70ms (approx)

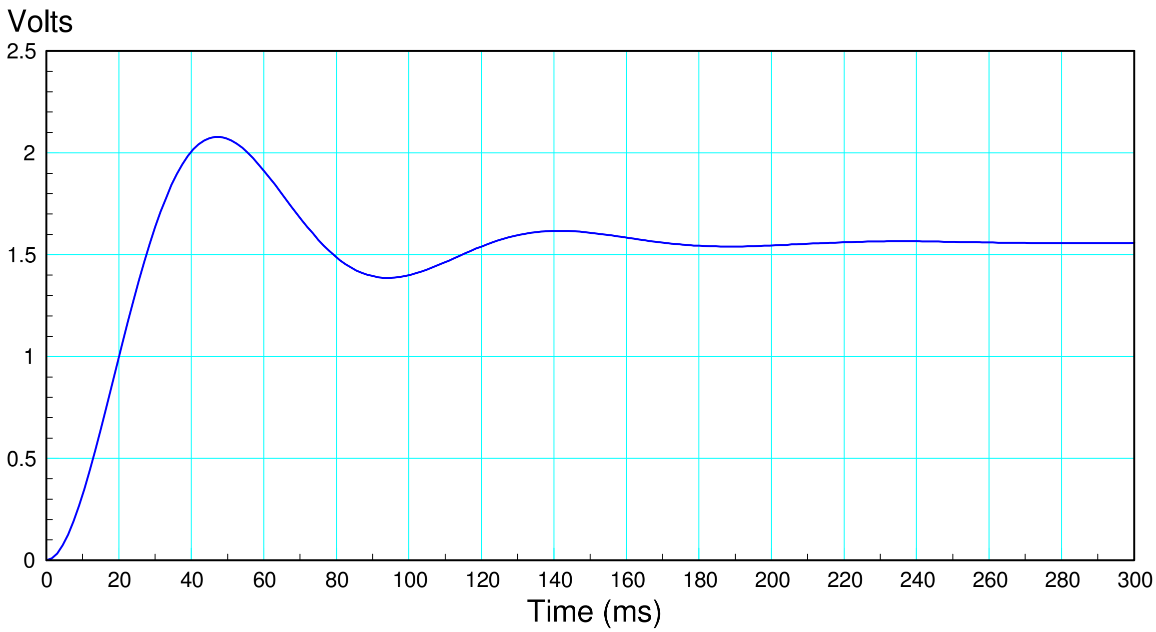
$$T_s = 70ms = \frac{4}{b}$$

$$b = 57.14$$

$$a = 2.8b = 160$$

$$G(s) \approx \left(\frac{160}{s+57.14}\right)$$

4) Determine the transfer function for a system with the following step response:



This is a 2nd-order system (oscillates), so G(s) is of the form

$$G(s) = \left(\frac{a}{s^2+bs+c}\right) = \left(\frac{a}{(s+\sigma+j\omega_d)(s+\sigma-j\omega_d)}\right)$$

Pull three pieces of information from the graph

DC gain = 1.6

Frequency of oscillation is 1 cycle every 92ms

$$\omega_d = 2\pi\left(\frac{1 \text{ cycle}}{92ms}\right) = 68.29$$

2% settling time = 160ms (approx)

$$\sigma = \left(\frac{4}{160ms}\right) = 25$$

So

$$G(s) \approx \left(\frac{a}{(s+25+j68.29)(s+25-j68.29)}\right)$$

Pick 'a' to make the DC gain 1.6

$$G(s) \approx \left(\frac{14{,}807}{(s+25+j68.29)(s+25-j68.29)}\right)$$