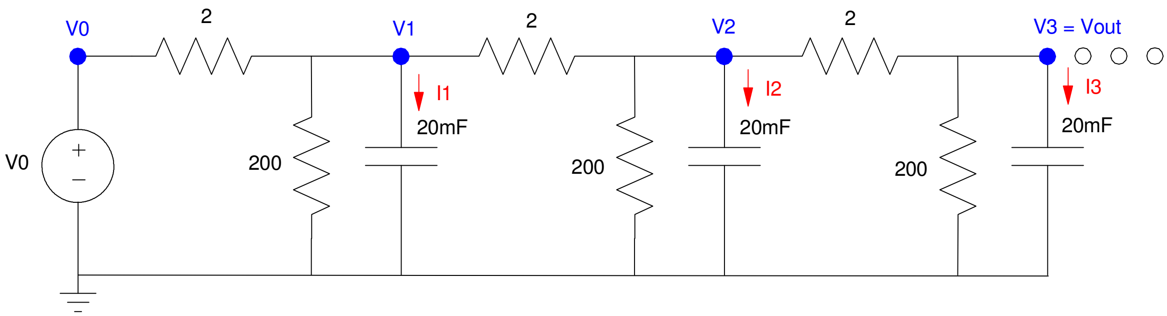
Homework #8: ECE 461/661

Gain, Lead, PID Compensation. Due Monday, October 17th

A 4th-order model for the following 10-stage RC filter is

$$G(s) = \left(\frac{903}{(s+0.81)(s+5.20)(s+13.59)(s+25.25)}\right)$$



1) Design a gain compensator (K(s) = k) which results in

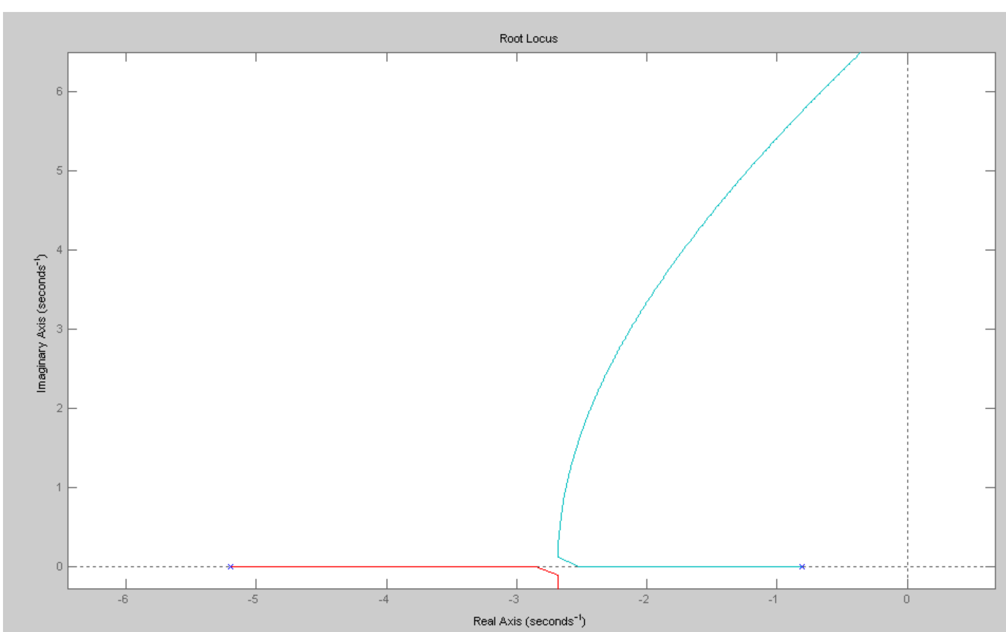
- The fastest system possible,
- With no overshoot for a step input (i.e. design for the breakaway point)

For this value of k, determine

- The closed-loop dominant pole(s)
- The 2% settling time,
- The error constant, Kp, and
- The steady-state error for a step input.

Check your design in Matlab or Simulink or VisSim

Translation: Place the closed-loop dominant pole at the breakaway point



The design point on the root locus is:

```
>> s = -2.682

s =   -2.6820
```

The gain, k, that puts you there

```
>> evalfr(G, s)

ans =   -0.7782

>> k = 1/abs(ans)

k =    1.2850
```

The resulting closed-loop system:

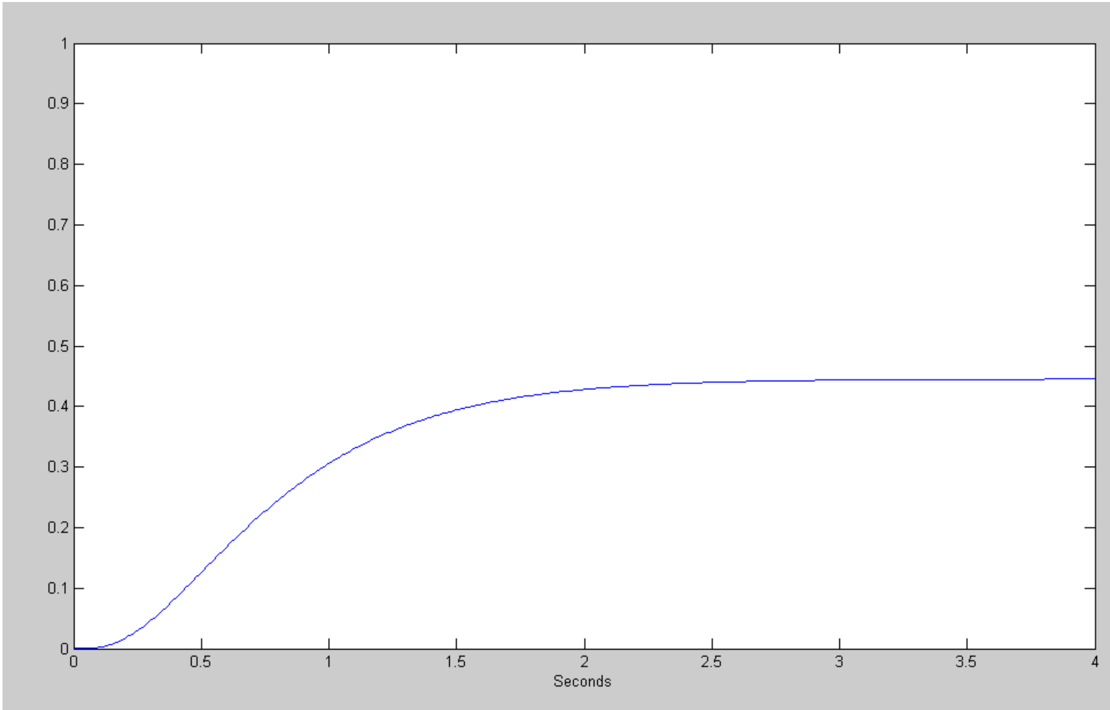
```
>> Gcl = minreal(G*k / (1+G*k))

               1160.3788
---------------------------------------
(s+2.682) (s+2.687) (s+14.44) (s+25.04)
```

Note that the dominant pole is where we placed it. The stats for the closed-loop system are then

```
>> Kp = evalfr(G*k, 0)

Kp =    0.8028

>> Estep = 1/(Kp+1)

Estep =    0.5547

>> Ts = 4/2.682

Ts =    1.4914
```

These match the step response of the closed-loop system

```
>> t = [0:0.01:4]';
>> y = step(Gcl, t);
>> plot(t,y);
>> xlabel('Seconds');
>> ylim([0,1])
```



Step response of the closed-loop system when k places the dominant pole at s = -2.6820

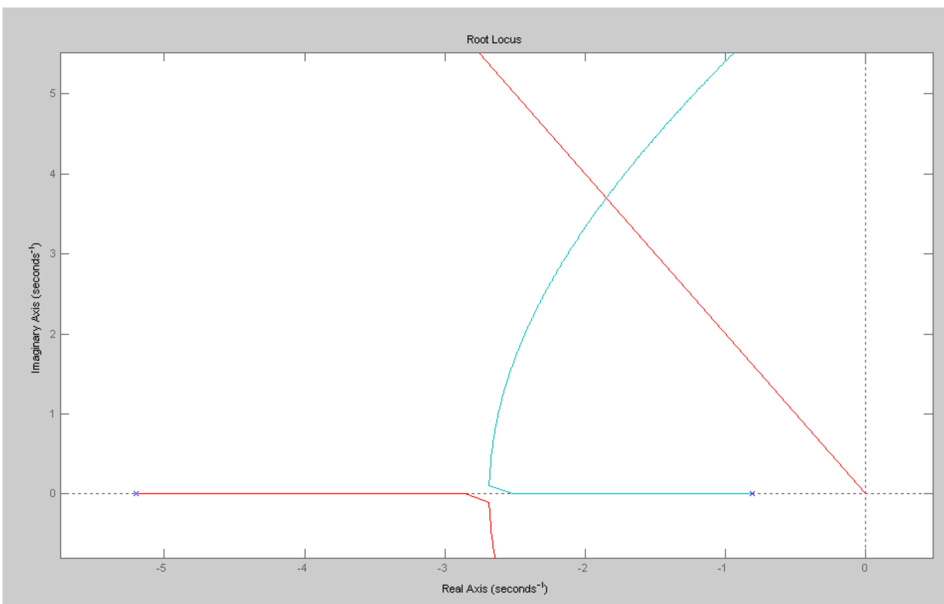
2) Design a gain compensator (K(s) = k) which results in 20% overshoot for a step input. For this value of k, determine

- The closed-loop dominant pole(s)
- The 2% settling time,
- The error constant, Kp, and
- The steady-state error for a step input.

Check your design in Matlab or Simulink or VisSim

Translation: Place the closed-loop dominant poles on the zeta = 0.4559 damping line

```
>> k = logspace(-2,2,1000)';
>> rlocus(G,k);
>> hold on
>> plot([0,-4],[0,8],'r')
>>
```



Root-Locus for G(s)

Pick the point on the root locus that meets the design specs (20% overshoot)

```
>> s = -1.85 + j*3.70;
```

Check: The angle of G(s) is 180 degrees (or the complex part of G(s) = 0)

```
>> evalfr(G, s)

ans =  -0.1614 + 0.0001i
```

The gain that puts you there is:

```
>> k = 1/abs(ans)

k =    6.1950
```

producing the following closed-loop system:

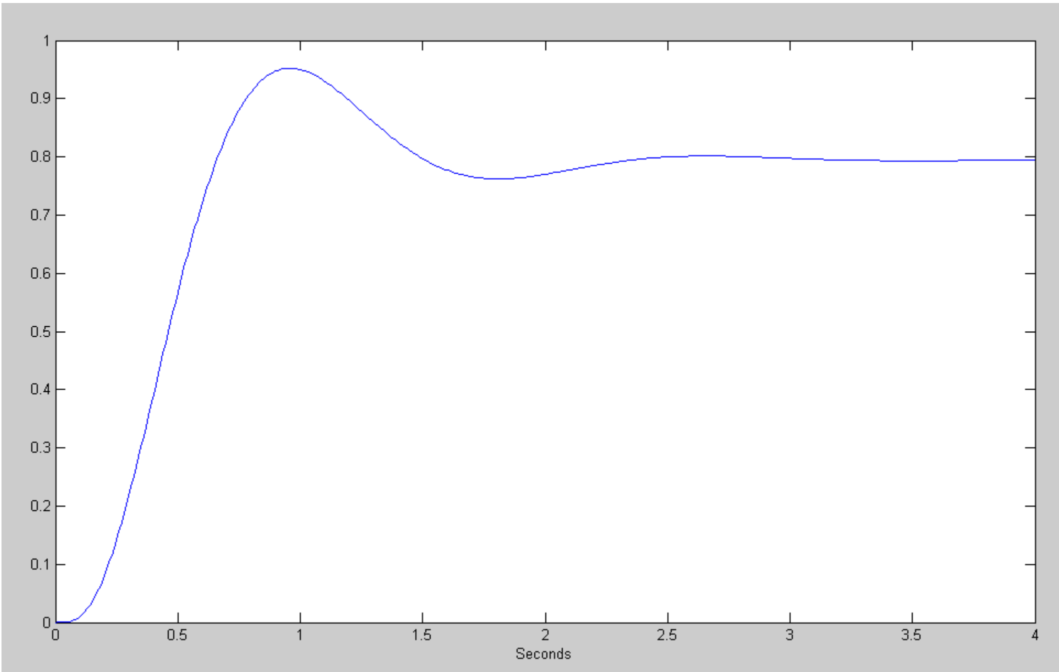
```
>> Gcl = minreal(G*k / (1+G*k))


                  5594.1247
-------------------------------------------
(s+17.13) (s+24.02) (s^2 + 3.699s + 17.11)


>> eig(Gcl)

  -1.8493 + 3.6997i
  -1.8493 - 3.6997i
 -17.1286
 -24.0228
```

The stats for the closed-loop system are:

```
>> Kp = evalfr(G*k, 0)

Kp =    3.8705

>> Estep = 1/(Kp+1)

Estep =    0.2053

>> Ts = 4/1.7493

Ts =    2.2866
```

These match what you get from the step response of the closed-loop system

```
>> t = [0:0.01:4]';
>> y = step(Gcl, t);
>> plot(t,y);
>> xlabel('Seconds');
```



Step Response for G(s) + Gain Compensation

3)) Design a lead compensator, $K(s) = k\left(\frac{s+a}{s+10a}\right)$, which results in 20% overshoot for a step input.

$$G(s) = \left(\frac{903}{(s+0.81)(s+5.20)(s+13.59)(s+25.25)}\right)$$

For this K(s), determine

- The closed-loop dominant pole(s)
- The 2% settling time,
- The error constant, Kp, and
- The steady-state error for a step input.

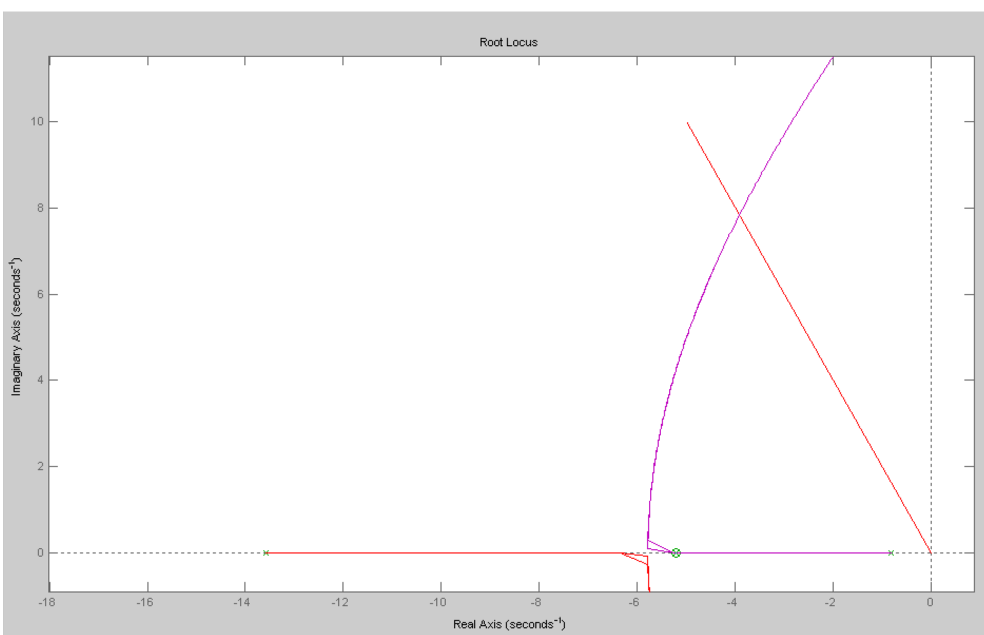
Check your design in Matlab or Simulink or VisSim

Give an op-amp circuit to implement K(s)

Let

$$K(s) = k\left(\frac{s+5.20}{s+52}\right)$$

```
>> K = zpk(-5.2,-52,1)

(s+5.2)
-------
(s+52)

>> GK = G*K;
>> k = logspace(-2,3,1000)';
>> rlocus(GK,k);
>> hold on
>> plot([0,-5],[0,10],'r')
```



Root-Locus for G(s) + Lead Compensator

From the root locus, the design point for 20% overshoot is

```
>> s = 3.9142 * (-1+j*2)

s =  -3.9142 + 7.8284i
```

Checking that this point is on the root locus (angles sum to 180 degrees or complex part of G(s) = 0)

```
>> evalfr(GK, s)

ans =  -0.0078 + 0.0000i
```

k is then the gain required to make |GK| = 1

```
>> k = 1/abs(ans)

k =  128.5172
```

which gives K(s) and the resulting closed-loop system:

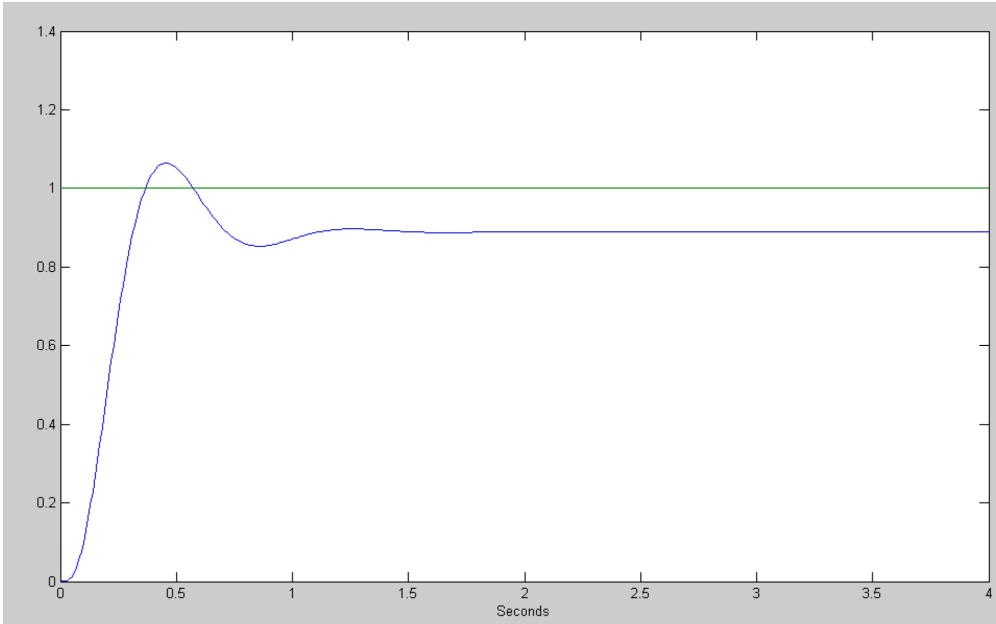
```
>> K = zpk(-5.2,-52,k)

128.5172 (s+5.2)
----------------
     (s+52)

>> GKcl = minreal(G*K / (1+G*K))

               116051.0708
-----------------------------------------
(s+34.64) (s+49.18) (s^2 + 7.828s + 76.6)

>> eig(GKcl)

  -3.9142 + 7.8284i
 -34.6378
 -49.1839
  -3.9142 - 7.8284i
```

The stats for the resulting closed-loop system are then:

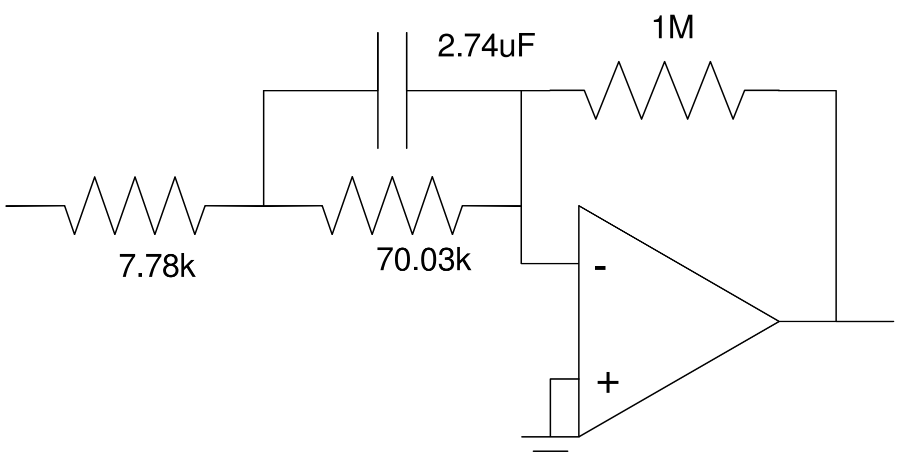
```
>> Kp = evalfr(G*K,0)

Kp =    8.0293

>> Estep = 1/(Kp+1)

Estep =    0.1108

>> Ts = 4/3.9142

Ts =    1.0219
```

The step response of the closed-loop system is:

```
>> t = [0:0.01:4]';
>> y = step(GKcl, t);
>> plot(t,y);
>> xlabel('Seconds');
```



Step response of the lead-compensated system



$$-128.51\left(\frac{s+5.2}{s+52}\right)$$

I Compensation

$$G(s) = \left(\frac{903}{(s+0.81)(s+5.20)(s+13.59)(s+25.25)} \right)$$

4) Design an I compensator, $K(s) = \frac{I}{s}$, which results in 20% overshoot for a step input. For this K(s), determine

- The closed-loop dominant pole(s)
- The 2% settling time,
- The error constant, Kp, and
- The steady-state error for a step input.

Check your design in Matlab or Simulink or VisSim

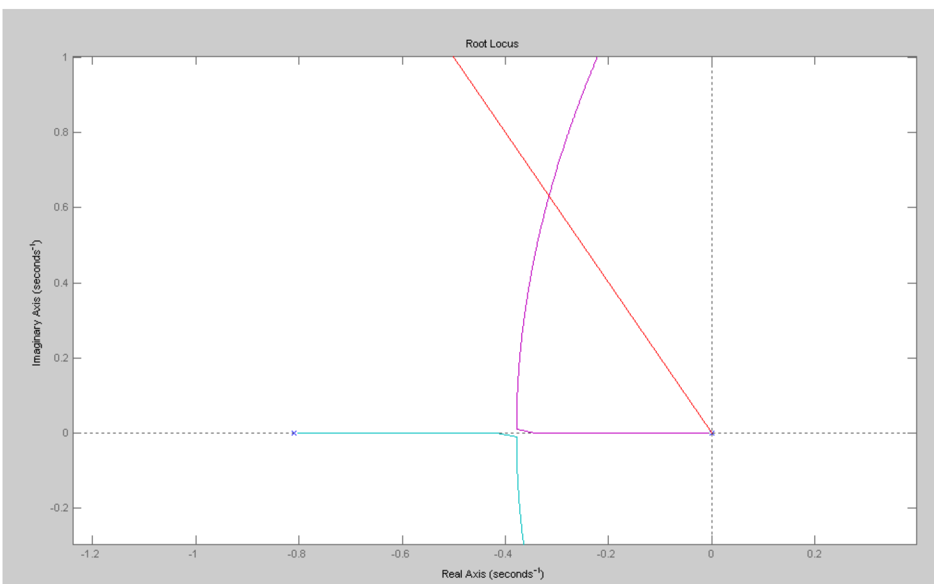
Give an op-amp circuit to implement K(s)

Start with the form for K(s)

```
>> K = zpk([],0,1)

1
-
s
```

Plot the root locus for G*K

```
>> k = logspace(-2,2,1000)';
>> rlocus(G*K,k);
>> hold on
>> plot([0,-2],[0,4],'r');
```



Root-Locus for G(s) + I compensator

From the root locus plot, the point that meets the design specs (20% overshoot) is

```
>> s = -0.3155 + j*0.6310;
```

7The gain, k, sets the gain of |GK|=1 at the design point

```
>> evalfr(G*K,s)

ans =  -0.9780 + 0.0002i

>> k = 1/abs(ans)

k = 1.0225

>> K = zpk([],0,k)

1.0225
------
  s
```

The closed-loop system is then

```
>> Gcl = minreal(G*K / (1+G*K))

                       923.3101
-------------------------------------------------------
(s+5.428) (s+13.54) (s+25.26) (s^2 + 0.6308s + 0.4976)
```

Note that one of the closed-loop poles is where we placed it:

```
>> eig(Gcl)

  -0.3154 + 0.6310i
  -0.3154 - 0.6310i
  -5.4277
 -13.5351
 -25.2564
```

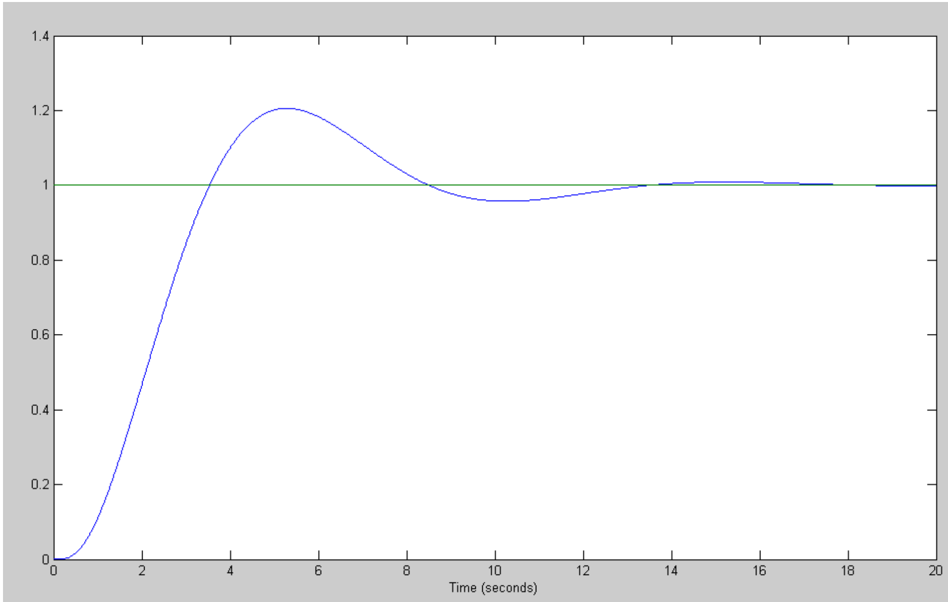
The stats for the closed-loop system are then:

```
>> Ts = 4/0.3154

Ts =   12.6823
```

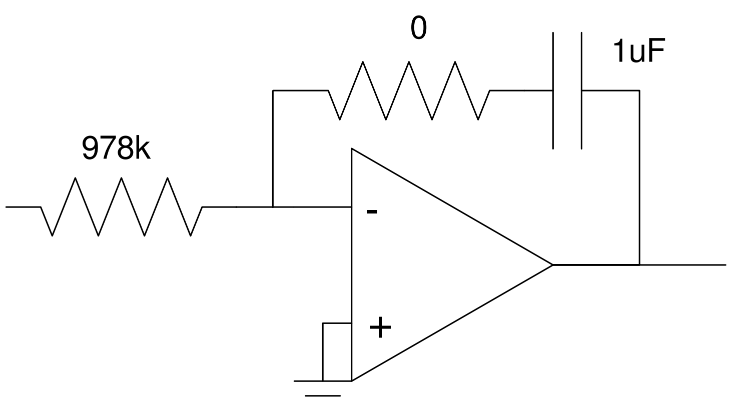
(along with no error for a step input (type-1 system))

This matches with the step response of the closed-loop system

```
>> t = [0:0.01:20]';
>> y = step(Gcl, t);
>> plot(t,y,t,0*y+1)
>> xlabel('Time (seconds)');
```



Step response of the I-compensated system



$$K(s) = -\left(\frac{1.0225}{s}\right)$$

PI Compensation

$$G(s) = \left(\frac{903}{(s+0.81)(s+5.20)(s+13.59)(s+25.25)} \right)$$

5) Design a PI compensator, $K(s) = k(\frac{s+a}{s})$, which results in 20% overshoot for a step input. For this K(s), determine

- The closed-loop dominant pole(s)
- The 2% settling time,
- The error constant, Kp, and
- The steady-state error for a step input.

Check your design in Matlab or Simulink or VisSim

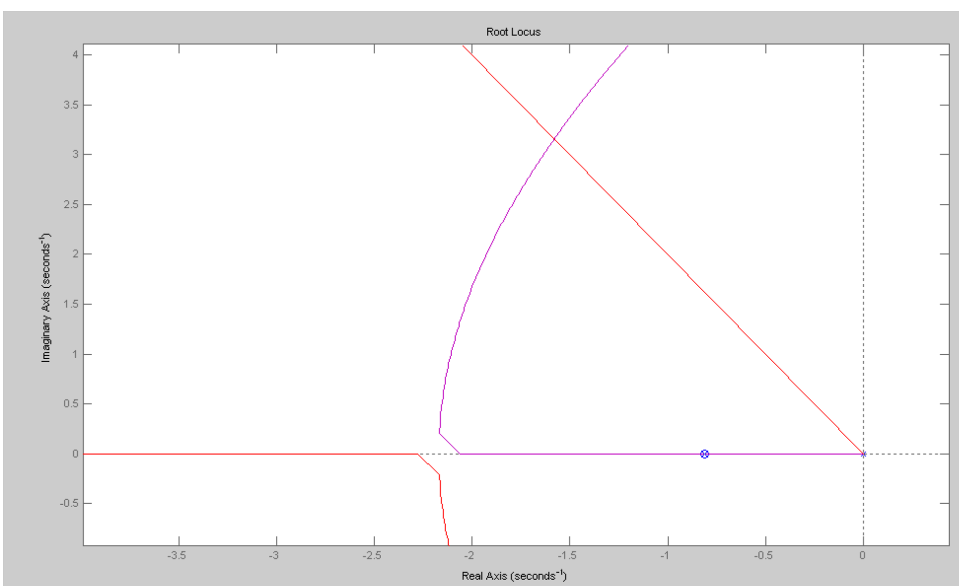
Give an op-amp circuit to implement K(s)

Pick the zero to cancel the slowest stable pole

```
>> K = zpk(-0.81,0,1)

(s+0.81)
--------
   s
```

Find k to place the closed-loop at the point on the root locus that meets the design specs (20% OS)

```
>> k = logspace(-2,2,1000)';
>> rlocus(G*K,k);
>> hold on
>> plot([0,-3],[0,6],'r');
```



Root-locus for G(s) + PI compensator

From the root locus plot

```
>> s = 1.5793 * (-1+j*2)

s =  -1.5793 + 3.1586i
```

The gain, k, that puts you there is

```
>> evalfr(G*K,s)

ans =  -0.1794 + 0.0000i

>> k = 1/abs(ans)

k =    5.5728
```

Giving the resulting K(s) and closed-loop system:

```
>> K = zpk(-0.81,0,k)

5.5728 (s+0.81)
---------------
       s

>> Gcl = minreal(G*K / (1+G*K))

                 5032.2161
------------------------------------------
(s+16.66) (s+24.22) (s^2 + 3.159s + 12.47)
```

Note that the dominant pole is where we placed it:

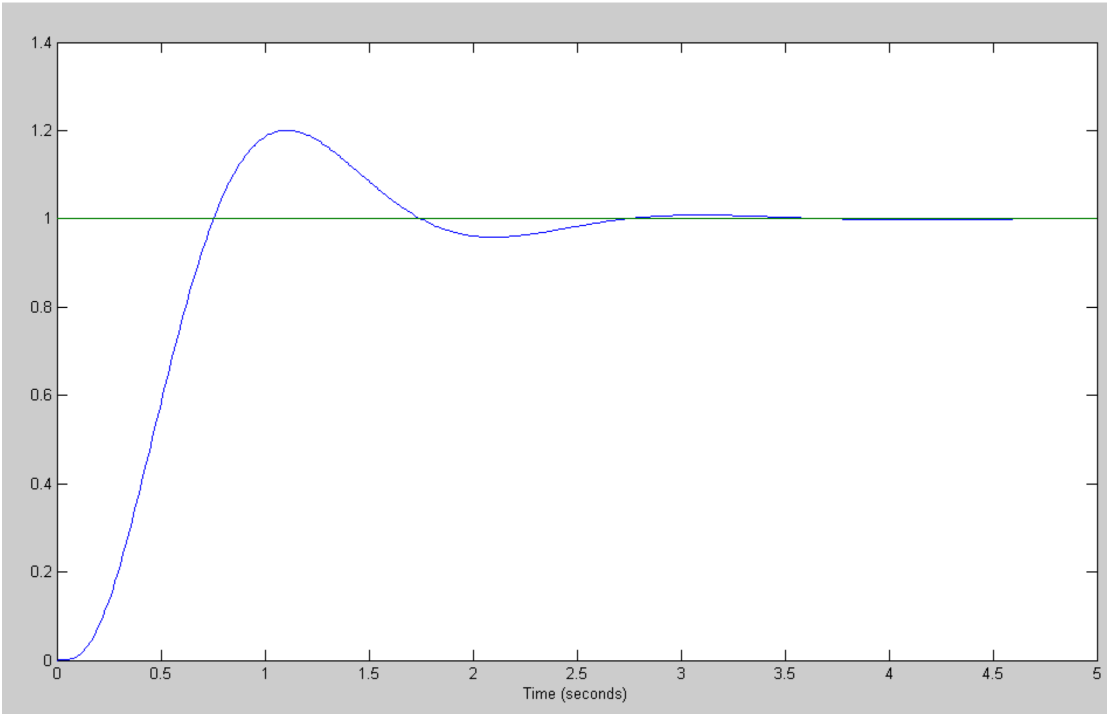
```
>> eig(Gcl)

  -1.5793 + 3.1586i
  -1.5793 - 3.1586i
 -16.6585
 -24.2229
```

The resulting system then has no error for a step input (type 1), 20% overshoot, and a 2% settling time of

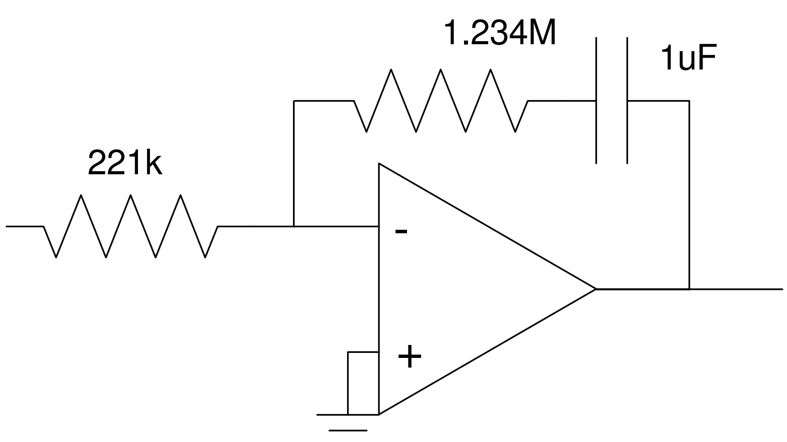
```
>> Ts = 4/1.5793

Ts =    2.5328
```

This matches the step response of the closed-loop system

```
>> t = [0:0.01:5]';
>> y = step(Gcl, t);
>> plot(t,y,t,0*y+1)
>> hold off
>> plot(t,y,t,0*y+1)
>> xlabel('Time (seconds)')
>> OS = max(y) - 1

OS =    0.2004
```



Step resose for G(s) + PI compensator



$$-5.5728\left(\frac{s+0.81}{s}\right)$$