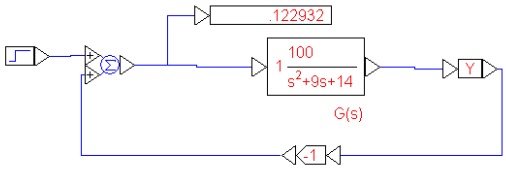
Homework #7: ECE 461/661

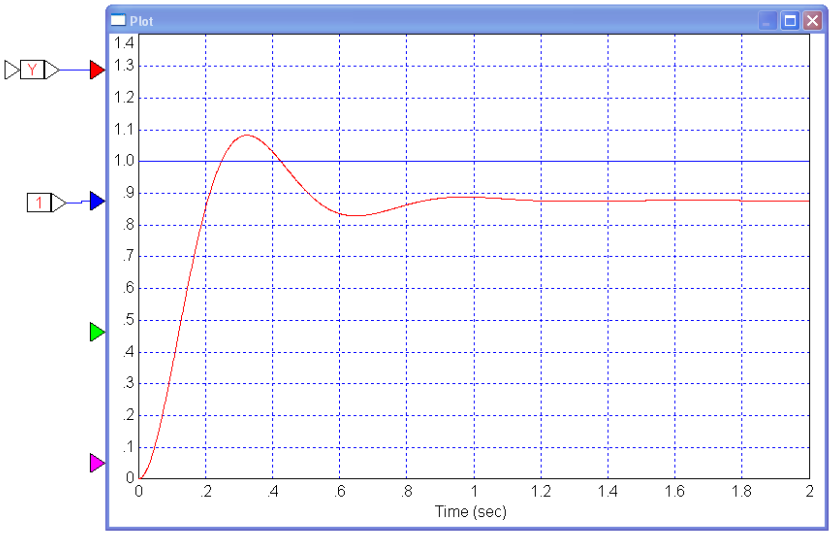
Error Constants, Routh Criteria, Skething a Root Locus. Due Monday, October 9th

Error Constants

1) Determine the error constants and steady-state error for the following systems

G(s)	System Type	Kp	Kv	Error for a unit step input
$\left(\frac{100}{(s+2)(s+7)}\right)$	**0**	**7.14**	**0**	**0.1228**
$\left(\frac{100}{s(s+2)(s+7)}\right)$	**1**	**inf**	**7.14**	**0**
$\left(\frac{100(s+5)}{s^2(s+2)(s+7)}\right)$	**2**	**inf**	**inf**	**0**
$\left(\frac{100}{(s-2)(s+7)}\right)$	**0**	**-7.14**	**0**	**-0.1628**





Routh Criteria

Determine the range of k that results in a negative definite polynomial (i.e. a stable system)

2) $(s-1)(s+10)(s+12)+2k=0$

Multiply out

$$s^3+21s^2+98s-120+2k=0$$

Place in a Routh table

1	98	0	
21	2k-120	0	
103.7143 - 0.0952k (a)	0	0	k < 1089
2k - 120 (b)	0	0	k > 60
0	0	0	

Result: 60 < k < 1089

(a) Element (3,1)

$$\frac{-\begin{vmatrix} 1 & 98 \\ 21 & 2k-120 \end{vmatrix}}{21} = 103.7143 - 0.0952k$$

(b) Element (4,1)

$$\frac{-\begin{vmatrix} 21 & 2k-120 \\ 103-0.09k & 0 \end{vmatrix}}{103-0.09k} = 2k-120$$

3) $(s+1)(s+3)(s+7)(s+8)+2k=0$

multiply out

$$s^4+19s^3+119s^2+269s+168+2k=0$$

Place in a Routh table

1	119	168+2k	
19	269	0	
104.8421 (a)	168 + 2k	0	
238.5542 - 0.3624k (b)	0		k < 658.17
168 + 2k	0		k > -84

Result: -84 < k < 658.17

(a)

$$\frac{-\begin{vmatrix} 1 & 119 \\ 19 & 269 \end{vmatrix}}{19} = 104.8421$$

(b)

$$\frac{-\begin{vmatrix} 19 & 269 \\ 104.8421 & 168+2k \end{vmatrix}}{104.8421} = 238.5542 - 0.3624k$$

Sketching a Root Locus

Sketch the root locus plot for the following systems for 0 < k < infinity. Also plot the

- real axis loci, break away points, jw crossings (if any), and asymptotes

4) $(s-1)(s+10)(s+12)+2k=0$

Real Axis Loci

(+1, -10), (-12, -infinity)

Breakaway Point(s)

s = -2.9531 (found by searching along s = x + j0.1 until the angle of G(s) is 180 degrees)

a little right of s = -4.5

Asymptotes

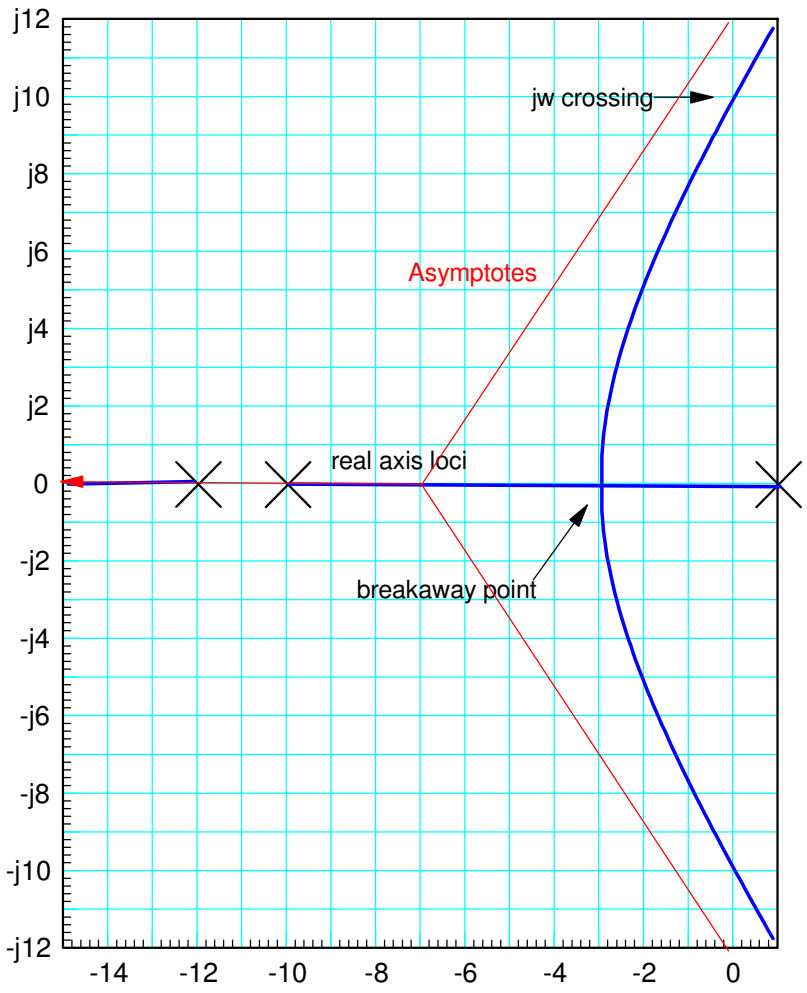
Three asymptotes

Angle = +60, -60, 180 degrees

Intercept = -7.000 (average of +1, -10, -12)

jw Crossing

s = j9.8995 (found by searching along s = jx until the angles add to zero)



5) $(s+1)(s+3)(s+7)(s+8)+2k=0$

Real Axis Loci:

(-1, -3), (-7, -8)

Breakaway Point(s)

s = -1.8318, -7.5476

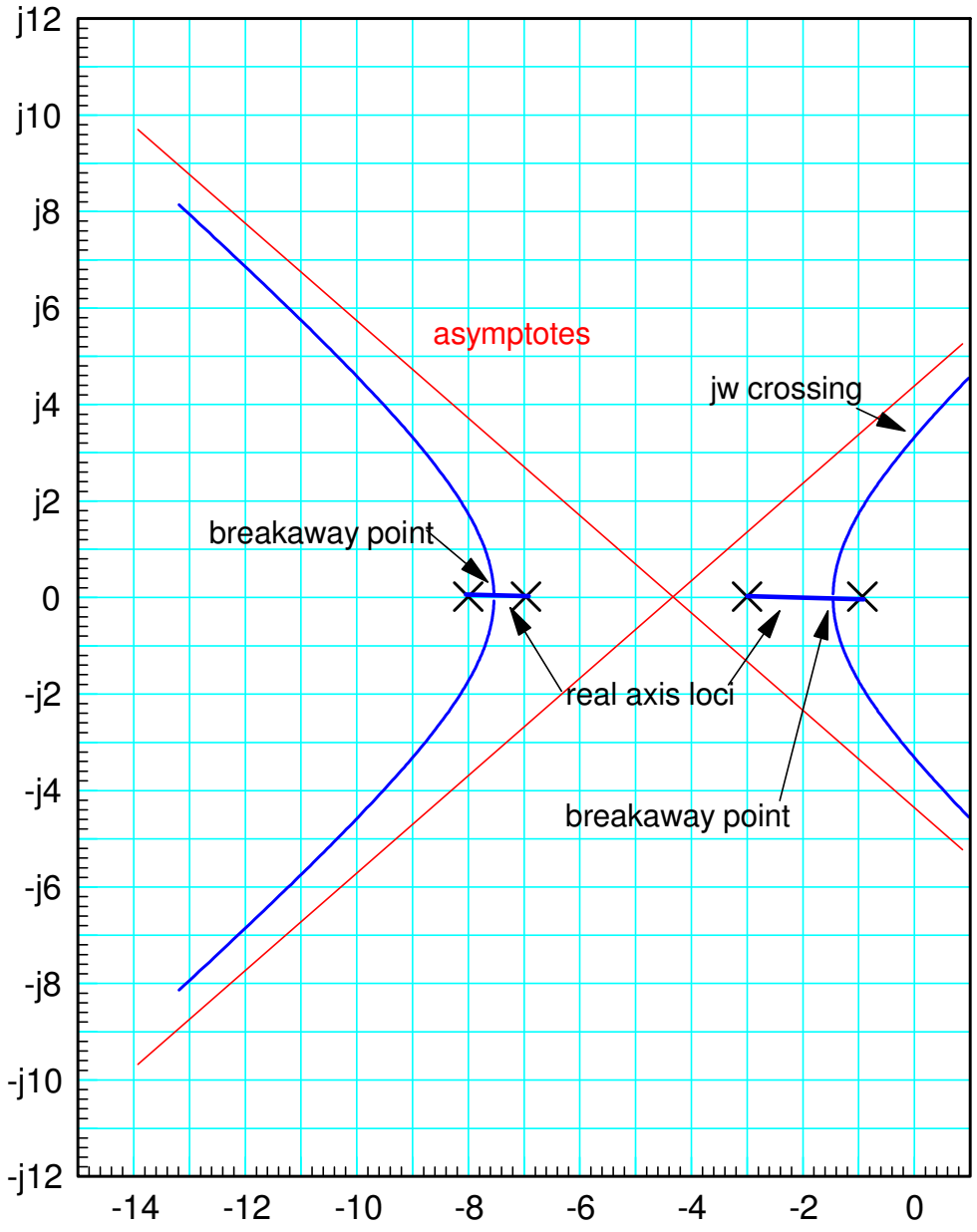
jw Crossing(s)

s = j3.7627

Asymptotes

four asymptotes

+/- 45 degrees, +/- 135 degrees



Root Locus with Complex Poles & Zeros

Sketch the root locus plot for the following systems for 0 < k < infinity. Also plot the

- real axis loci, break away points, jw crossings (if any), asymptotes, and departure/approach angle

6) $G(s) = \left(\frac{10}{s(s+8)(s+1+j4)(s+1-j4)} \right)$

Real Axis Loci: (0, -8)

Breakaway Point: s = -5.6450

jw Crossing: s = j9.7107

Departure Angle

$$\angle\left(\frac{10}{s(s+8)(s+1+j4)(s+1-j4)} \right)_{s \to -1+j4} = 180^0$$

$$\angle\left(\frac{10}{s(s+8)(s+1+j4)} \right)_{s=-1+j4} - \theta = 180^0$$

$$136.219^0 - \theta = 180^0$$

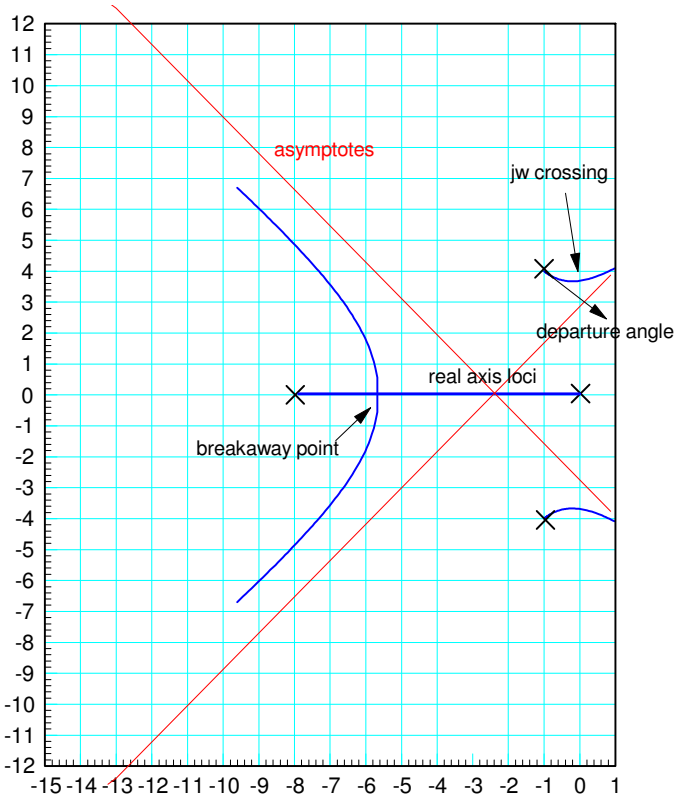
$$\theta = -43.78^0$$

Asymptotes

four asymptotes

Asymptote Intersect = -2.5 (average of poles)

Asymptote angles = +/- 45 degrees, +/- 135 degrees



7) $G(s) = \left(\frac{(s+j2)(s-j2)}{s(s+1)(s+5)(s+6)} \right)$

Real Axis Loci: (0, -1), (-5, -6)

Breakaway Point: s = -0.4235

jw Crossing: none

Approach Angle

$$\angle \left(\frac{(s+j2)(s-j2)}{s(s+1)(s+5)(s+6)} \right)_{s \to j2} = 180^0$$

$$\angle \left(\frac{(s+j2)}{s(s+1)(s+5)(s+6)} \right)_{s=j2} + \theta = 180^0$$

$$-103.67^0 + \theta = 180^0$$

$$\theta = -77.63^0$$

Asymptotes

two asymptotes (4 poles - 2 zeros)

Asymptote Intersect = -6 (poles - zeros)/(two asymptotes)

Asymptote angles = +/- 90 degrees