Homework #9: ECE 461/661

Meeting Specs, Delays, Unstable Systems. Due Monday, October 24th
20 points per problem

Meeting Design Specs

1) Assume

$$G(s) = \left(\frac{903}{(s+0.81)(s+5.20)(s+13.59)(s+25.25)}\right)$$

Design a compensator, K(s), For the 4th-order model that results in

- No error for a step input
- A 2% settling time of 2 seconds, and
- 20% overshoot for the step response

Translation:

Make it a type-1 system

Place the closed-loop dominant pole at s = -2 + j4

Compensator Design:

Let K(s)

- Add two zeros, cancelling the slowest stable poles
- Add a pole at s = 0 to make it type-1, and
- Add another pole to push the root locus through s = -2 + j4

$$K(s) = k\left(\frac{(s+0.81)(s+5.20)}{s(s+a)}\right)$$

giving

$$GK = \left(\frac{903k}{s(s+13.59)(s+25.25)(s+a)}\right)$$

Analyze what we know

$$\left(\frac{903}{s(s+13.59)(s+25.25)}\right)_{s=-2+j4} = 0.6981\angle -145.3677^0$$

For the angles to add up to 180 degrees,

$$\angle\left(\frac{1}{s+a}\right) = -34.6323^0$$

$$\angle(s+a) = 34.63.23^0$$

Doing some trig

$$a = \left(\frac{4}{\tan(34.6323^0)}\right) + 2$$

$$a = 7.7913$$

So, K(s) and GK are

$$K(s) = k\left(\frac{(s+0.81)(s+5.20)}{s(s+7.7913)}\right)$$

$$GK = \left(\frac{930k}{s(s+7.7913)(s+13.59)(s+25.25)}\right)$$

To find k, at any point on the root locus

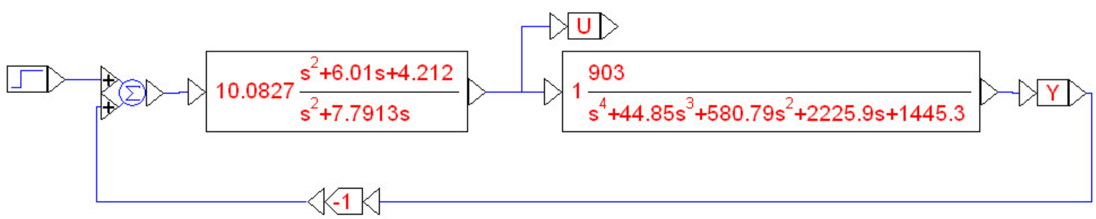
$$GK = -1$$

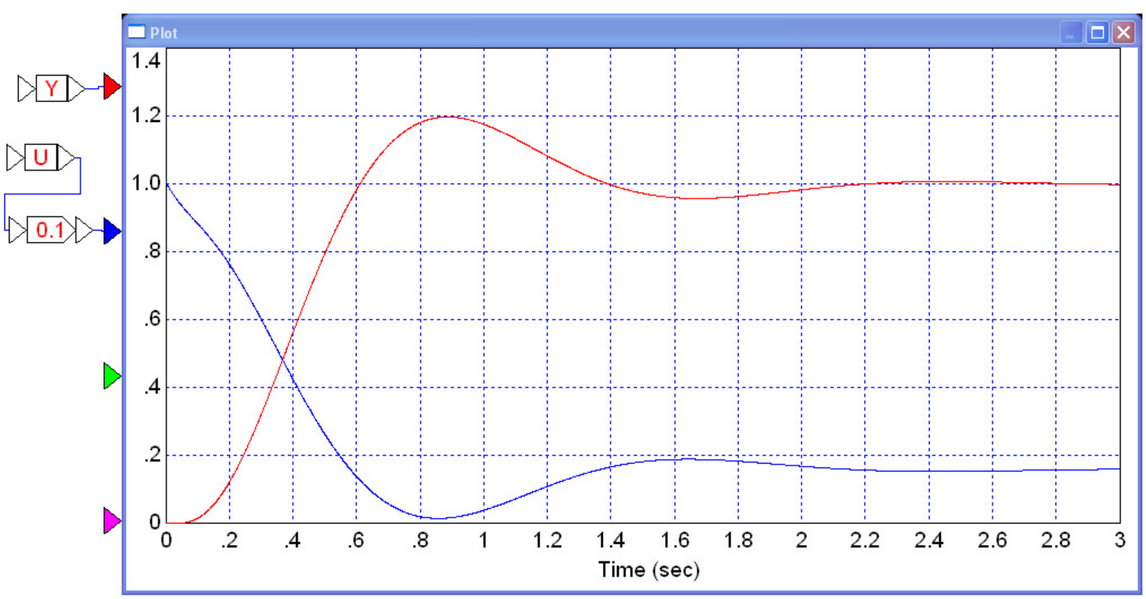
$$\left(\frac{903k}{s(s+7.7913)(s+13.59)(s+25.25)}\right)_{s=-2+j4} = 0.0992k\angle 180^0$$

$$k = \frac{1}{0.0992} = 10.0827$$

$$\boxed{K(s) = 10.0827\left(\frac{(s+0.81)(s+5.20)}{s(s+7.7913)}\right)}$$

Check your design in Matlab or Simulink or VisSim



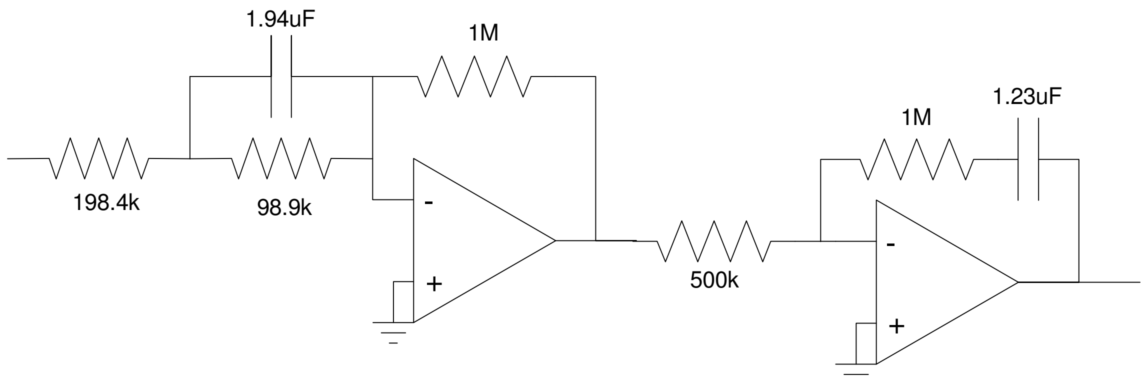


Give an op-amp circuit to implement K(s)

$$K(s) = 10.0827\left(\frac{(s+0.81)(s+5.20)}{s(s+7.7913)}\right)$$

Rewrite as

$$K(s) = 5.0414\left(\frac{s+5.20}{s+7.7913}\right) \cdot 2\left(\frac{s+0.81}{s}\right)$$



Systems with Delays

2) Assume a 100ms delay is added to the system

$$G(s) = \left(\frac{903}{(s+0.81)(s+5.20)(s+13.59)(s+25.25)}\right) e^{-0.1s}$$

Design a compensator, K(s), For the 4th-order model that results in

- No error for a step input
- A 2% settling time of 2 seconds, and
- 20% overshoot for the step response

Same as problem #1. Let

$$K(s) = k\left(\frac{(s+0.81)(s+5.20)}{s(s+a)}\right)$$

$$GK = \left(\frac{903k}{s(s+a)(s+13.59)(s+25.25)}\right) e^{-0.1s}$$

Evaluate what we know:

$$\left(\left(\frac{903}{s(s+13.59)(s+25.25)}\right) e^{-0.1s}\right)_{s=-2+j4} = 0.8526k\angle -168.2860^0$$

For the angle to add up to 180 degrees

$$\angle(s+a) = 11.7140^0$$

$$a = \left(\frac{4}{\tan\left(11.7140^0\right)}\right) + 2$$

$$a = 21.2915$$

This results in

$$\left(\left(\frac{903k}{s(s+21.2915)(s+13.59)(s+25.25)}\right) e^{-0.1s}\right)_{s=-2+j4} = -1$$

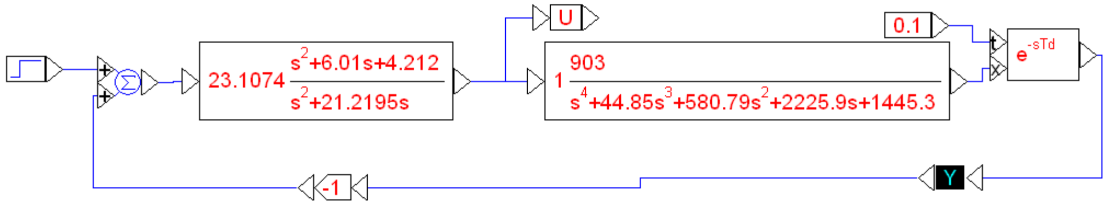
$$0.0433k\angle 180^0 = -1$$

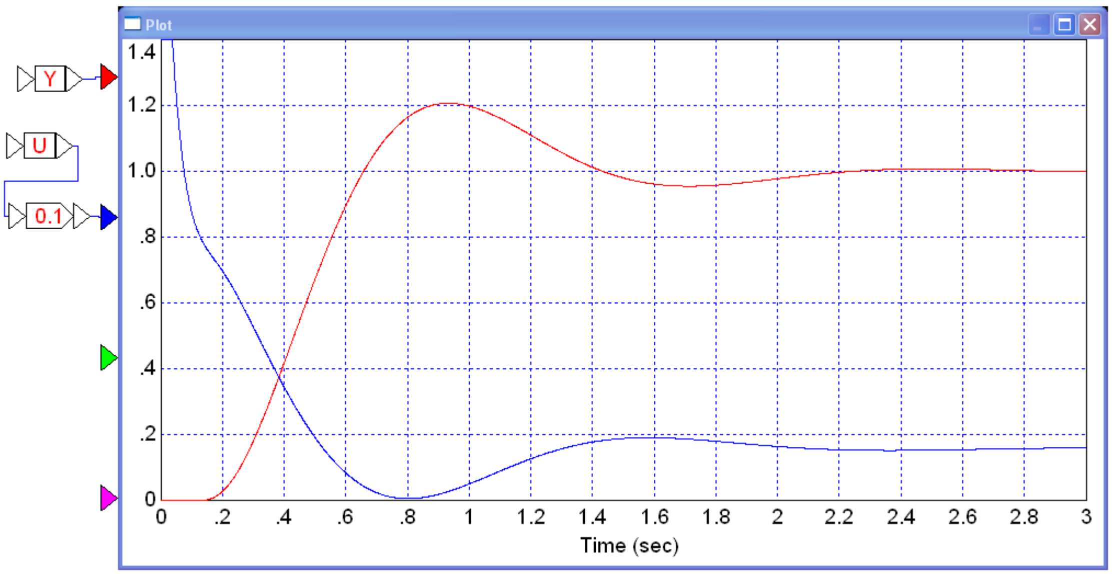
$$k = \frac{1}{0.0433} = 23.1074$$

and

$$K(s) = 23.1074\left(\frac{(s+0.81)(s+5.20)}{s(s+21.2915)}\right)$$

Check your design in Matlab or Simulink or VisSim



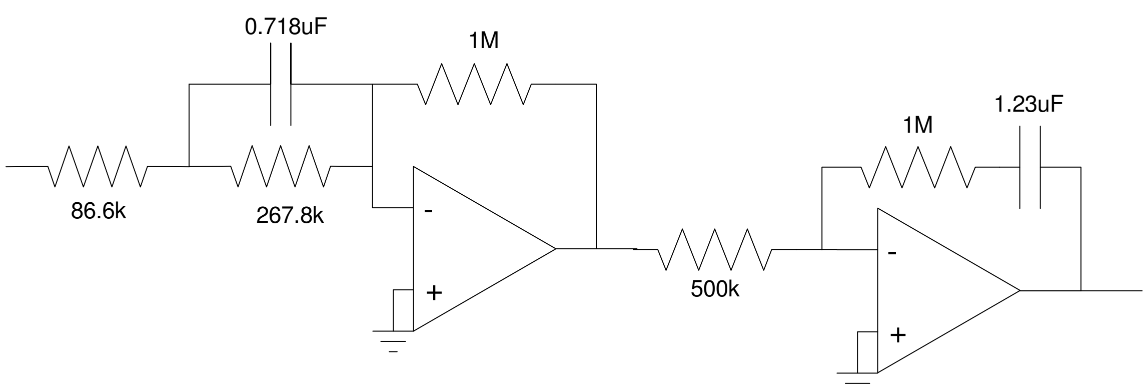


Give an op-amp circuit to implement K(s)

$$K(s) = 23.1074\left(\frac{(s+0.81)(s+5.20)}{s(s+21.2915)}\right)$$

Rewrite as

$$K(s) = 11.5537\left(\frac{s+5.20}{s+21.2915}\right)\cdot 2\left(\frac{s+0.81}{s}\right)$$



Unstable Systems

3) Assume the slow pole was unstable

$$G(s) = \left(\frac{903}{(s-0.81)(s+5.20)(s+13.59)(s+25.25)}\right)$$

Design a compensator, K(s), For the 4th-order model that results in

- No error for a step input
- A 2% settling time of 2 seconds, and
- 20% overshoot for the step response

Check your design in Matlab or Simulink or VisSim

Step 1: Stabilize the system. Let

$$K_1(s) = k$$

Place the closed-loop dominant pole at s = -1 (arbitrary)

$$\left(\frac{903}{(s-0.81)(s+5.20)(s+13.59)(s+25.25)}\right)_{s=-1} = -0.3891$$

$$k = \frac{1}{0.3891} = 2.5073$$

Now find the closed-loop system. In Matlab

```
>> G = zpk([],[0.81,-5.20,-13.59,-25.25],903)

                   903
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(s-0.81) (s+5.2) (s+13.59) (s+25.25)

>> k1 = 2.5703;
>> G2 = minreal(G*k1 / (1 + G*k1))

               2320.9809
-----------------------------------
(s+1) (s+2.343) (s+15.05) (s+24.84)
```

Now design a second feedback system to meet the design specs. Let

$$K_2 = k\left(\frac{(s+1)(s+2.343)}{s(s+a)}\right)$$

$$G_2K_2 = \left(\frac{2320.9809k}{s(s+a)(s+15.05)(s+24.84)}\right)$$

Evaluate what we know at s = -2 + j4

$$\left(\frac{2320.9809}{s(s+15.05)(s+24.84)}\right)_{s=-2+j4} = 1.6398\angle -143.5396^0$$

For the total angle to add up to 180 degrees

$$\angle(s+a) = 36.4604^0$$

$$a = \left(\frac{4}{36.4604^0}\right) + 2 = 7.4135$$

To find k

$$G_2K_2 = \left(\frac{2320.9809k}{s(s+7.4135)(s+15.05)(s+24.84)}\right)_{s=-2+j4} = 0.2436k\angle 180^0$$

k is then

$$k = \frac{1}{0.2436} = 4.1047$$

and

$$\boxed{K_2 = 4.1047\left(\frac{(s+1)(s+2.343)}{s(s+7.4135)}\right)}$$

Checking in VisSim