Homework #7: ECE 461/661

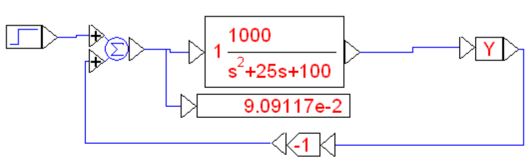
Error Constants, Routh Criteria, Skething a Root Locus. Due Monday, October 10th

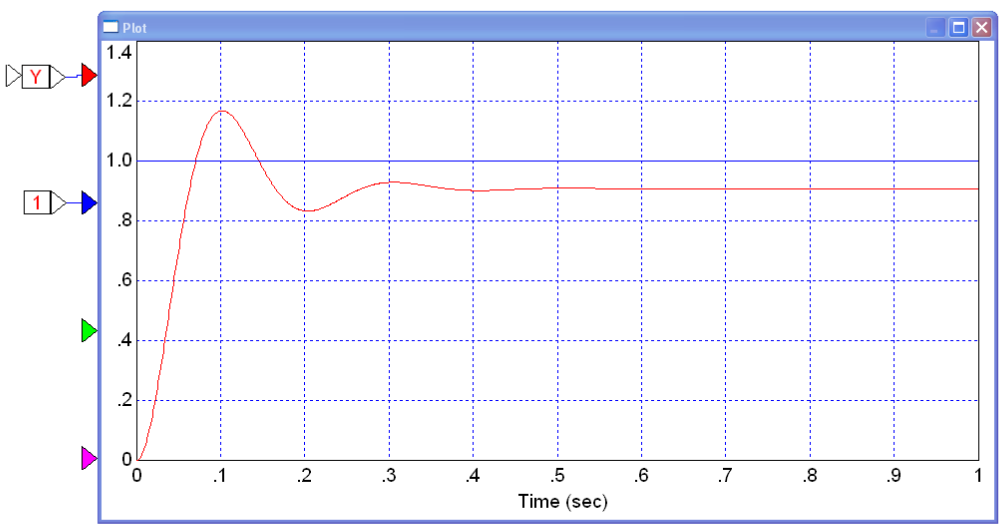
Error Constants

1) Determine the error constants and steady-state error for the following systems

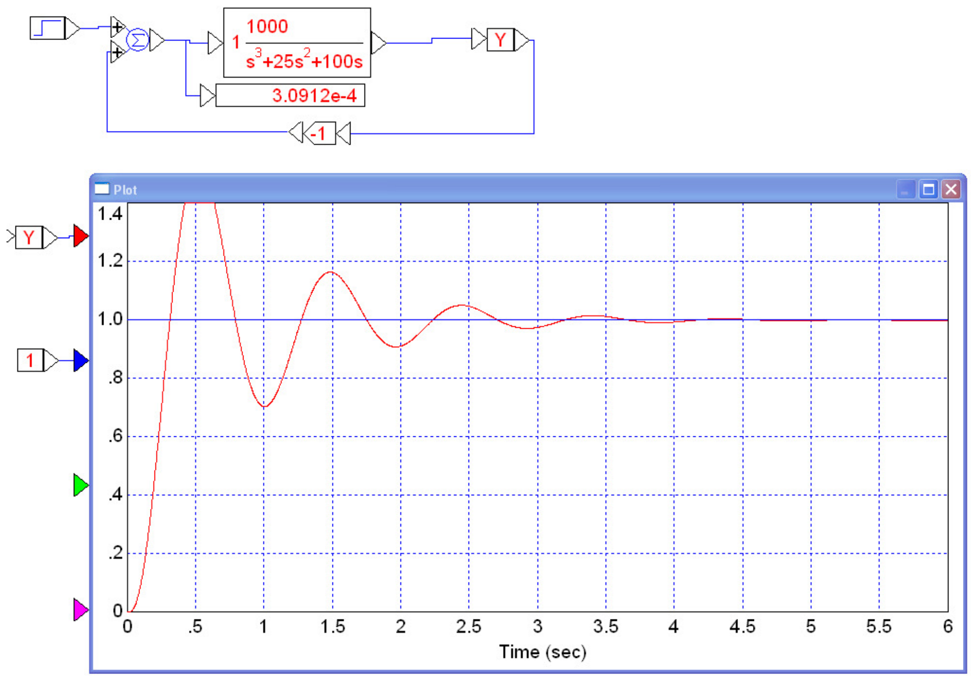
G(s)	System Type	Kp	Kv	Error for a unit step input
$\left(\frac{1000}{(s+5)(s+20)}\right)$	0	10	0	1/11
$\left(\frac{1000}{s(s+5)(s+20)}\right)$	1	infinity or n/a	10	0
$\left(\frac{1000(s+1)}{s^2(s+5)(s+20)}\right)$	2	infinity or n/a	infinity or n/a	0
$\left(\frac{1000}{(s-5)(s+20)}\right)$	0	-10	0	-1/9

a) error = 1/11

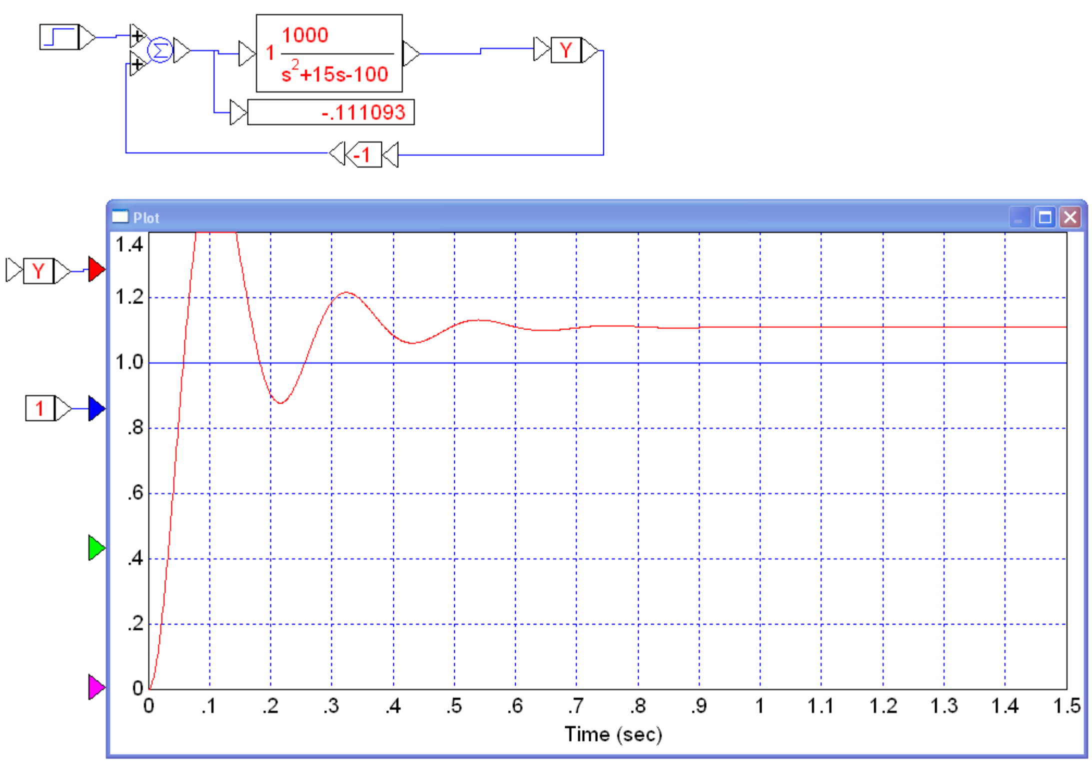




b) error = 0



d) error = -1/9



Routh Criteria

Determine the range of k that results in a negative definite polynomial (i.e. a stable system)

2) $(s-1)(s+8)(s+10)+2k=0$

multiply out

$$s^3+17s^2+62s+(2k-80)=0$$

1	62	0	range of k
17	2k-80	0	
66.7059 - 0.1176k (a)	0 (b)	0	k < 567.00
2k-80 (c)	0	0	k > 40.00
0	0	0	

ans: 40 < k < 567

(a)

$$\frac{-\begin{vmatrix} 1 & 62 \\ 17 & 2k-80 \end{vmatrix}}{17}=66.7059-0.1176k$$

(b)

$$\frac{-\begin{vmatrix} 1 & 0 \\ 17 & 0 \end{vmatrix}}{17}=0$$

(c)

$$\frac{-\begin{vmatrix} 17 & 2k-80 \\ 66.70-0.1176k & 0 \end{vmatrix}}{66.70-0.1176k}=2k-80$$

3) $(s+1)(s+6)(s+8)(s+10)+2k=0$

Multiply out

$$s^4+25s^3+212s^2+668s+2k+480=0$$

1	212	2k+480	range of k
25	668	0	
185.28 (a)	2k + 480 (b)	0	
732.7668 - 0.2699k (c)	0 (d)	0	k < 2235.34
2k + 480	0	0	k > -240
0	0	0	

ans: 240 < k < 2235.34

(a)

$$\frac{-\begin{vmatrix} 1 & 212 \\ 25 & 668 \end{vmatrix}}{25} = 185.28$$

(b)

$$\frac{-\begin{vmatrix} 1 & 2k+480 \\ 25 & 0 \end{vmatrix}}{23} = 2k+480$$

(c)

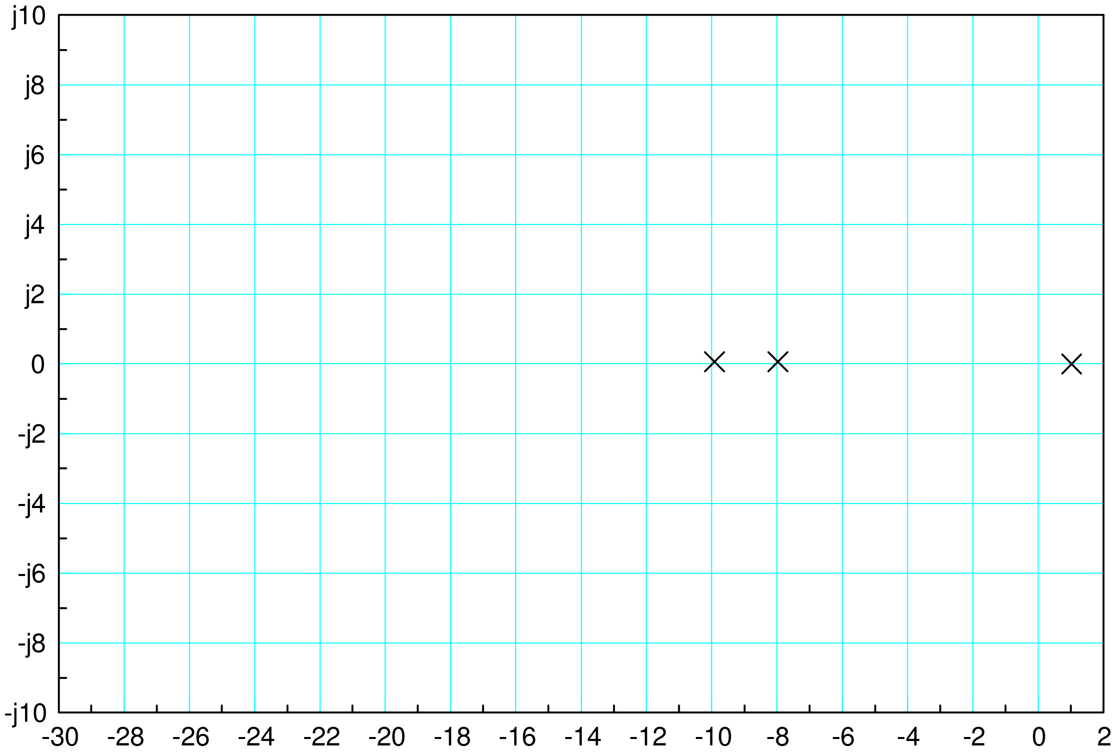
$$\frac{-\begin{vmatrix} 25 & 668 \\ 185.28 & 2k+480 \end{vmatrix}}{185.28} = 603.2332-0.2699k$$

Sketching a Root Locus

Sketch the root locus plot for the following systems for 0 < k < infinity. Also plot the

- real axis loci, break away points, jw crossings (if any), and asymptotes

4) $$(s-1)(s+8)(s+10)+2k=0$$



Real Asix Loci

(+1, -8),   (-10, -infinity)

Asymptotes

3 asymptotes

+/- 60 degrees, 180 degrees

intersect = - 17/3

Breakaway Point

s = -2.2829

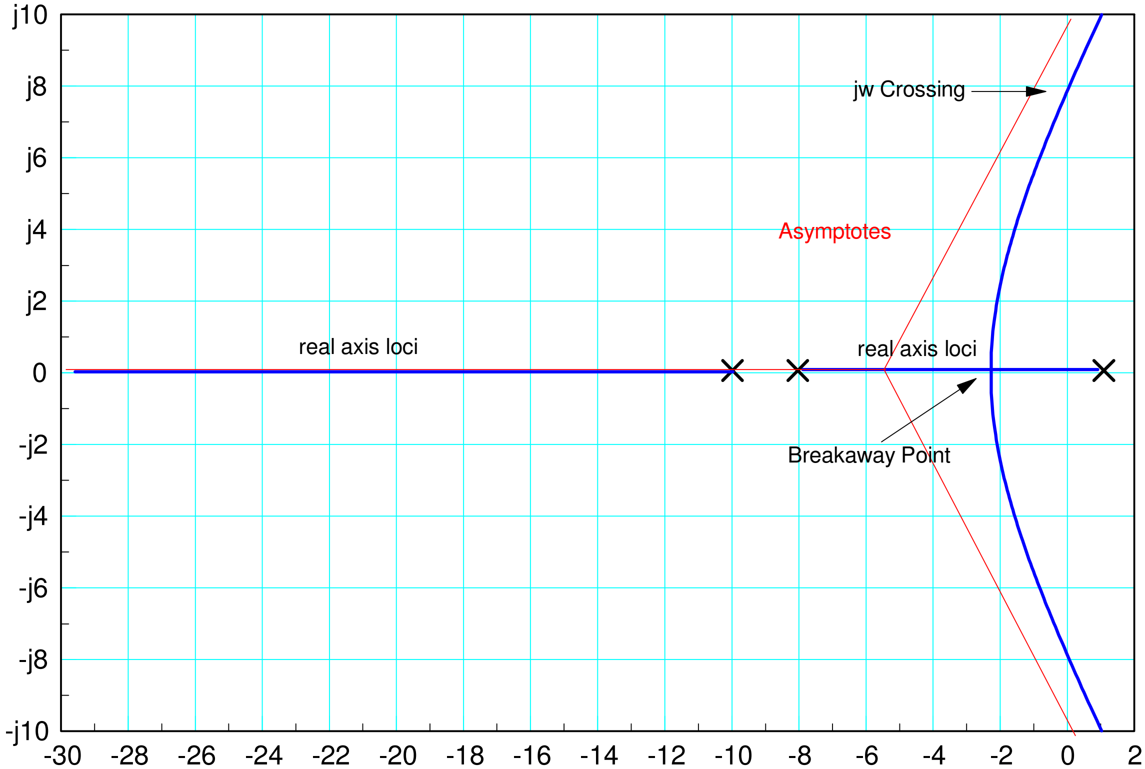
found numerically by searching along the line s = x + j0.1

jw Crossing

s = j7.8739

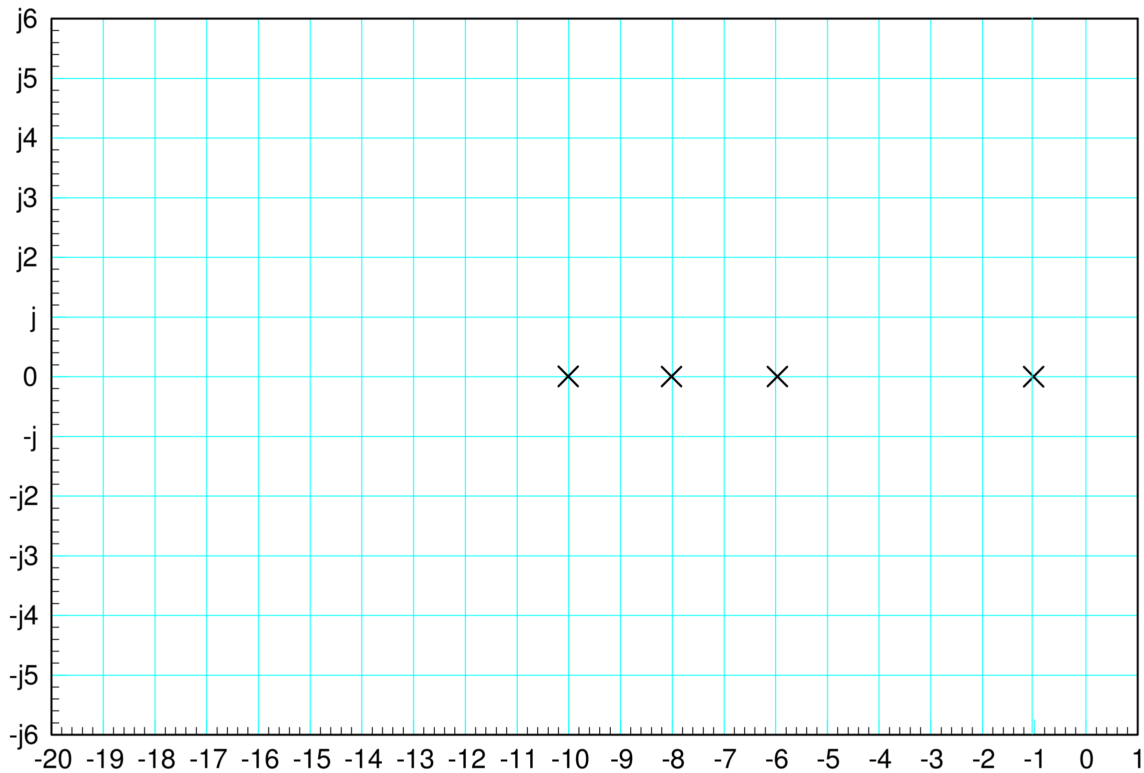
found numerically by searching along the line s = jx

Resulting Root Locus:



j10
j8
j6
j4
j2
0
-j2
-j4
-j6
-j8
-j10
-30
-28
-26
-24
-22
-20
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
jw Crossing
Asymptotes
real axis loci
real axis loci
Breakaway Point

5) $(s+1)(s+6)(s+8)(s+10)+2k=0$



Real Axis Loci

(-1, -6), (-8, -10)

Asymptotes

4 asymptotes

+/- 45 degrees, +/- 135 degrees

intersect = -25/4

Breakaway Points

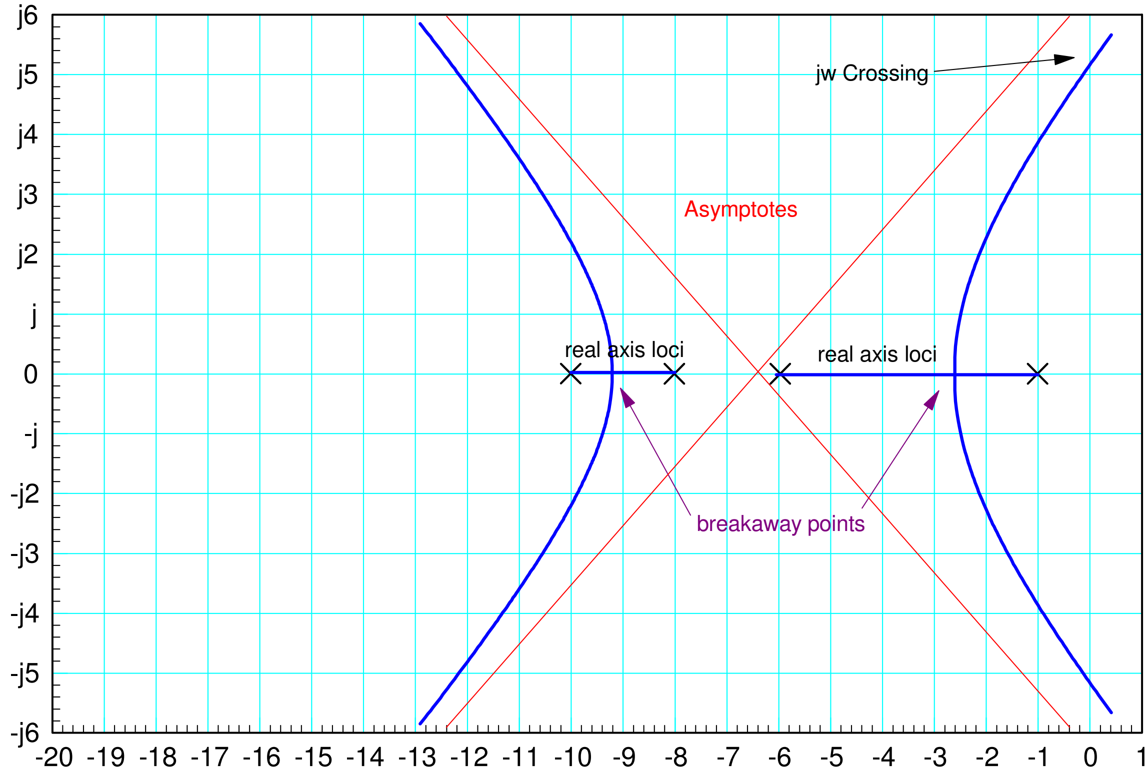
s = -2.6186 *found numerically by searching along the line s = x + j0.1*

s = -9.2094

jw Crossing

s = j5.1691 *found numerically by searching along the line s = jx*

Resulting root locus



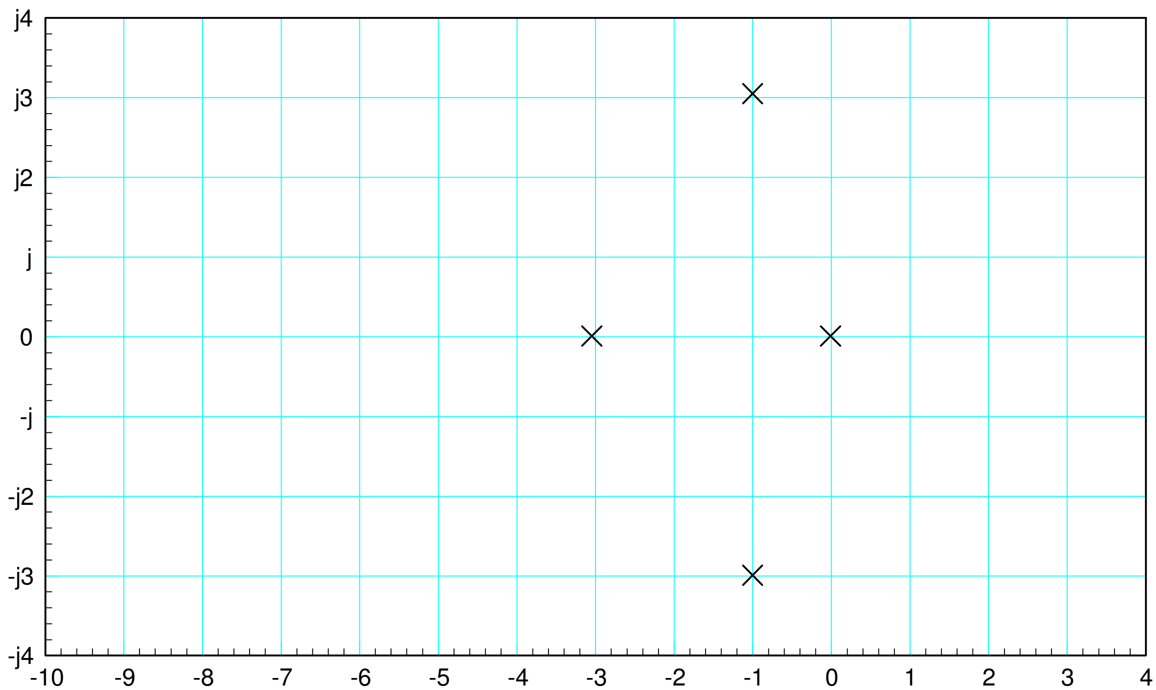
jw Crossing
Asymptotes
real axis loci
real axis loci
breakaway points
j6
j5
j4
j3
j2
j
0
-j
-j2
-j3
-j4
-j5
-j6
-20 -19 -18 -17 -16 -15 -14 -13 -12 -11 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1

Root Locus with Complex Poles & Zeros

Sketch the root locus plot for the following systems for 0 < k < infinity. Also plot the

- real axis loci, break away points, jw crossings (if any), asymptotes, and departure/approach angle

6) $$G(s) = \left(\frac{10}{s(s+3)(s+1+j3)(s+1-j3)}\right)$$



Real Axis Loci

(0, -3)

Asymptotes

4 asymptotes

+/- 45 degrees, +/- 135 degrees

Intersect = -5/4

Departure Angle

-74.74 degrees

evaluate G(s) at s = -0.9999 + j3 (angle from the pole at -1+j3 is zero degrees)

resulting angle = 105.2591 degrees

you need to add another 74.7049 degrees to get to 180 degrees (subtract a negagive 74.74 deg)

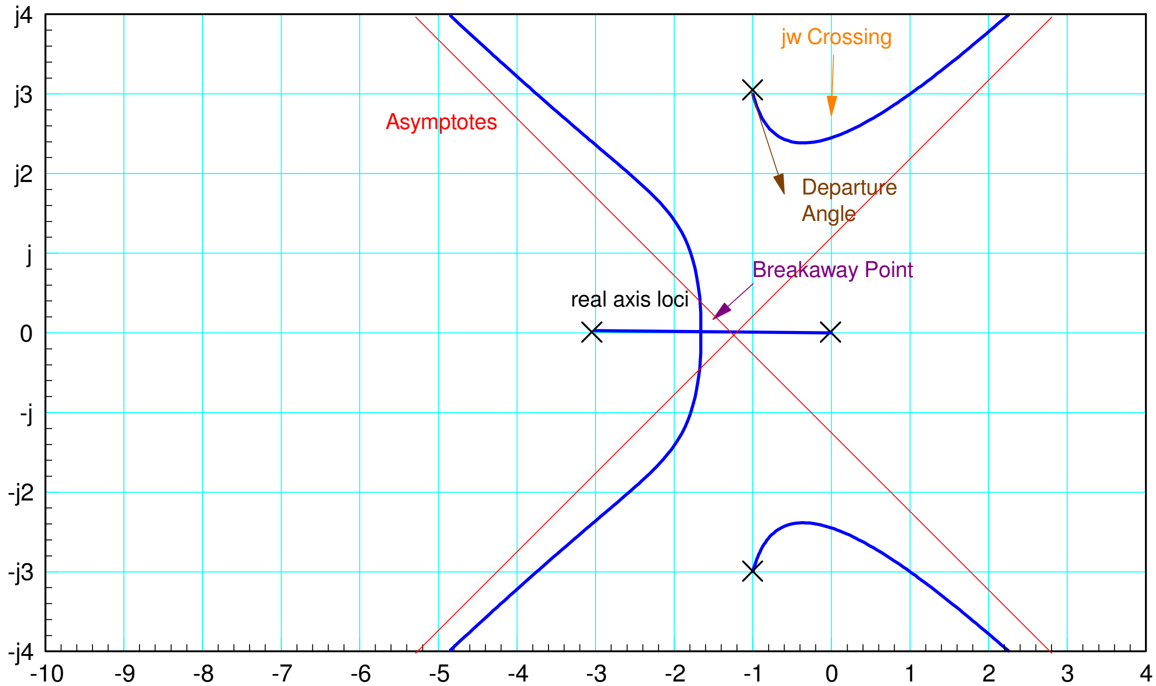
Breakaway Point

s = -1.6556 *found using numerical methods by searching along the line s = x + j0.1*

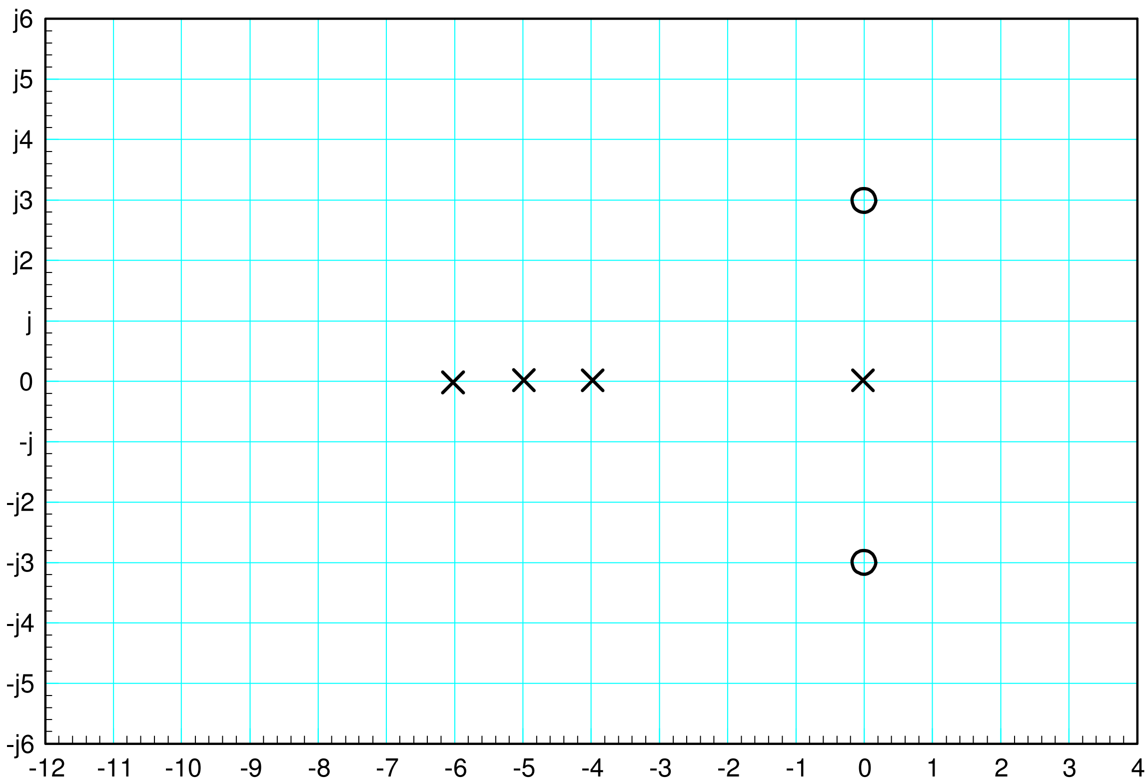
jw Crossing

s = j2.4495 *found using numerical methods by searching along the line s = jx*

Resulting Root Locus



7) $G(s) = \left(\frac{(s+j3)(s-j3)}{s(s+4)(s+5)(s+6)} \right)$



Real Axis Loci

(0, -4), (-5, -6)

Asymptotes

4 poles - 2 zeros = 2 asymptotes

Angle = +/- 90 derees

Intercept = -15/2

Breakaway Point

s = -1.0099 *found numerically by searching along the line s = x + j0.1*

jw Crossing

s = j2.8284 *found numerically by searching along the line s = jx*

Approach Angle

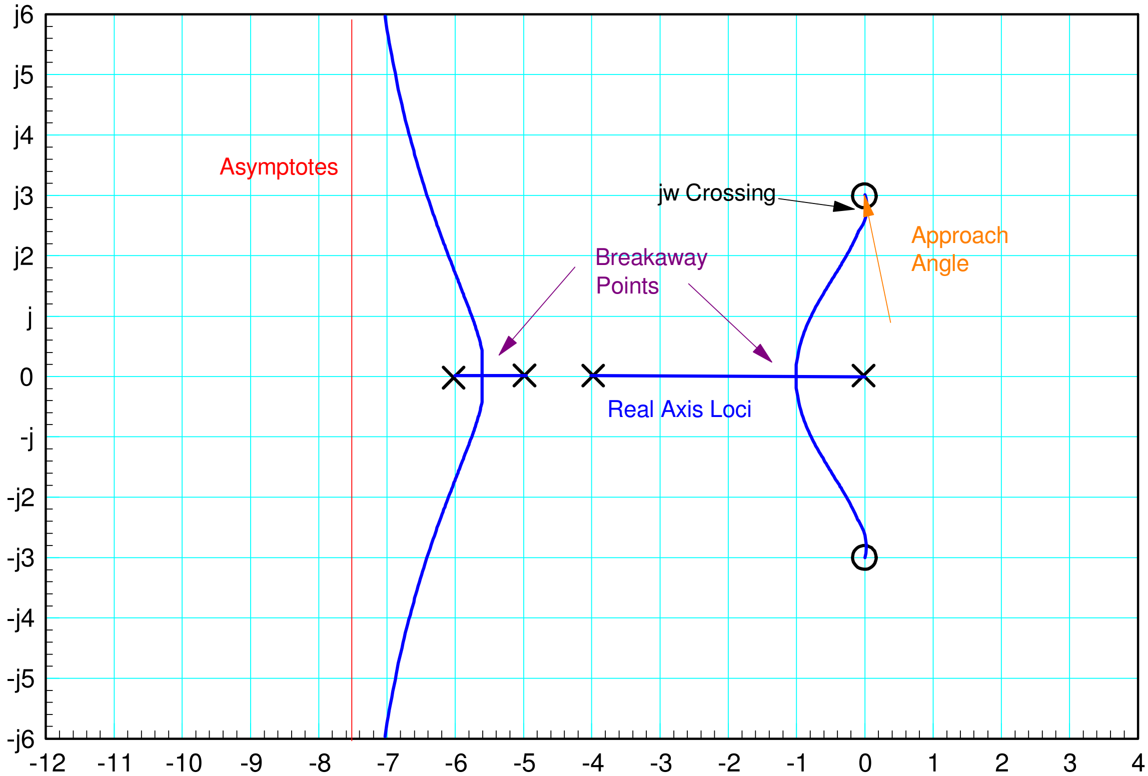
-85.6038 degrees

evaluate G(s) at s = 0.0001 + j3 (angle from zero at +j3 is zero degrees)

resulting angle is -94.3962 degrees

you need to subtract another 85.6038 degrees to make the net angle 180 degrees

Net Result



j6
j5
j4
j3
j2
j
0
-j
-j2
-j3
-j4
-j5
-j6
-12
-11
-10
-9
-8
-7
-6
-5
-4
-3
-2
-1
0
1
2
3
4
Asymptotes
jw Crossing
Approach
Angle
Breakaway
Points
Real Axis Loci