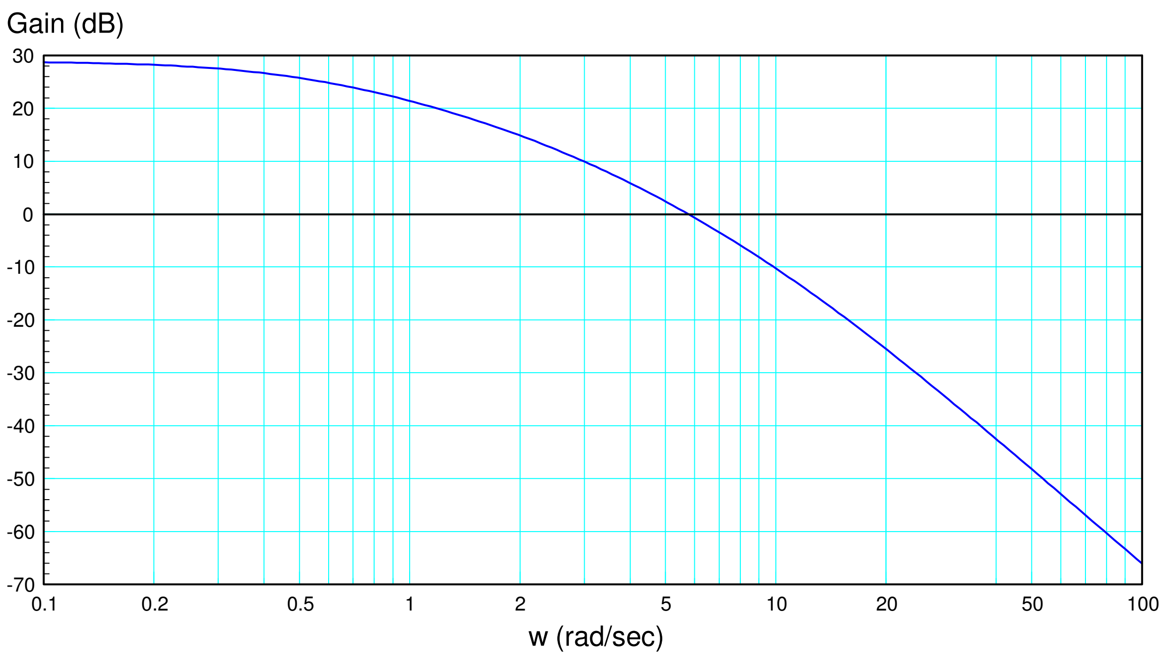
Homework #12: ECE 461/661

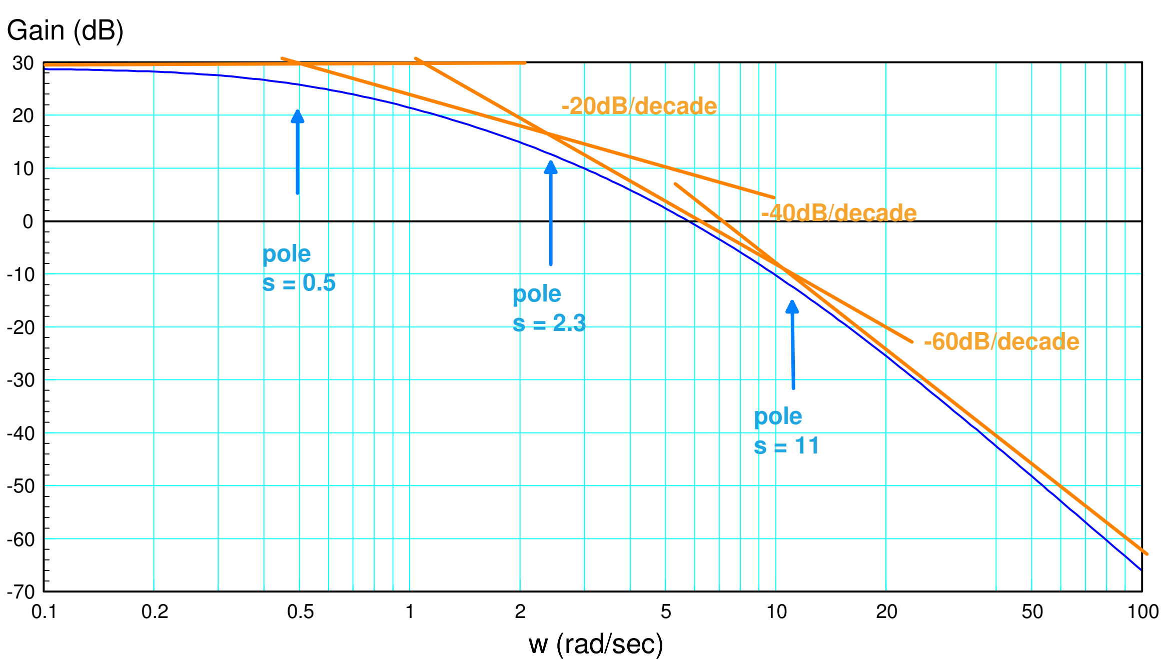
Bode Plots. Nichols charts and gain & lead compensation.   Due Monday, November 22nd

Bode Plots

1) Determine the system, G(s), with the following gain vs. frequency



Step 1: Draw in the asymptotes. Make sure they are multiples of 20dB/decade (showin in orange)



Each corner corresponds to a pole

$$G(s) \approx \left(\frac{k}{(s+0.5)(s+2.3)(s+11)} \right)$$

Pick 'k' to match the gain at some frequency. At s = 0 (off the chart to the left)

$$G(s=0) = 30dB = 10^{30/20} = 31.622$$

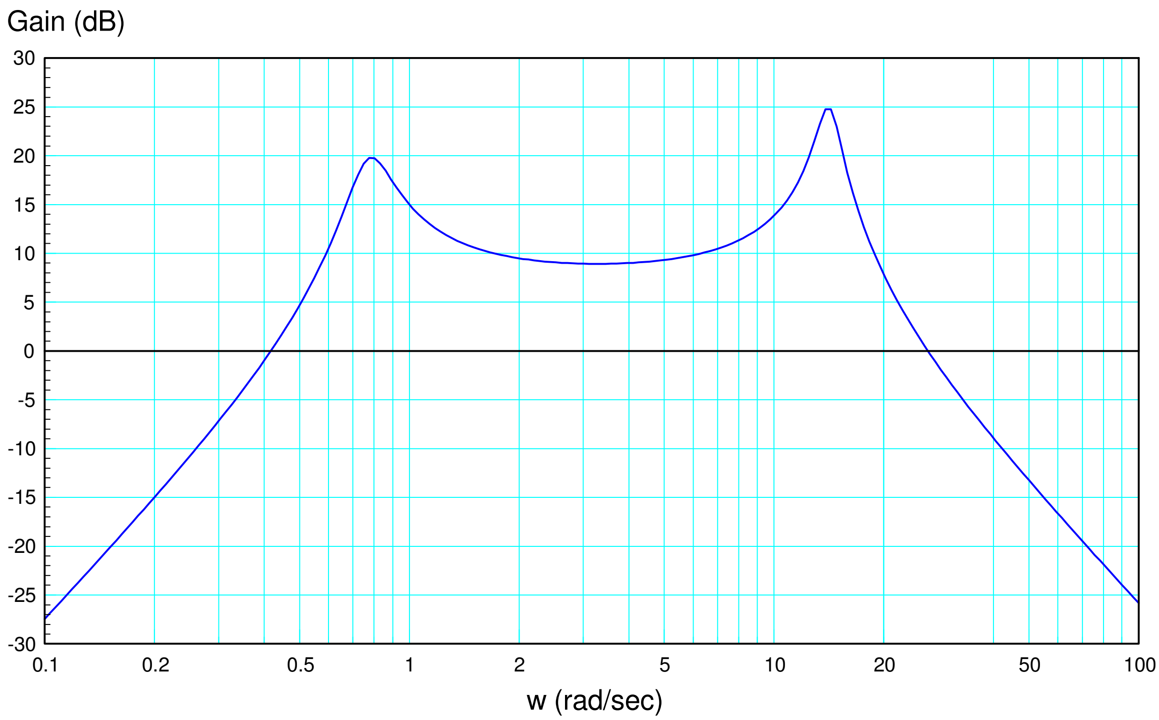
$$\left(\frac{k}{(s+0.5)(s+2.3)(s+11)} \right)_{s=0} = 31.622$$

$$k = 400.03$$

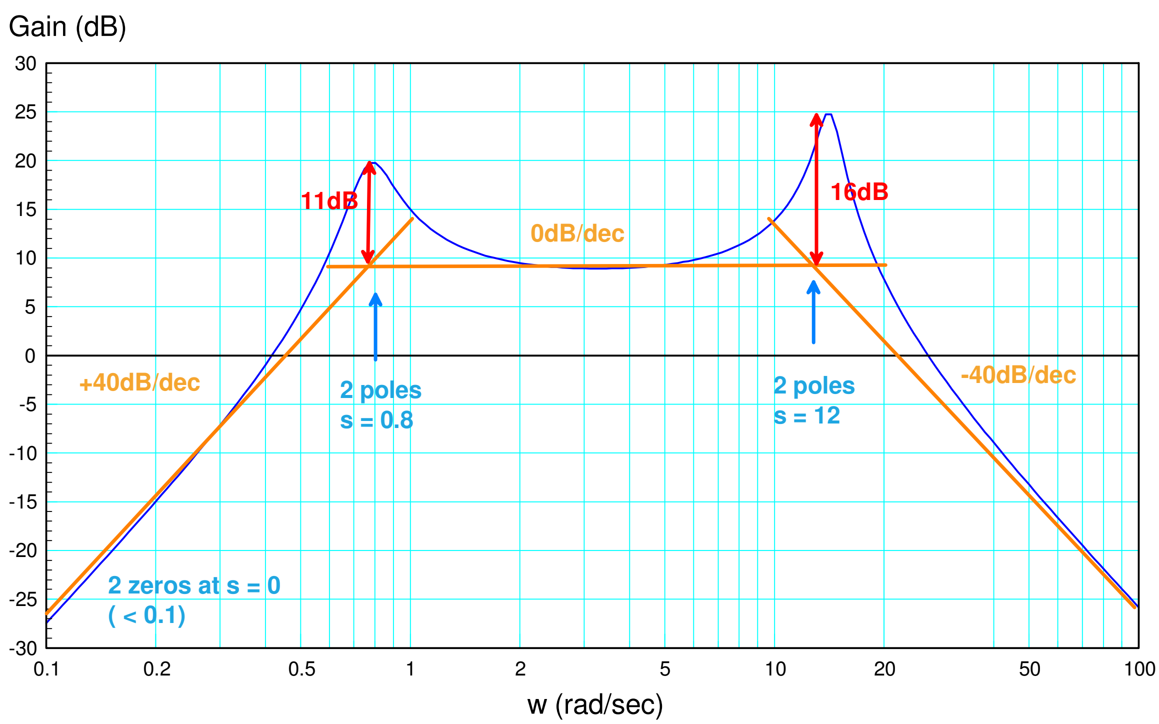
giving

$$G(s) \approx \left(\frac{400.03}{(s+0.5)(s+2.3)(s+11)} \right)$$

2) Determine the system, G(s), with the following gain vs. frequency



Step 1: Draw the asymptotes at multiples of 20dB/decade (shown in orange)



Each corner is a pole (gain drops) or a zero (gain increases)

Step 2: Determine the angle of the poles (the damping ratio)

At 0.8 rad/sec

$$\frac{1}{2\zeta} = 11dB = 10^{11/20} = 3.5481$$

$$\zeta = 0.1409$$

$$\theta = \arccos(\zeta) = 81.89^0$$

At 12 rad/sec

$$\frac{1}{2\zeta} = 16dB = 10^{16/20} = 6.3096$$

$$\zeta = 0.0793$$

$$\theta = \arccos(\zeta) = 85.45^0$$

G(s) is then

$$G(s) \approx \left(\frac{ks^2}{\left(s+0.8\angle\pm 81.89^0\right)\left(s+12\angle\pm 85.45^0\right)} \right)$$

To find k, match the gain at some frequency.

At s = j3, |G(s)| = +9dB (from the graph)

$$G(s) \approx \left(\frac{ks^2}{\left(s+0.8\angle\pm 81.89^0\right)\left(s+12\angle\pm 85.45^0\right)} \right)_{s=j3} = 9dB = 10^{9/20} = 2.8184$$

Using the Bode approximation (the straignt line asymptotes), at 3 rad/sec

$$G(j3) \approx \left(\frac{k \cdot 3^2}{(3)(3)(12)(12)} \right) = \frac{k}{12^2} = 2.8184$$

$$k = 405.84$$

$$G(s) \approx \left(\frac{105.84 \cdot s^2}{\left(s+0.8\angle\pm 81.89^0\right)\left(s+12\angle\pm 85.45^0\right)} \right)$$

Nichols Charts

3) The gain vs. frequency of a system is measured

w (rad/sec)	2	3	4	5	6	10
Gain (dB)	1.73	-2.62	-6.13	-9.19	-11.93	-21.03
Phase (deg)	-125	-141	-154	-167	-178	-213

Using this data

- Transfer it to a Nichols chart
- Determine the maximum gain that results in a stable system
- Determine the gain, k, that results in a maximum closed-loop gain of Mm = 1.5

Step 1: Transfer the data to a Nichols chart

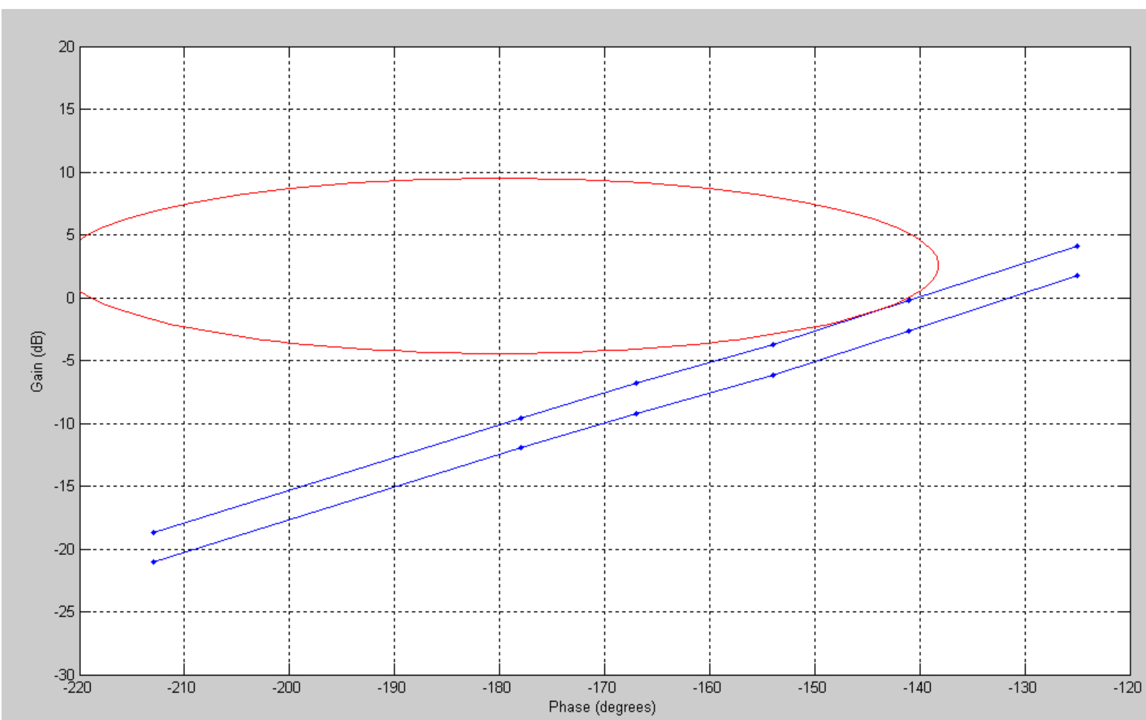
- Lower blue line below

Max Gain for Stability:

- When the phase is 180 degrees, the gain must be less than one
- k < 12.5dB

Gain for Mm = 1.5

- Slide G(jw) up until you're tangent to the M-circle
- Upper blue line below
- k = 1.315



Gain and Lead Compensation

Problem 4 & 5) Assume

$$G(s) = \left(\frac{903}{s(s+5.20)(s+13.59)(s+25.25)}\right)$$

4) Design a gain compensator that results in a 50 degree phase margin.

- Check the resulting step response in Matlab

Translation: At some frequency

$$Gk(j\omega) = 1\angle -130^0$$

Search s = jw until the phase is -130 degrees

$$G(j2.3857) = 0.1890\angle -130^0$$

so

$$k = \frac{1}{0.1890} = 5.2896$$

Step Response

50 degree phase margin means...

$$M_m \approx \left|\frac{1\angle -130^0}{1+1\angle -130^0}\right| = 1.1831$$

Which means

$$M_m = \frac{1}{2\zeta\sqrt{1-\zeta^2}} = 1.1831$$

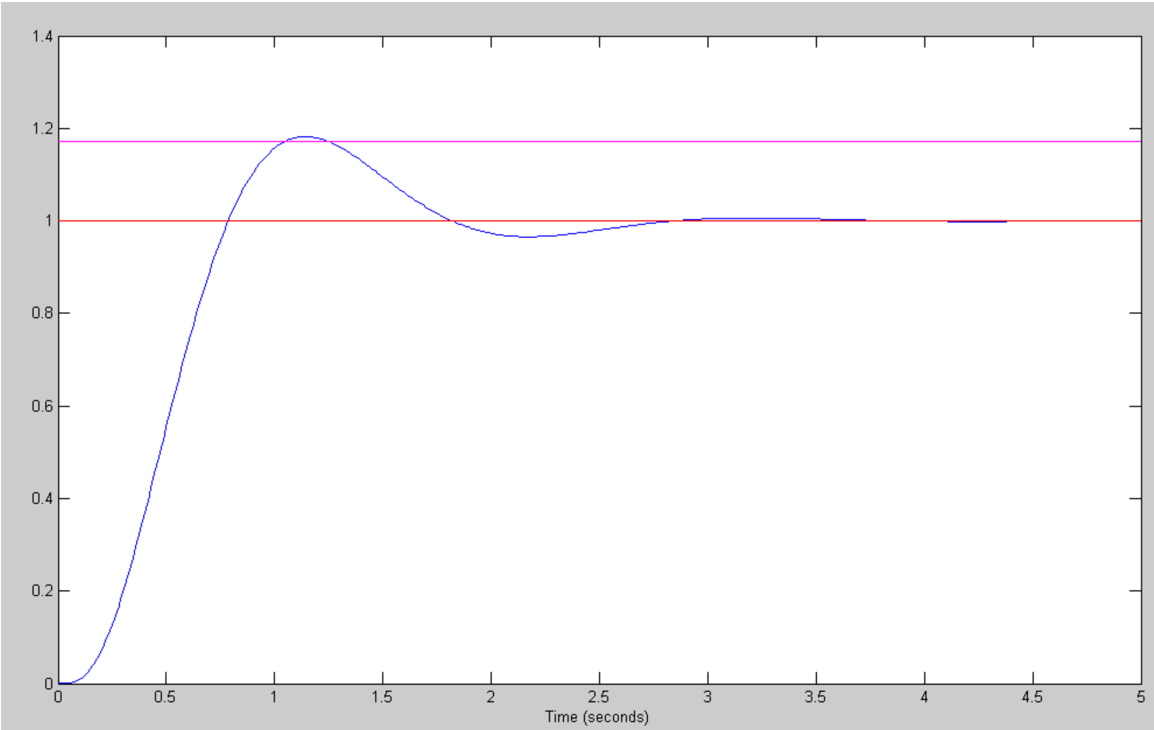
$$\zeta = 0.4825$$

Which means

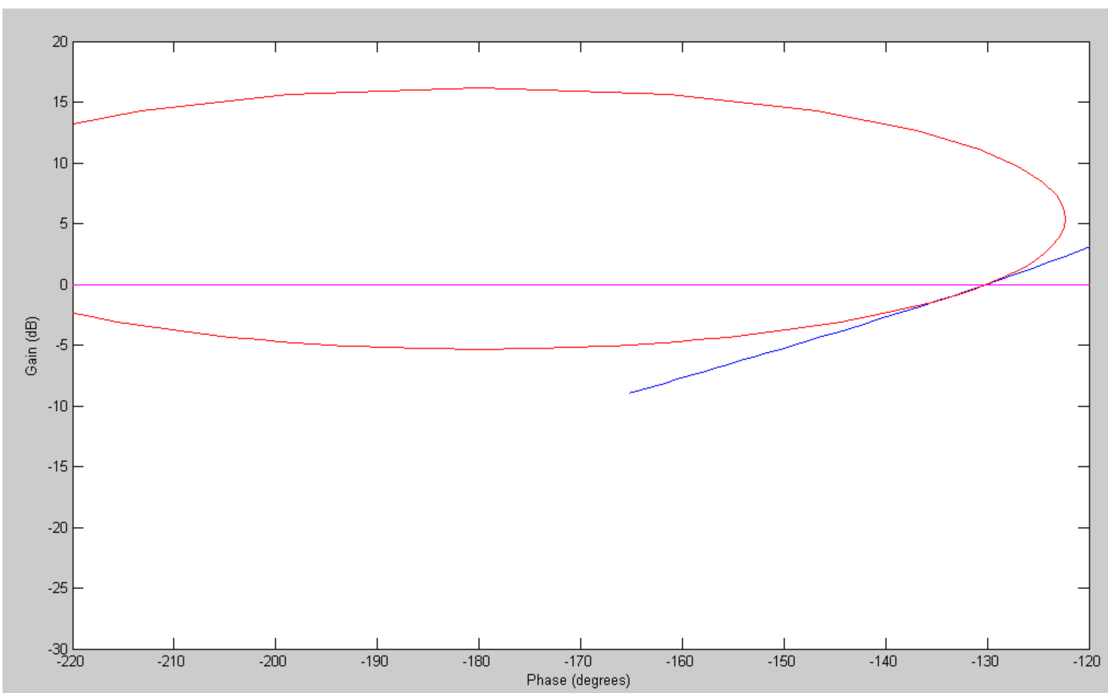
$$OS = \exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right) = 17.72\%$$

Checking in Matlab

```
>> G = zpk([],[0,-5.20,-13.59,-25.25],903);
>> k = 5.2896;

>> Gcl = minreal(G*k / (1 + G*k));

>> t = [0:0.01:5]';
>> y = step(Gcl,t);
>> plot(t,y)
>> DC = evalfr(Gcl, 0)

DC =    1.0000

>> OS = max(y) / DC

OS = 1.1820

>>
```



Step Response of Gain Compensator with a 50-degree Phase Margin
Overshoot is 18% (should be 17%)



Nichols Chart for a 50-degree Phase Margin
G*k intersects the Mm circle at -130 degrees

5) Design a lead compensator that results in a 50 degree phase margin.

- Check the resulting step response in Matlab

$$G(s) = \left(\frac{903}{s(s+5.20)(s+13.59)(s+25.25)}\right)$$

From problem #4, the resonance with gain compensation is at 2.3857 rad/sec. To increase the phase margin, pick the zero of the lead compensator in the range of

$$zero = (1..3)x\left(2.3857\frac{rad}{\sec}\right)$$

$$2.38 < zero < 7.24$$

Pick the zero to be s = -5.20 to cancel the pole (just like root locus). Let

$$K(s) = k\left(\frac{s+5.20}{s+52}\right)$$

$$GK = \left(\frac{903k}{s(s+13.59)(s+25.25)(s+52)}\right)$$

Search along the jw axis until the phase is -130 degrees

$$GK(j5.4478) = 0.0084\angle -130^0$$

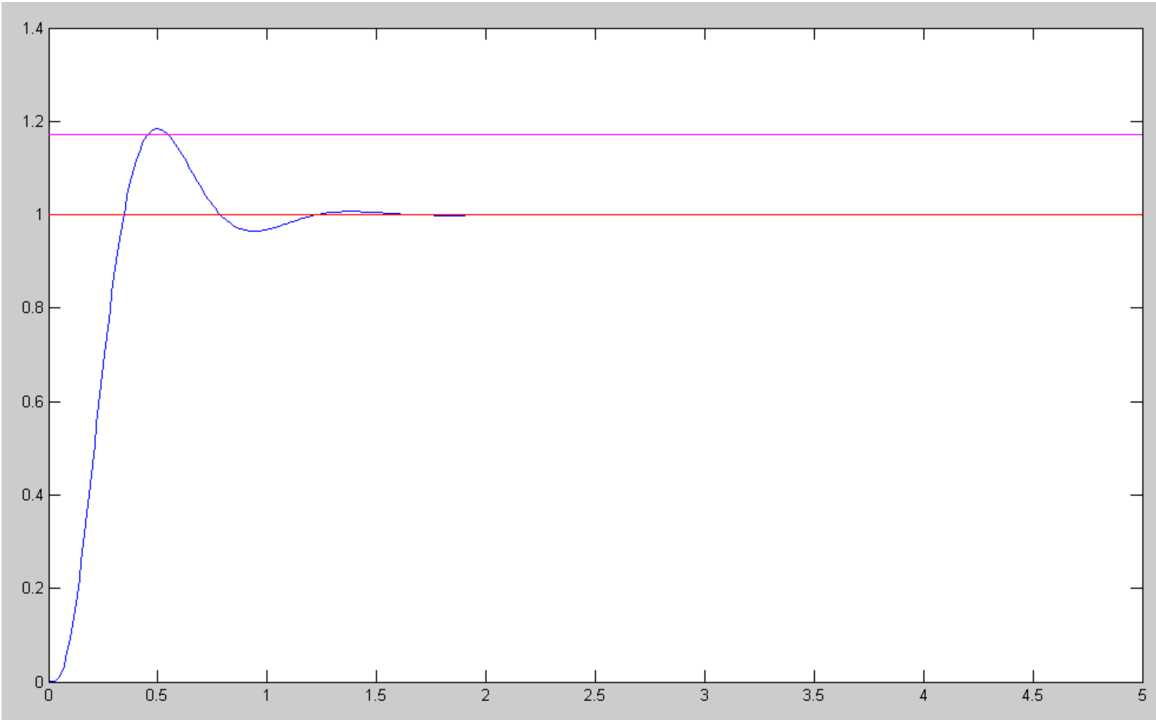
$$k = \frac{1}{0.0084} = 119.29$$

and

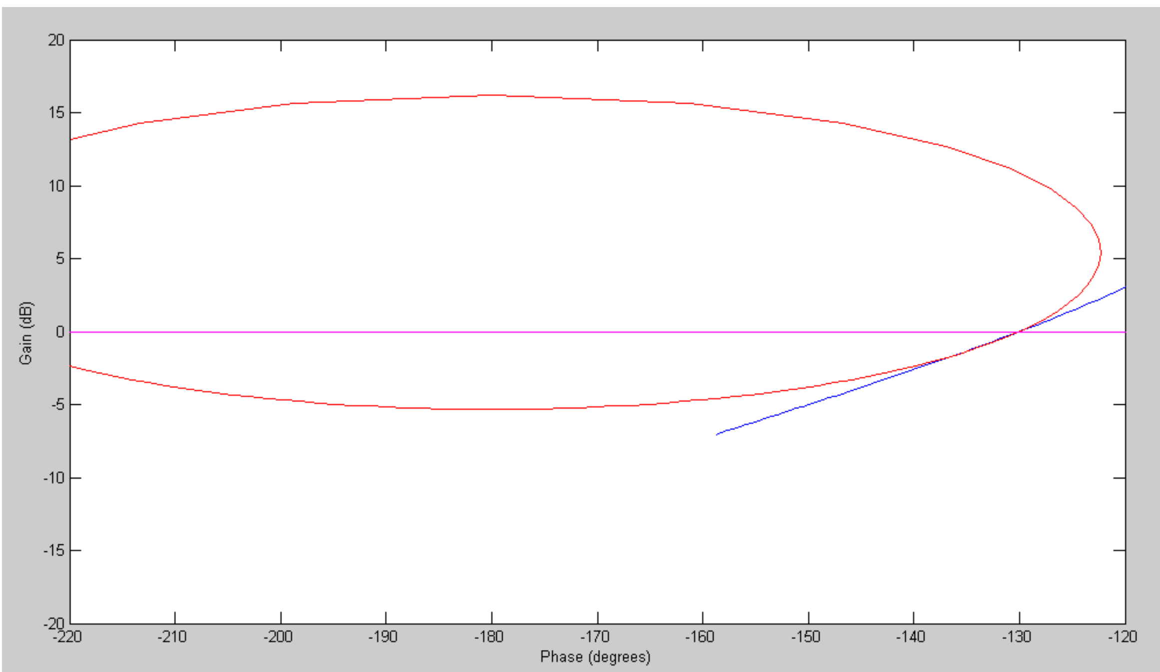
$$K(s) = 119.29\left(\frac{s+5.2}{s+52}\right)$$

Plotting the step response

```
>> G = zpk([],[0,-5.20,-13.59,-25.25],903);
>> K = zpk(-5.2,-52,119.29)

119.29 (s+5.2)
--------------
    (s+52)

>> GKcl = minreal(G*K / (1 + G*K))

               107718.87
----------------------------------------
(s+33.89) (s+49.5) (s^2 + 7.444s + 64.2)

>> t = [0:0.01:5]';
>> y = step(GKcl, t);
>> DC = evalfr(GKcl, 0)

DC =    1.0000

>> OS = max(y) / DC

OS =    1.1837
```



Step Response of Gain Compensator with a 50-degree Phase Margin
Overshoot is 18% (should be 17%)



Nichols Chart for lead compensated system
G(jw) intersects the M-circle at -130 degrees

Problem 6 & 7) Assume a 200ms delay is added

$$G(s) = \left(\frac{903}{s(s+5.20)(s+13.59)(s+25.25)} \right) e^{-0.2s}$$

6) Design a gain compensator that results in a 50 degree phase margin.

- Check the resulting step response in Matlab

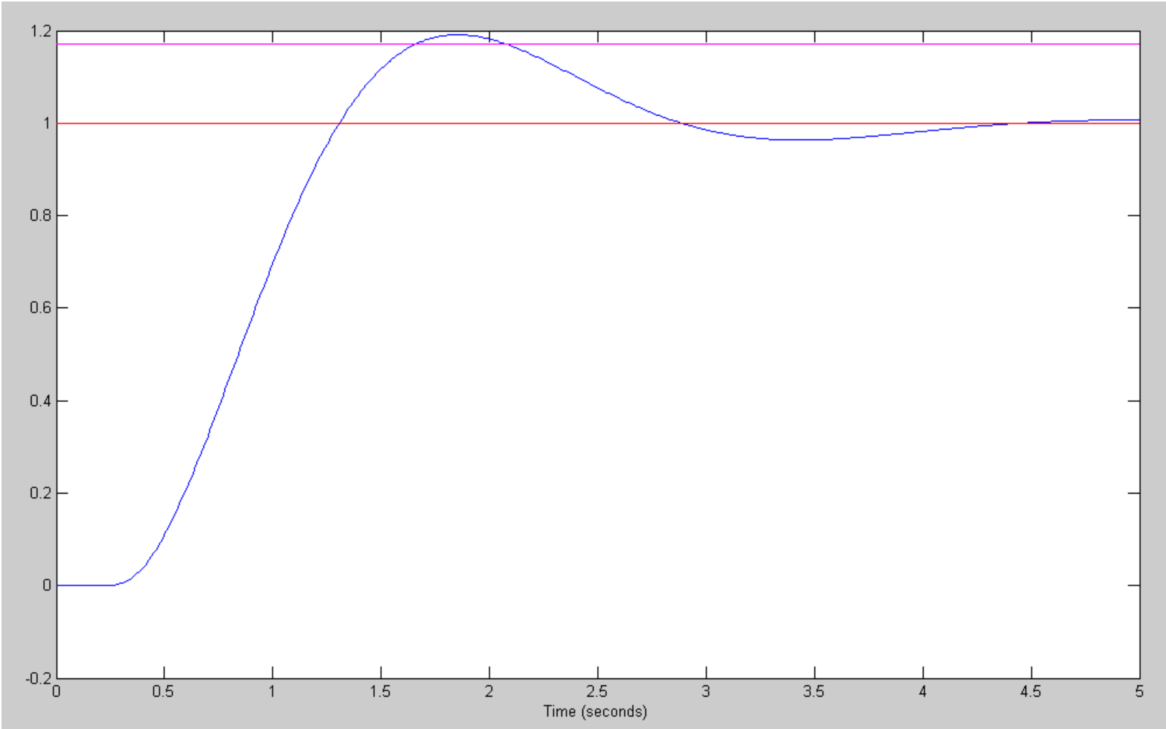
Same procedure as before. Search along the jw axis until G(jw) has a phase of -130 degrees

$$G(j1.3941) = 0.3483 \angle -130^0$$

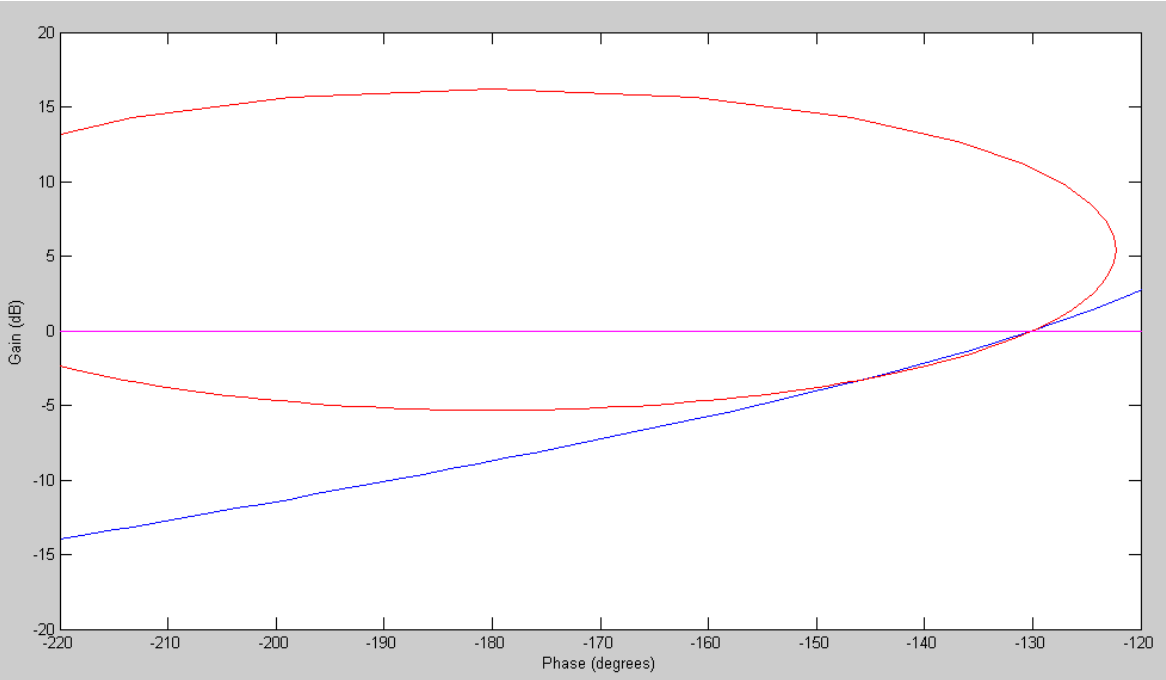
$$k = \frac{1}{0.3483} = 2.8714$$

Checking the step response in Matlab

```
>> G = zpk([],[0,-5.20,-13.59,-25.25],903);

>> [num,den] = pade(0.2, 4);
>> D = tf(num,den)

s^4 - 100 s^3 + 4500 s^2 - 1.05e005 s + 1.05e006
------------------------------------------------
s^4 + 100 s^3 + 4500 s^2 + 1.05e005 s + 1.05e006

>> k = 2.8714;

>> Gcl = minreal(G*D*k / (1 + G*D*k));
>> t = [0:0.01:5]';
>> y = step(Gcl, t);
>> max(y)

ans =

    1.1910

>> plot(t,y,'b',t,0*t+1,'r',t,0*t+1.172,'m')
>>
```



Step Response when G(s) has a 200ms delay (50 degree phase margin)
Overshoot = 19% (shoulr be 17%)



Nichols chart for gain compensated system
G(jw) intersects the M-circle at -130 degrees

7) Design a lead compensator that results in a 50 degree phase margin.

- Check the resulting step response in Matlab

$$G(s) = \left(\frac{903}{s(s+5.20)(s+13.59)(s+25.25)}\right) e^{-0.2s}$$

From problem #6, the resonance is at 1.3941 rad/sec. This means you need to add phase at this frequency. Pick the zero of the lead compensator in the range of

$$zero = (1..3)x(1.3931 \tfrac{rad}{\sec})$$

$$1.3941 < zero < 4.18$$

Let the zero be 3 rad/sec (nice round number in this range)

$$K(s) = k\left(\frac{s+3}{s+30}\right)$$

giving

$$GK = \left(\frac{903(s+3)k}{s(s+5.2)(s+13.59)(s+25.25)(s+30)(s+52)}\right) e^{-0.2s}$$

Search along the jw axis until the phase is -130 degrees

$$GK(j2.7630) = 0.0213\angle -130^0$$

$$k = \frac{1}{0.0213} = 46.883$$

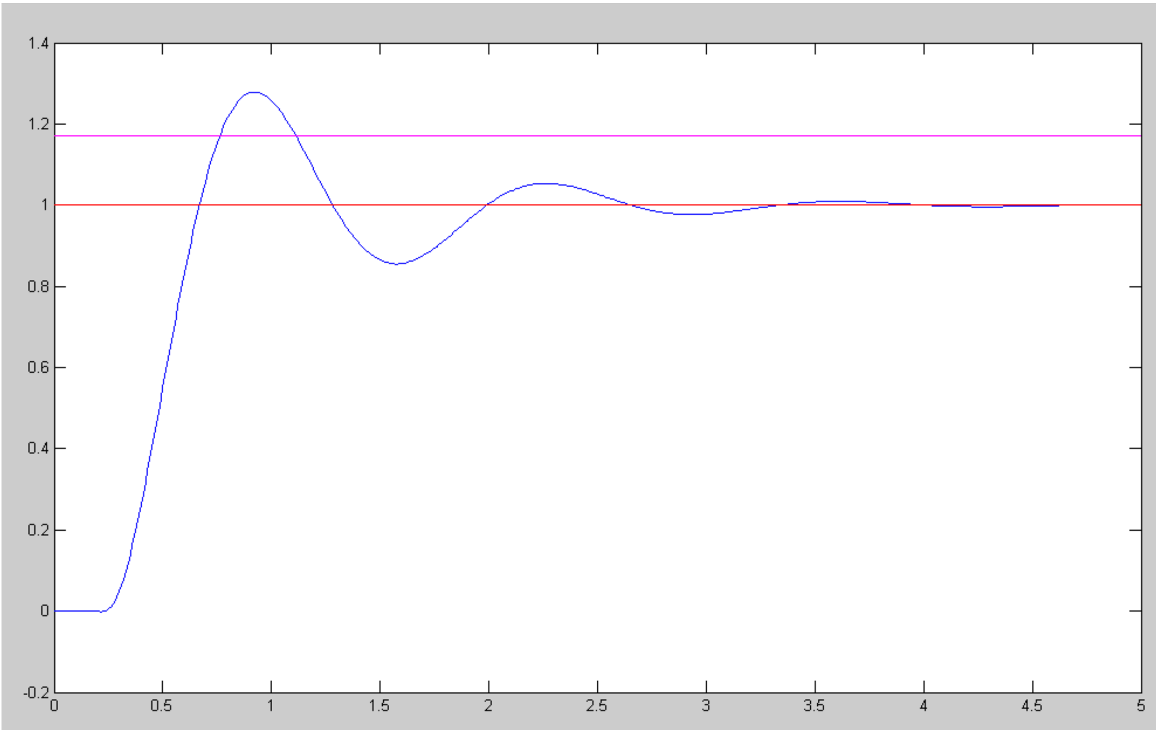
and

$$K(s) = 46.883\left(\frac{s+3}{s+30}\right)$$

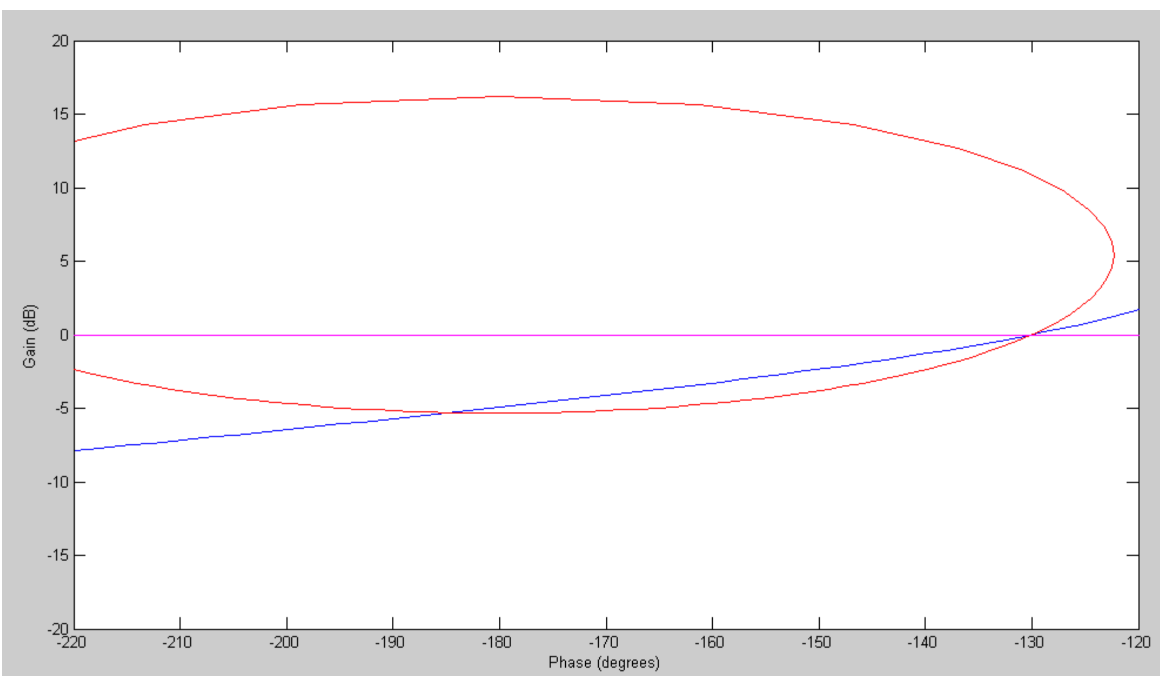
Checking the step response in Matlab

```
>> G = zpk([],[0,-5.20,-13.59,-25.25],903)

              903
-----------------------------
s (s+5.2) (s+13.59) (s+25.25)

>> [num,den] = pade(0.2, 4);
>> D = tf(num,den)

s^4 - 100 s^3 + 4500 s^2 - 1.05e005 s + 1.05e006
-------------------------------------------------
s^4 + 100 s^3 + 4500 s^2 + 1.05e005 s + 1.05e006

>> K = zpk(-3,-30,46.883)

46.883 (s+3)
------------
   (s+30)

>> Gcl = minreal(G*D*K / (1 + G*D*K));
>> t = [0:0.01:5]';
>> y = step(Gcl, t);
>> OS = max(y) / DC

OS =    1.2783
```

```
>> plot(t,y,'b',t,0*t+1,'r',t,0*t+1.172,'m')
>>
```



Step Response of Lead Compensated System with a 200ms delay
Overshoot = 28.8% (vs 17%)



Nichols Chart: G*D*K(jw) intersects the M-circle at -130 degeres