Homework #4: ECE 461 / 661

1st and 2nd Order Approximations. Due Monday, September 11th

LaPlace Transforms

1) Assume X and Y are related by the following transfer function

$$Y = \left(\frac{2s+20}{(s+1)(s+3)(s+10)}\right)X$$

a) What is the differential equation relating X and Y?

Cross multiply and multiply out

$$((s+1)(s+3)(s+10))Y = (2s+20)X$$

$$(s^3 + 14s^2 + 43s + 30)Y = (2s+20)X$$

Note that 'sY' means 'the derivative of y'

$$y''' + 14y'' + 43y' + 30y = 2x' + 20x$$

b) Determine y(t) assuming

$$x(t) = 4\cos(3t) + 2\sin(3t)$$

Use phasor analysis

$$s = j3$$

$$X = 4 - j2$$

$$Y = \left(\frac{2s+20}{(s+1)(s+3)(s+10)}\right)X$$

$$Y = \left(\frac{2s+20}{(s+1)(s+3)(s+10)}\right)_{s=j3} \cdot (4 - j2)$$

In Matlab

```
>> s = j*3;
>> Y = (2*s+20)/( (s+1)*(s+3)*(s+10) ) * (4-j*2)

Y =  -0.5333 - 0.4000i
```

or another way to do it in Matlab:

```
>> G = zpk(-10,[-1,-3,-10],2)

      2 (s+10)
-------------------
(s+1) (s+3) (s+10)
```

```
>> Y = evalfr(G, j*3) * (4 - j*2)

Y =   -0.5333 - 0.4000i
```

meaning

$$y(t) = -0.5333\cos(3t) + 0.4000\sin(3t)$$

c) Determine y(t) assuming x(t) is a unit step input

In this case, use LaPlace transforms since x(t) = 0 for t<0

$$Y = \left(\frac{2s+20}{(s+1)(s+3)(s+10)}\right)X$$

$$Y = \left(\frac{2s+20}{(s+1)(s+3)(s+10)}\right)\left(\frac{1}{s}\right)$$

Use partial fractions

$$Y = \left(\frac{0.667}{s}\right) + \left(\frac{-1}{s+1}\right) + \left(\frac{0.333}{s+3}\right) + \left(\frac{0}{s+10}\right)$$

Take the inverse LaPlace transform

$$y(t) = (0.667 - e^{-t} + 0.333e^{-3t} + 0e^{-10t})u(t)$$

Note: Matlab can do partial fractions

```
>> G = zpk(-10,[-1,-3,-10,0],2)

      2 (s+10)
--------------------
s (s+1) (s+3) (s+10)

>> s = 0 + 1e-9;
>> evalfr(G,s) * (s + 0)

ans =    0.6667

>> s = -1 + 1e-9;
>> evalfr(G,s) * (s + 1)

ans =   -1.0000

>> s = -3 + 1e-9;
>> evalfr(G,s) * (s + 3)

ans =    0.3333

>> s = -10 + 1e-9;
>> evalfr(G,s) * (s + 10)

ans = -3.1746e-012
```

2) Assume X and Y are related by the following transfer function:

$$Y = \left(\frac{3000}{(s+3+j7)(s+3-j7)(s+30)}\right)X$$

a) Use 2nd-order approximations to determine

- The 2% settling time
- The percent overshoot for a step input
- The steady-state output for a step input (x(t) = u(t))

Keep the dominant pole (s = -3 +/- j7)

From the 2nd-order approximations

$$s = -3 + j7$$

$$T_s = \frac{4}{-real(s)} = \frac{4}{3}$$

$$\theta = \arctan\left(\frac{7}{3}\right) = 66.80^0$$

$$\zeta = \cos\theta = 0.3939$$

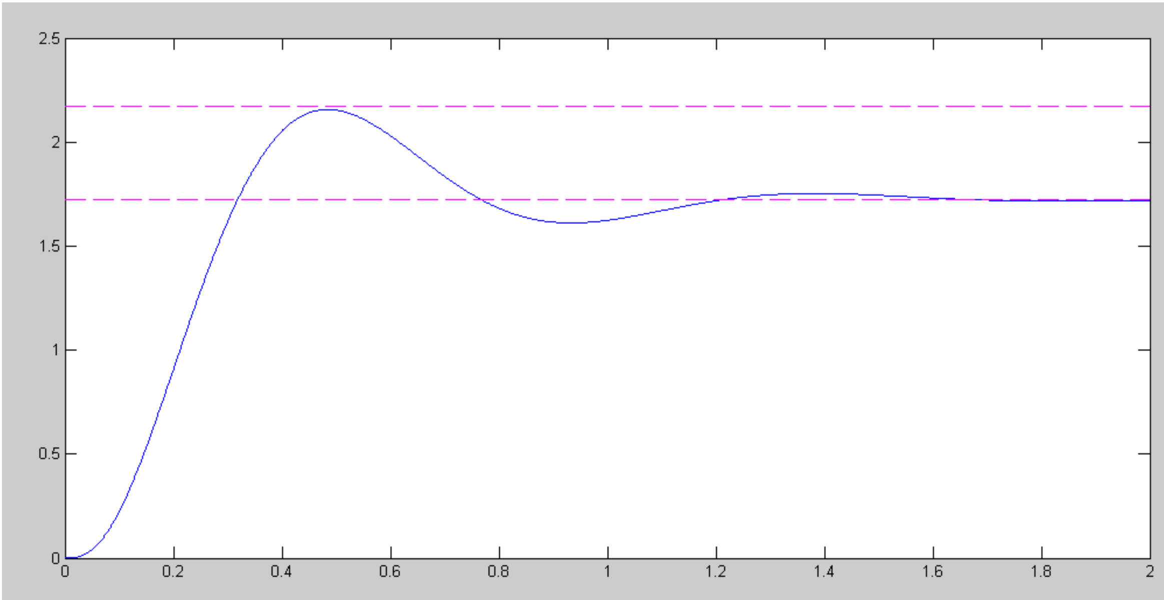
$$OS = \exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right) = 26.02\%$$

$$\text{DC gain} = \left(\frac{3000}{(s+3+j7)(s+3-j7)(s+30)}\right)_{s=0} = 1.7241$$

b) Check your answers using the 3rd order model and Matlab, Simulink, of VisSim (your pick)

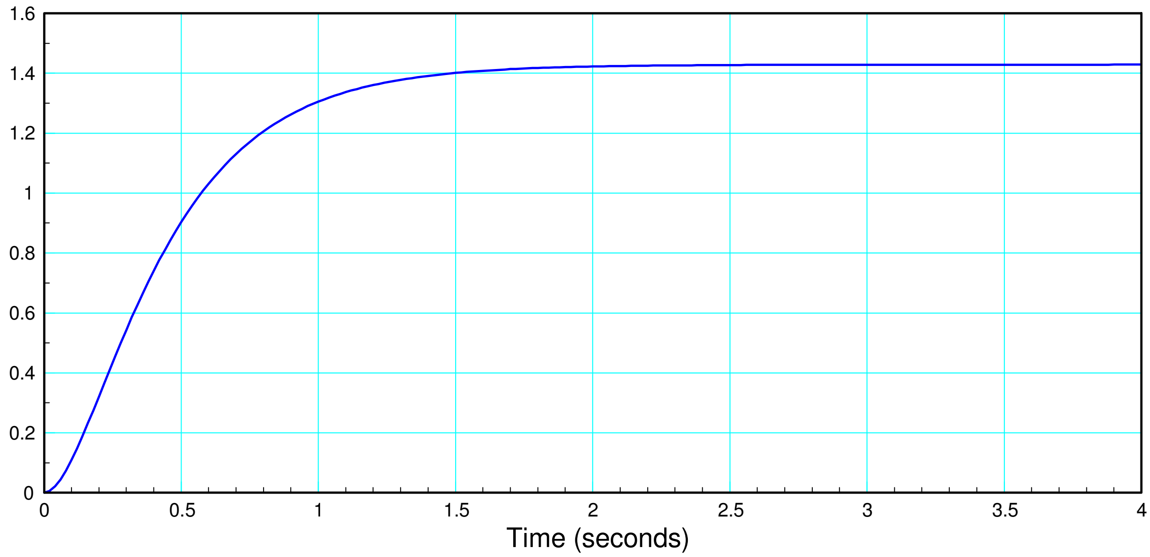
Using Matlab:

```
>> G = zpk([],[-3-j*7,-3+j*7,-30],3000)

          3000
----------------------
(s+30) (s^2 + 6s + 58)

>> DC = evalfr(G,0)

DC =    1.7241

>> t = [0:0.001:2]';
>> y = step(G,t);
>> OS = max(y) / DC

OS =    1.2510

>> plot(t,y,t,0*t+DC,'m--',t,0*t+1.2602*DC,'m--')
```



Step Response along with the DC gain & 1.26 * DC Gain

3) Determine the transfer function for a system with the following step response:



There is no oscillation, so this looks like a 1st-order system

$$G(s) = \left(\frac{a}{s+b}\right)$$

The DC gain is 1.42

$$\left(\frac{a}{s+b}\right)_{s=0} = \left(\frac{a}{b}\right) = 1.42$$

The 2% settling time is abot 1.5 secods (ish)

$$\left(\frac{4}{b}\right) = 1.5$$

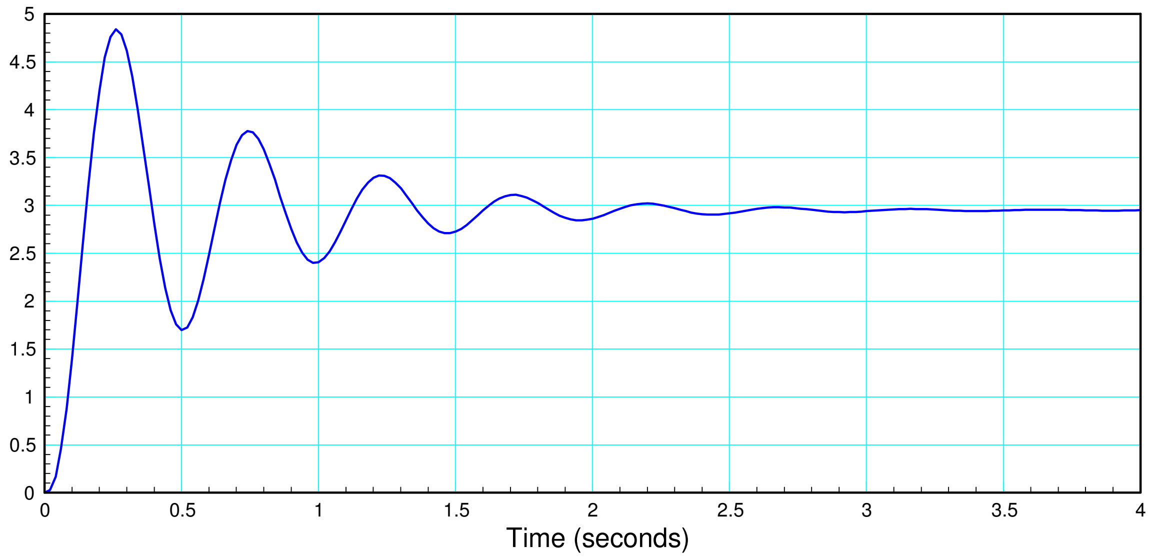
$$b = \left(\frac{4}{1.5}\right) = 2.67$$

$$a = 1.42b = 3.78$$

so

$$\boxed{G(s) \approx \left(\frac{3.78}{s+2.67}\right)}$$

4) Determine the transfer function for a system with the following step response:



This does have oscillations, meaning the dominant poles are complex

$$G(s) = \left(\frac{a}{(s+b+jc)(s+b-jc)}\right)$$

The DC gain is 2.9

$$G(s=0) = \left(\frac{a}{(s+b+jc)(s+b-jc)}\right)_{s=0} = 2.9$$

The frequency of oscillation is

$$c = \omega_d \approx \left(\frac{4 \text{ cycles}}{1.9 \text{ sec}}\right) 2\pi = 13.23$$

The 2% settling time is about 2.5 seconds

$$b = \sigma \approx \frac{4}{2.5} = 1.6$$

so

$$G(s) = \left(\frac{a}{(s+1.6+j13.23)(s+1.6-j13.23)}\right)$$

Pick 'a' to make the DC gain 2.9

$$G(s) = \left(\frac{515.02}{(s+1.6+j13.23)(s+1.6-j13.23)}\right)$$