Homework #11: ECE 461/661

Digital PID Control. Due Monday, November 14th

PID Control

Assume T = 0.1 seconds:

$$G(s) = \left(\frac{903}{(s+0.81)(s+5.20)(s+13.59)(s+25.25)} \right)$$

1) Design a digital I controller

$$K(z) = k\left(\frac{z}{z-1}\right)$$

that results in 20% overshoot in the step response.

Simulate the step response of the closed-loop system (VisSim or Simulink preferred with K(z)*G(s))

Root Locus: Convert G(s) to G(z). From homework #10:

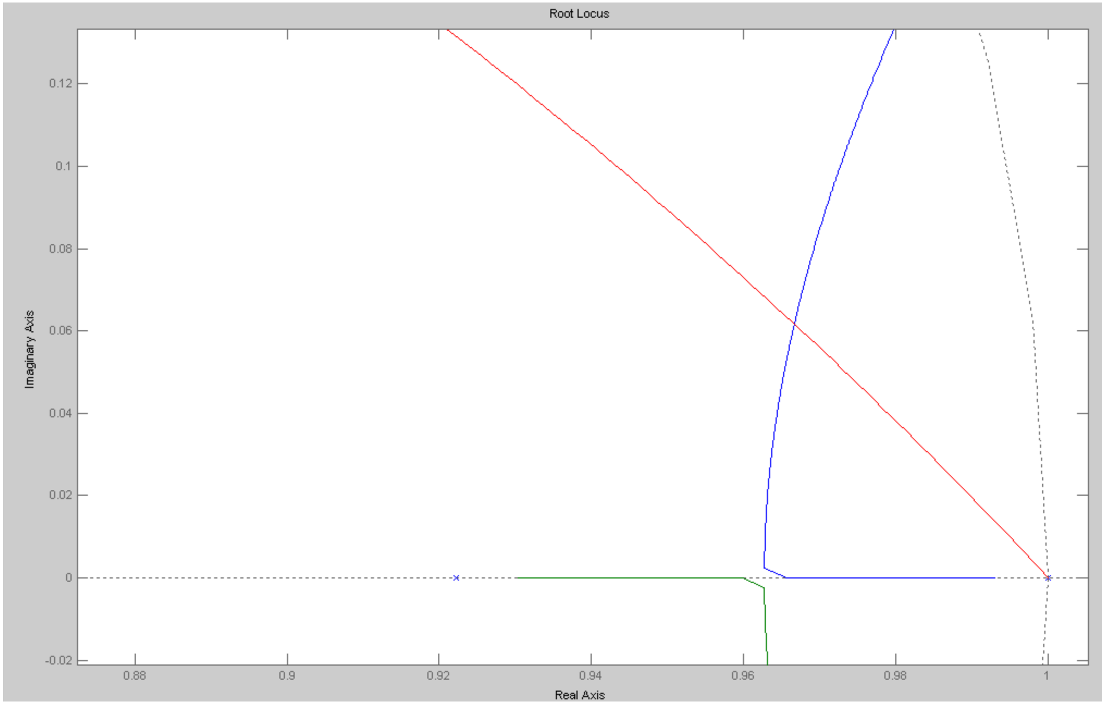
$$G_z(z) \approx \left(\frac{0.0135z^2}{(z-0.9222)(z-0.5945)(z-0.2569)(z-0.0801)} \right)$$

Add K(z)

$$GK \approx \left(\frac{0.0135z^2}{(z-0.9222)(z-0.5945)(z-0.2569)(z-0.0801)} \right)\left(\frac{kz}{z-1}\right)$$

Draw the root locus and find the spot where it intersects the 20% overshoot damping line:

```
>> s = [-0.81,-5.20,-13.59,-25.25]';
>> T = 0.1;
>> Gz = zpk([0,0],exp(s*T),0.0135,0.1)

                 0.0135 z^2
--------------------------------------------
(z-0.9222) (z-0.5945) (z-0.2569) (z-0.08006)

Sampling time (seconds): 0.1

>> Kz = zpk(0,1,1,0.1)

  z
-----
(z-1)

Sampling time (seconds): 0.1

>> GzKz = minreal(Gz*Kz)

                    0.0135 z^3
--------------------------------------------------
(z-0.9222) (z-1) (z-0.5945) (z-0.2569) (z-0.08006)

Sampling time (seconds): 0.1

>> k = logspace(-2,2,1000)';
>> rlocus(GzKz,k);
>> hold on
>> s = [0:0.01:5]' * (-1+j*2);
>> z = exp(s*T);
>> plot(real(z),imag(z),'r')
```



Root Locus for G(z) * K(z)

Find the spot on the root locus that intersects the damping line

```
>> z = 0.9667 + j*0.0617;
>> evalfr(GzKz, z)

ans =  -9.6297 - 0.0071i

>> k = 1/abs(ans)

k =    0.1038
```

and

$$K(z) = 0.1038\left(\frac{z}{z-1}\right)$$

Method #2: At any point on the root locus:

$$G(s) \cdot ZOH \cdot K(z) = -1$$

$$ZOH = e^{-sT/2}$$

$$\left(\frac{903}{(s+0.81)(s+5.20)(s+13.59)(s+25.25)} \right) \cdot e^{-0.05s} \cdot \left(\frac{kz}{z-1} \right) = -1$$

Search along the line s = (-1 + j2) until the angles add up to 180 degrees

```
>> s = 0.1*( -1 + j*2);
>> z = exp(s*T);
>> evalfr(Gs,s) * exp(-s*T/2) * (z / (z-1))

ans = -22.6811 -22.0203i

>> s = s*1.1;
>> z = exp(s*T);
>> evalfr(Gs,s) * exp(-s*T/2) * (z / (z-1))

ans = -21.5057 -19.4167i
```

time passes

```
>> s = s*1.001;
>> z = exp(s*T);
>> evalfr(Gs,s) * exp(-s*T/2) * (z / (z-1))

ans =  -9.7872 - 0.0042i

>> k = 1/abs(ans)

k =    0.1022

s =  -0.3154 + 0.6307i
z =   0.9670 + 0.0611i
```

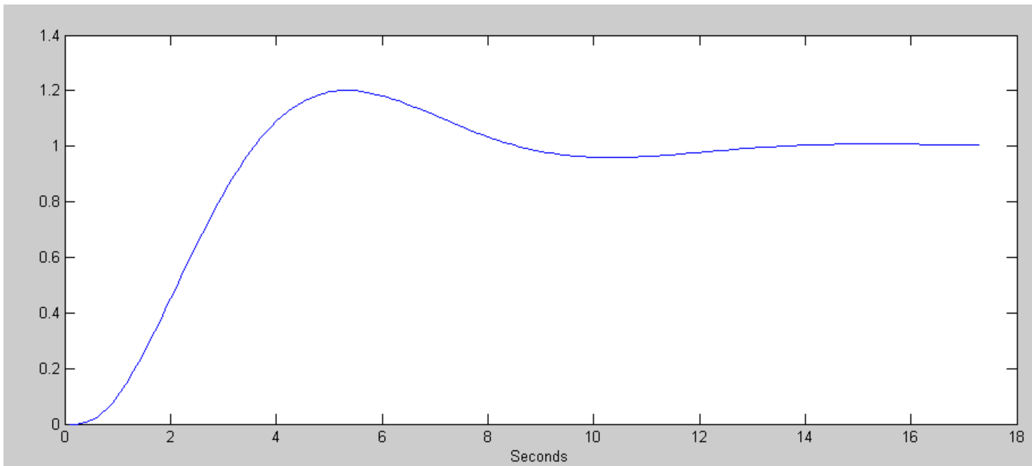
$$K(z) = 0.1022\left(\frac{z}{z-1}\right)$$

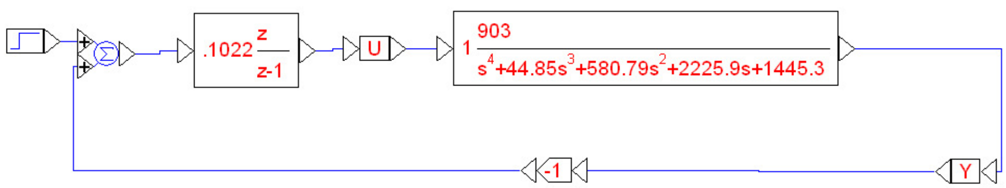
Plot the closed-loop system's step response

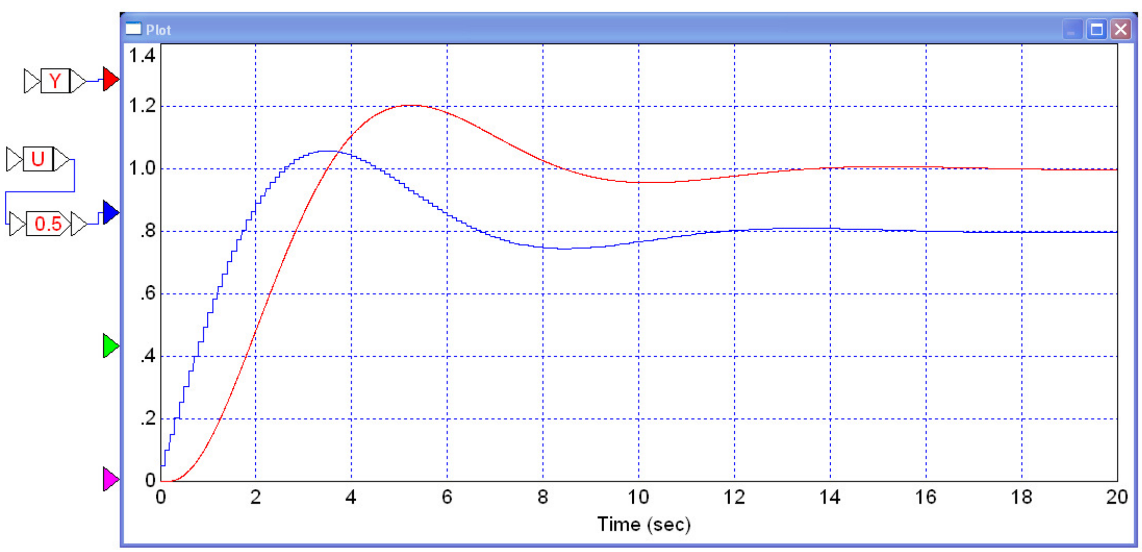
In Matlab

```
>> Kz = zpk(0,1,k,0.1)

0.10217 z
---------
  (z-1)

Sampling time (seconds): 0.1

>> Gclz = minreal(Gz*Kz / (1 + Gz*Kz))


                     0.0013794 z^3
--------------------------------------------------------

(z-0.5828) (z-0.2577) (z-0.08005) (z^2 - 1.933z + 0.938)
```

```
>> y = step(Gclz)

>> size(y)

   173     1

>> t = [1:173] * T;
>> plot(t,y)
>> xlabel('Seconds');
```







2) Assume T = 0.1 seconds and

$$G(s) = \left(\frac{903}{(s+0.81)(s+5.20)(s+13.59)(s+25.25)} \right)$$

Design a digital PI controller

$$K(s) = k\left(\frac{z-a}{z-1}\right)$$

that results in 20% overshoot in the step response.

Simulate the step response of the closed-loop system (VisSim or Simulink preferred with K(z)*G(s))

Using the second method

Pick 'a' to cancel the pole at s = -0.81

$$K(z) = k\left(\frac{z-0.9222}{z-1}\right)$$

Search until the angles add up to 180 degrees

$$G(s) \cdot e^{-sT/2} \cdot K(z) = -1$$

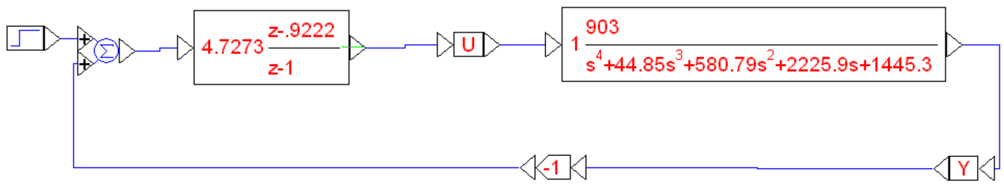
$$\left(\frac{903}{(s+0.81)(s+5.20)(s+13.59)(s+25.25)} \right) \cdot \exp\left(\frac{-sT}{2}\right) \cdot k\left(\frac{z-0.9222}{z-1}\right) = -1$$

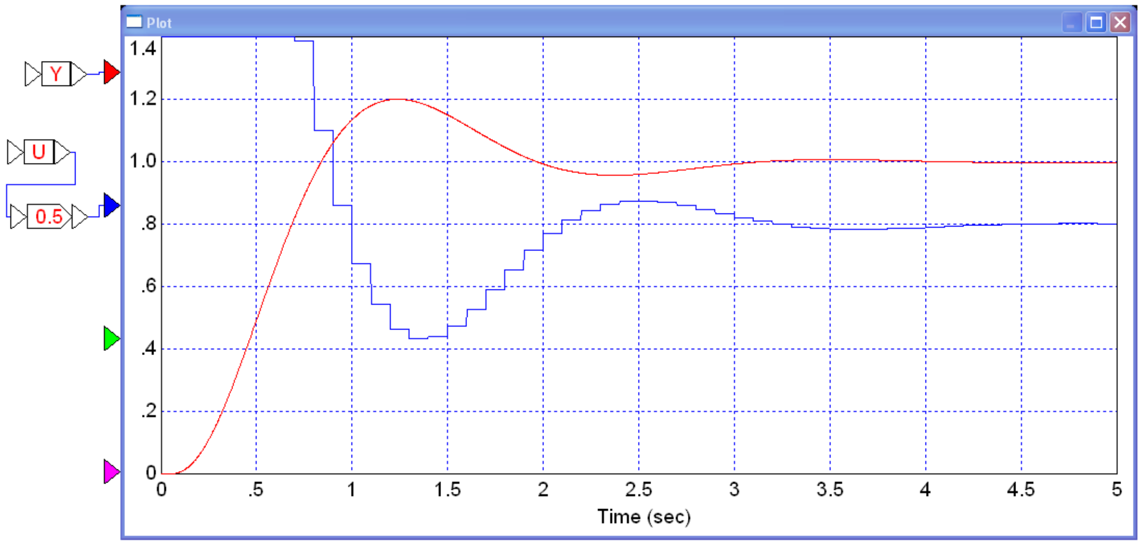
Itterate until you find s

```
>> s = -1 + j*2;
>> z = exp(s*T);
>> evalfr(Gs,s) * exp(-s*T/2) * ((z-0.9222) / (z-1))

ans =  -0.2677 - 0.0900i

>> s = s * 1.1;
>> z = exp(s*T);
>> evalfr(Gs,s) * exp(-s*T/2) * ((z-0.9222) / (z-1))

ans =  -0.2527 - 0.0621i
:
:
:
>> s = s * 1.01;
>> z = exp(s*T);
>> evalfr(Gs,s) * exp(-s*T/2) * ((z-0.9222) / (z-1))

ans =  -0.2115 - 0.0008i

>> k = 1/abs(ans)

k =    4.7273

s =  -1.3850 + 2.7701i

z =   0.8375 + 0.2381i
```

which tells you that

$$K(z) = 4.7273\left(\frac{z-0.9222}{z-1}\right)$$


4.7273
z-.9222
z-1
U
1
903
s^4+44.85s^3+580.79s^2+2225.9s+1445.3
Y
-1


Plot
Y
U
0.5
1.4
1.2
1.0
.8
.6
.4
.2
0
0
.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Time (sec)

3) Assume T = 0.1 seconds and

$$G(s) = \left(\frac{903}{(s+0.81)(s+5.20)(s+13.59)(s+25.25)}\right)$$

Design a digital PID controller

$$K(z) = k\left(\frac{(z-a)(z-b)}{z(z-1)}\right)$$

that results in 20% overshoot in the step response.

Pick 'a' and 'b' to cancel the poles at s = {-0.81, -5.20}

$$K(z) = k\left(\frac{(z-0.9222)(z-0.5945)}{z(z-1)}\right)$$

Using the second method, find the point where

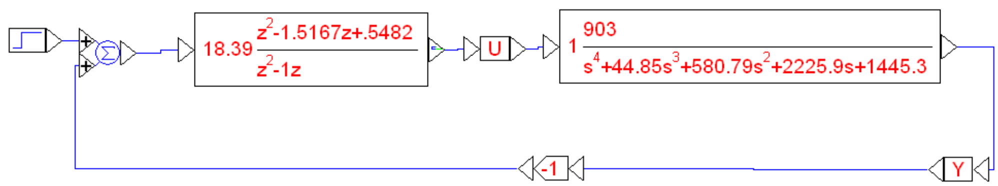
$$G(s) \cdot \exp\left(\frac{-sT}{2}\right) \cdot K(z) = -1$$

$$\left(\frac{903}{(s+0.81)(s+5.20)(s+13.59)(s+25.25)}\right) \cdot \exp\left(\frac{-sT}{2}\right) \cdot k\left(\frac{(z-0.9222)(z-0.5945)}{z(z-1)}\right) = -1$$

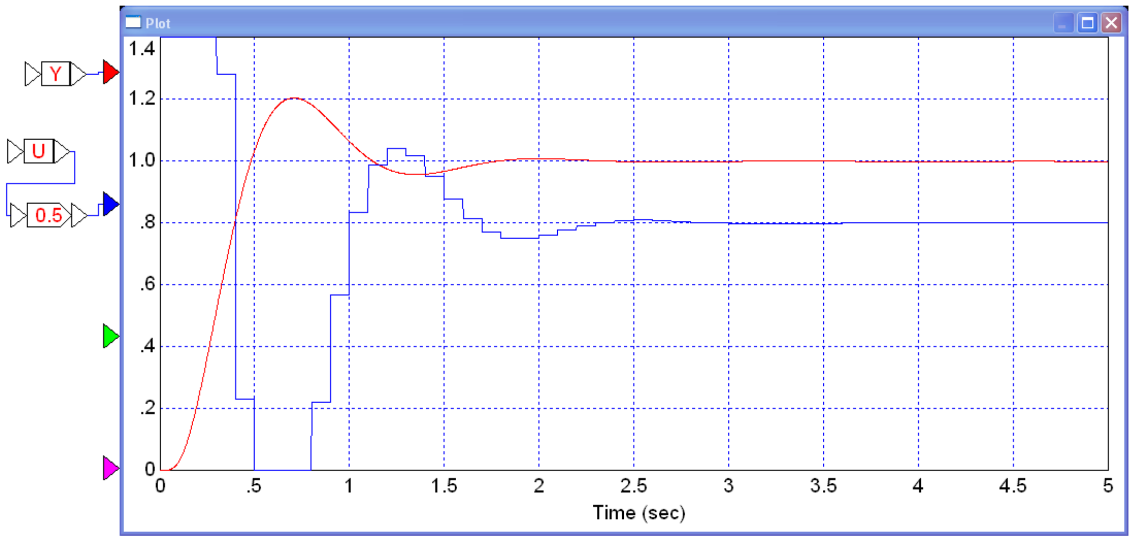
```
>> s = -1 + j*2;
>> z = exp(s*T);
>> evalfr(Gs,s) * exp(-s*T/2) * ((z-0.9222)*(z-0.5945) / (z*(z-1)) )

ans =  -0.0836 - 0.0670i

>> s = s * 1.1;
>> z = exp(s*T);
>> evalfr(Gs,s) * exp(-s*T/2) * ((z-0.9222)*(z-0.5945) / (z*(z-1)) )

ans =  -0.0800 - 0.0585i
:
:
:
>> s = s * 1.01;
>> z = exp(s*T);
>> evalfr(Gs,s) * exp(-s*T/2) * ((z-0.9222)*(z-0.5945) / (z*(z-1)) )

ans =  -0.0544 - 0.0001i

>> k = 1/abs(ans)

k =   18.3964

s =  -2.4586 + 4.9172i
z =   0.6894 + 0.3692i
```

meaning

$$K(z) = 18.3964\left(\frac{(z-0.9222)(z-0.5945)}{z(z-1)}\right)$$



18.39
z2-1.5167z+.5482
z2-1z
U
1
903
s4+44.85s3+580.79s2+2225.9s+1445.3
-1
Y



Plot
Y
U
0.5
1.4
1.2
1.0
.8
.6
.4
.2
0
.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Time (sec)

Meeting Design Specs

4) Assume a sampling rate of T = 0.1 seconds and

$$G(s) = \left(\frac{903}{(s+0.81)(s+5.20)(s+13.59)(s+25.25)}\right)$$

Design a digital controller that results in

- No error for a step input
- 20% overshoot for the step response, and
- A 2% settling time of 2 seconds

Simulate the step response of the closed-loop system (VisSim or Simulink preferred with K(z)*G(s))

Let K(z)

- Cancel the poles at s = {-0.81, -5.20}.
- Add a pole at z = +1 (making it type-1),
- Add a pole at z = +a

$$K(z) = k\left(\frac{(z-0.9222)(z-0.5945)}{(z-1)(z-a)}\right)$$

At s = -2 + j4

$$G(s) \cdot \exp\left(\frac{-sT}{2}\right) \cdot K(z) = -1$$

Evaluate what we know

```
>> Gs

                  903
------------------------------------
(s+0.81) (s+5.2) (s+13.59) (s+25.25)


>> s = -2 + j*4;
>> z = exp(s*T)
    0.7541 + 0.3188i

>> GDK = evalfr(Gs, s) * exp(-s*T/2) * ( (z-0.9222)*(z-0.5945) / (z-1) )

GDK =  -0.0420 - 0.0298i

>> angle(GDK)*180/pi

ans = -144.6031
```

To make the angles add up to 180 degrees, (z-a) must contribute -35.3969 degrees

$$\angle(z-a) = 35.3969^0$$

$$a = 0.7541 - \left(\frac{0.3188}{\tan\left(35.3969^0\right)}\right)$$

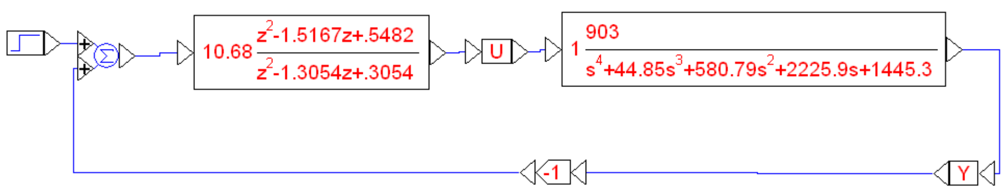
```
>> a = real(z) - imag(z) / tan(35.3969*pi/180)

a =    0.3054
```

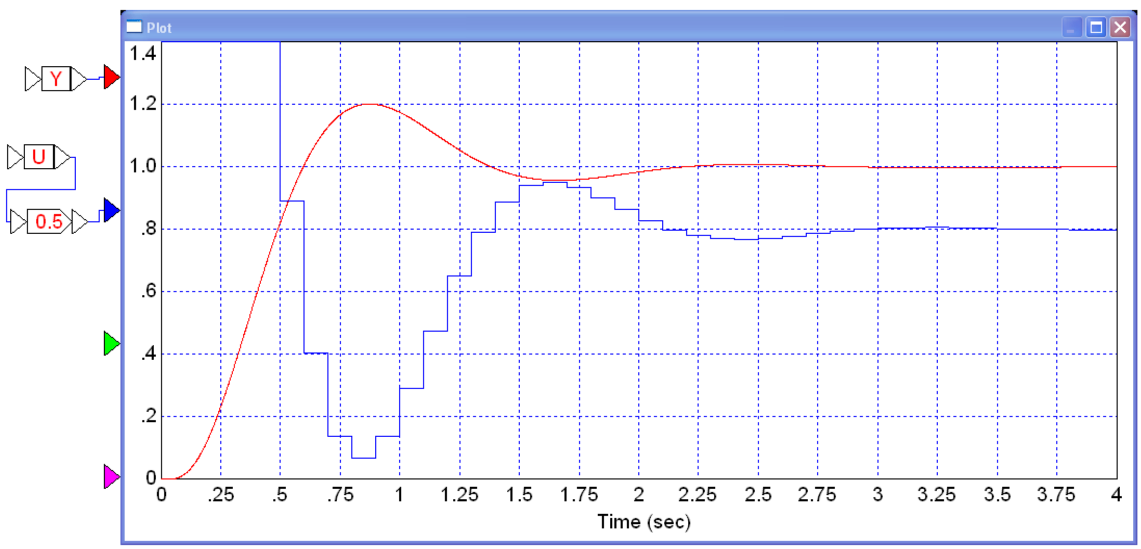
To find the gain, k,

```
>> GDK = evalfr(Gs, s) * exp(-s*T/2) * ( (z-0.9222)*(z-0.5945) /
((z-1)*(z-0.3054))   )

GDK =  -0.0936 - 0.0000i

>> k = 1/abs(GDK)

k =   10.6859
```

giving

$$K(z) = 10.6859\left(\frac{(z-0.9222)(z-0.5945)}{(z-1)(z-a)}\right)$$





5) Assume

$$G(s) = \left(\frac{903}{(s+0.81)(s+5.20)(s+13.59)(s+25.25)} \right)$$

Design a digital controller with T = 0.2 seconds that results in

- No error for a step input
- 20% overshoot for the step response, and
- A 2% settling time of 2 seconds

Simulate the step response of the closed-loop system (VisSim or Simulink preferred with K(z)*G(s))

Note: Changing the sampling rate is a big deal: it means a complete redesign of K(z)

Let K(z)

- Cancel the poles at s = {-0.81, -5.20}.
- Add a pole at z = +1 (making it type-1),
- Add a pole at z = +a

Converting from the s-plane to the z-plane:

```
>> s = [-0.81,-5.20]';
>> T = 0.2;
>> z = exp(s*T)

    0.8504
    0.3535
```

$$K(z) = k\left(\frac{(z-0.8504)(z-0.3535)}{(z-1)(z-a)} \right)$$

At s = -2 + j4

$$G(s) \cdot \exp\left(\frac{-sT}{2}\right) \cdot K(z) = -1$$

Evaluate what we know

```
>> s = -2 + j*4;
>> z = exp(s*T)

z =   0.4670 + 0.4809i

>> GDK = evalfr(Gs, s) * exp(-s*T/2) * ( (z-0.8504)*(z-0.3535) / (z-1))

GDK =  -0.0597 - 0.0462i

>> angle(GDK)*180/pi

ans = -142.2897
```

The angle is off by 37.7103 degrees, meaning

$$\angle(z+a) = 37.7103^0$$

$$a = 0.4670 - \left(\frac{0.4809}{\tan\left(37.7103^0\right)} \right)$$

```
>> a = real(z) - imag(z) / tan(37.7103*pi/180)

a =   -0.1549
```

To find k

```
>> GDK = evalfr(Gs, s) * exp(-s*T/2) * ( (z-0.8504)*(z-0.3535) / (
(z-1)*(z+0.1549) ) )

GDK =  -0.0960 + 0.0000i

>> k = 1/abs(GDK)

k =   10.4125
```

$$K(z) = 10.4125\left(\frac{(z-0.8504)(z-0.3535)}{(z-1)(z+0.1549)}\right)$$