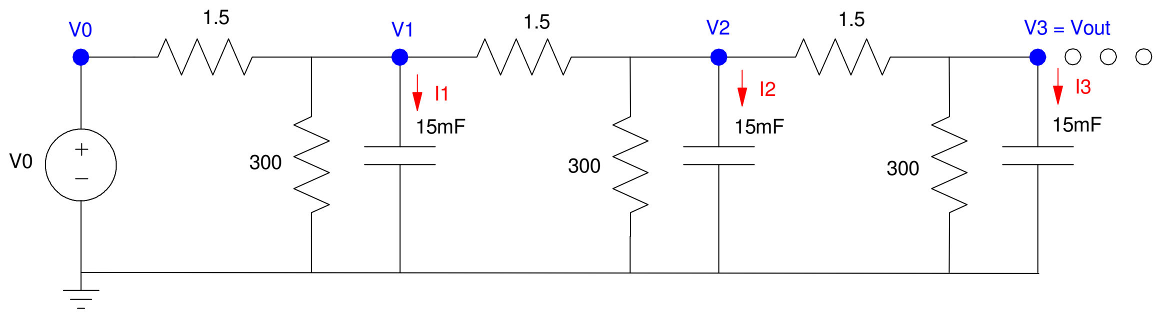
Homework #8: ECE 461/661

Gain, Lead, PID Compensation. Due Monday, October 16th

A 4th-order model for the following 10-stage RC filter is

$$G(s) = \left(\frac{9111}{(s+1.215)(s+9.025)(s+23.95)(s+44.67)}\right)$$



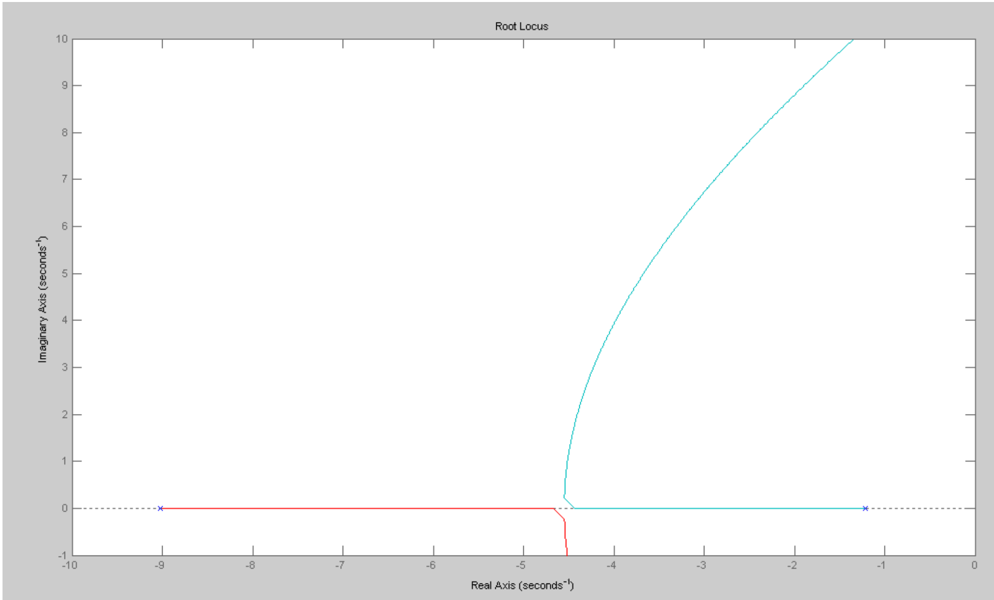
1) Design a gain compensator (K(s) = k) which results in
- The fastest system possible,
- With no overshoot for a step input (i.e. design for the breakaway point)

For this value of k, determine
- The closed-loop dominant pole(s)
- The 2% settling time,
- The error constant, Kp, and
- The steady-state error for a step input.

Check your design in Matlab or Simulink or VisSim

Translation: Place the closed-loop dominant poles at the breakaway point. In Matlab:

```
>> G = zpk([],[-1.215,-9.025,-23.95,-44.67],9111)

                  9111
------------------------------------------
(s+1.215) (s+9.025) (s+23.95) (s+44.67)

>> k = logspace(-2,2,2000)';
>> rlocus(G,k);
>> xlim([-10,0]);
>> ylim([-1,10]);
```



s = -4.5490

At any point on the root locus, G*K = -1

$$\left(\frac{9111}{(s+1.215)(s+9.025)(s+23.95)(s+44.67)}\right)_{s=-4.5490} = -0.7838$$

Pick k to make the gain one

$$k = \frac{1}{0.7838} = 1.2758$$

in Matlab

```
>> evalfr(G,s)

ans =   -0.7844

>> k = 1/abs(ans)

k =    1.2749
```

This results in...

Closed-Loop Dominant Poles:

s = {-4.5490, -4.5490}

2% Settling Time:

$$T_s = \frac{4}{4.5490} = 0.8793 \text{ seconds}$$

Kp:

$$K_p = (GK)_{s=0}$$

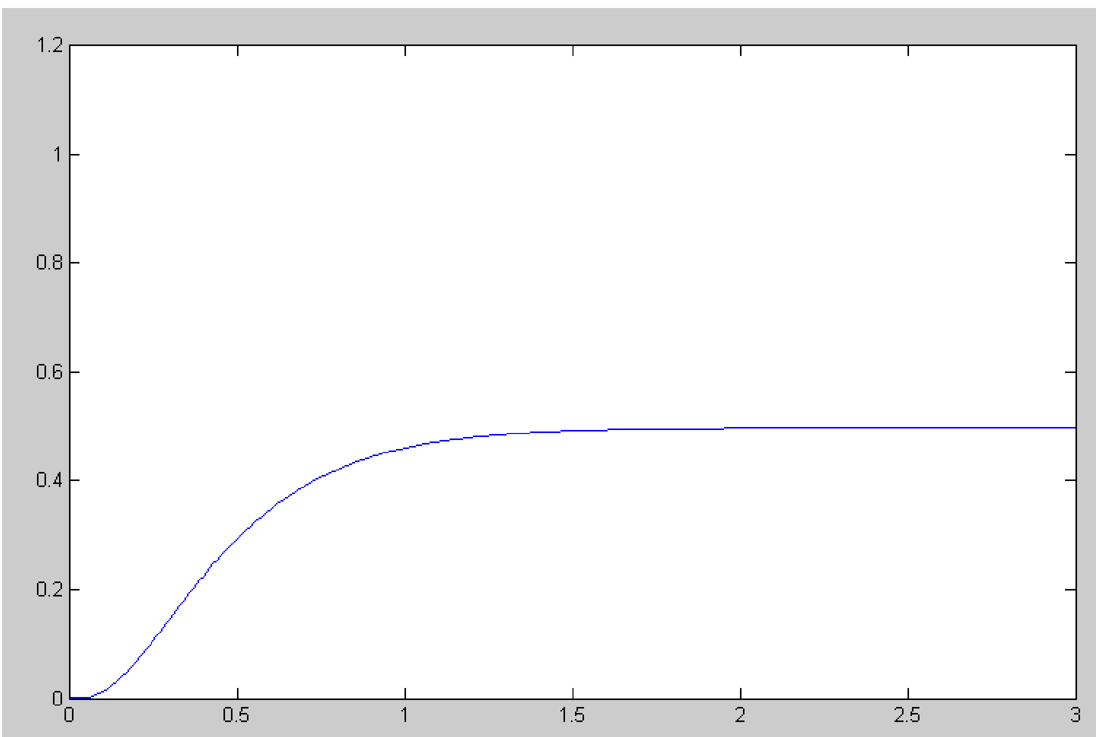
$$K_p = G(0) \cdot K(0) = 0.7766 \cdot 1.2857$$

$$K_p = 0.9985$$

Steady-State Error

$$E = \frac{1}{K_p+1} = 0.5004$$

```
>> Gcl = minreal(G*k / (1+G*k))

                 11615.8645
--------------------------------------
(s+4.549) (s+4.55) (s+25.47) (s+44.29)

>> t = [0:0.01:3]';
>> ya = step(Gcl,t);
>> plot(t,y)
```



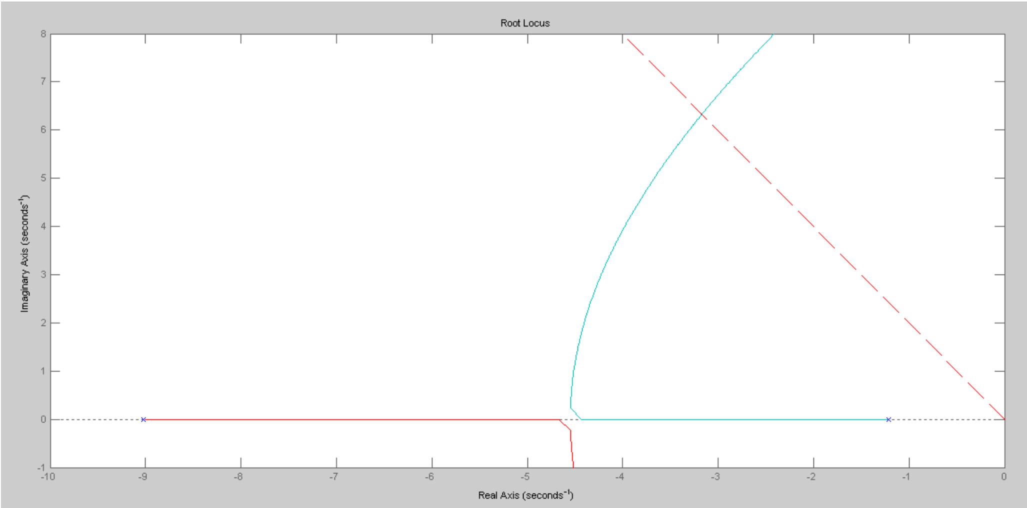
2) Design a gain compensator (K(s) = k) which results in 20% overshoot for a step input. For this value of k, determine

- The closed-loop dominant pole(s)
- The 2% settling time,
- The error constant, Kp, and
- The steady-state error for a step input.

Check your design in Matlab or Simulink or VisSim

Start with the root locus along with the damping line

```
>> G = zpk([],[-1.215,-9.025,-23.95,-44.67],9111)

                    9111
---------------------------------------
(s+1.215) (s+9.025) (s+23.95) (s+44.67)

>> k = logspace(-2,2,2000)';
>> rlocus(G,k);
>> hold on
>> plot([0,-5],[0,10],'r--')
>> xlim([-10,0]);
>> ylim([-1,8]);
```



Find the spot on the root locus which intersects the damping line

```
>> s = -3.2674 + j*6.3374;
```

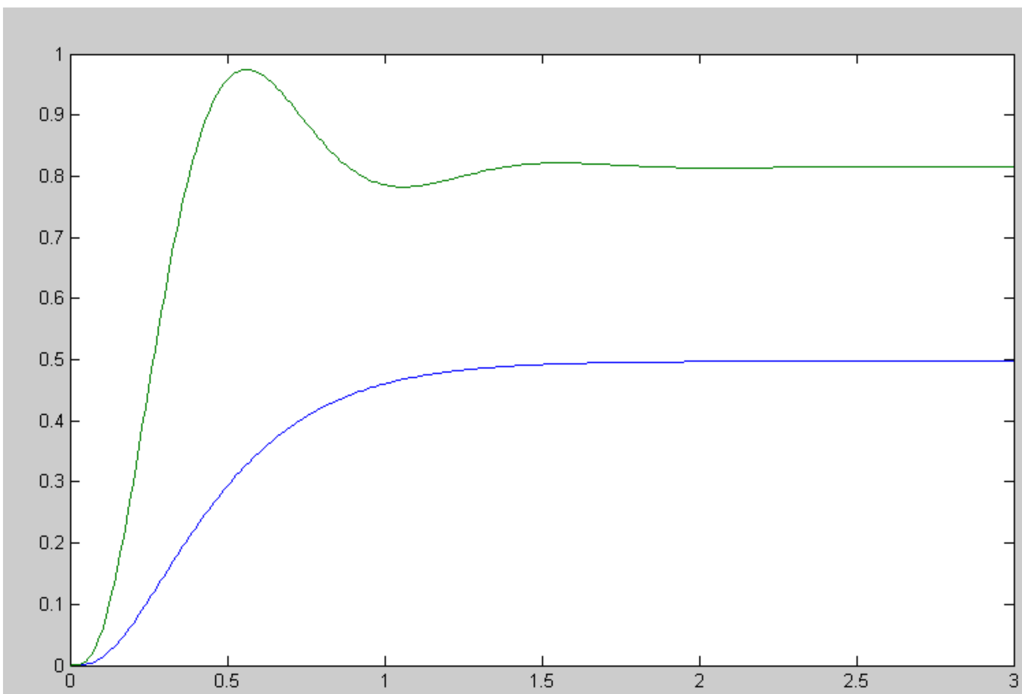
Find k so that G*K = -1 at this point

```
>> evalfr(G,s)
ans =  -0.1762 + 0.0044i

>> k = 1/abs(ans)
k =    5.6721

>> Gcl = minreal(G*k / (1+G*k));
```

```
>> eig(Gcl)

  -3.1809 + 6.3020i
  -3.1809 - 6.3020i
 -29.8020
 -42.6963
```

This results in

```
>> Ts = 4/3.1890

Ts =    1.2543

>> Kp = evalfr(G,0) * k

Kp =    4.4052

>> Estep = 1 / (Kp + 1)

Estep =    0.1850

>> t = [0:0.01:5]';
>> y2 = step(Gcl,t);
>> plot(t,y1, t, y2)
```



3)) Design a lead compensator, $K(s) = k\left(\frac{s+a}{s+10a}\right)$, which results in 20% overshoot for a step input. For this K(s), determine

- The closed-loop dominant pole(s)
- The 2% settling time,
- The error constant, Kp, and
- The steady-state error for a step input.

Check your design in Matlab or Simulink or VisSim

Give an op-amp circuit to implement K(s)

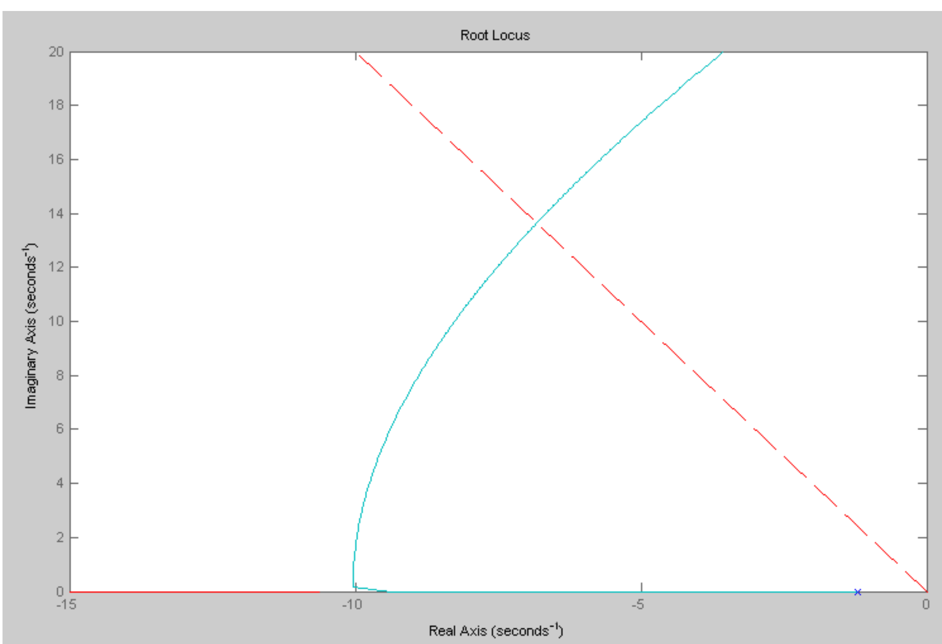
Cancel the 2nd slowest stable pole

$$G(s) = \left(\frac{9111}{(s+1.215)(s+9.025)(s+23.95)(s+44.67)}\right)$$

$$K(s) = k\left(\frac{s+9.025}{s+90.25}\right)$$

Draw the root locus and find the spot where it intersects the damping line

```
>> GK = zpk([],[-1.215,-90.25,-23.95,-44.67],9111)

                  9111
---------------------------------------
(s+1.215) (s+23.95) (s+44.67) (s+90.25)

>> k = logspace(-2,2,2000)';
>> rlocus(GK,k*10)
>> hold on
>> plot([0,-10],[0,20],'r--')
>> xlim([-15,0])
>> ylim([0,20])
```



Find the spot on the root locus where it intersects the damping line. Pick 'k' to make the gain equal to 1.000 at this point

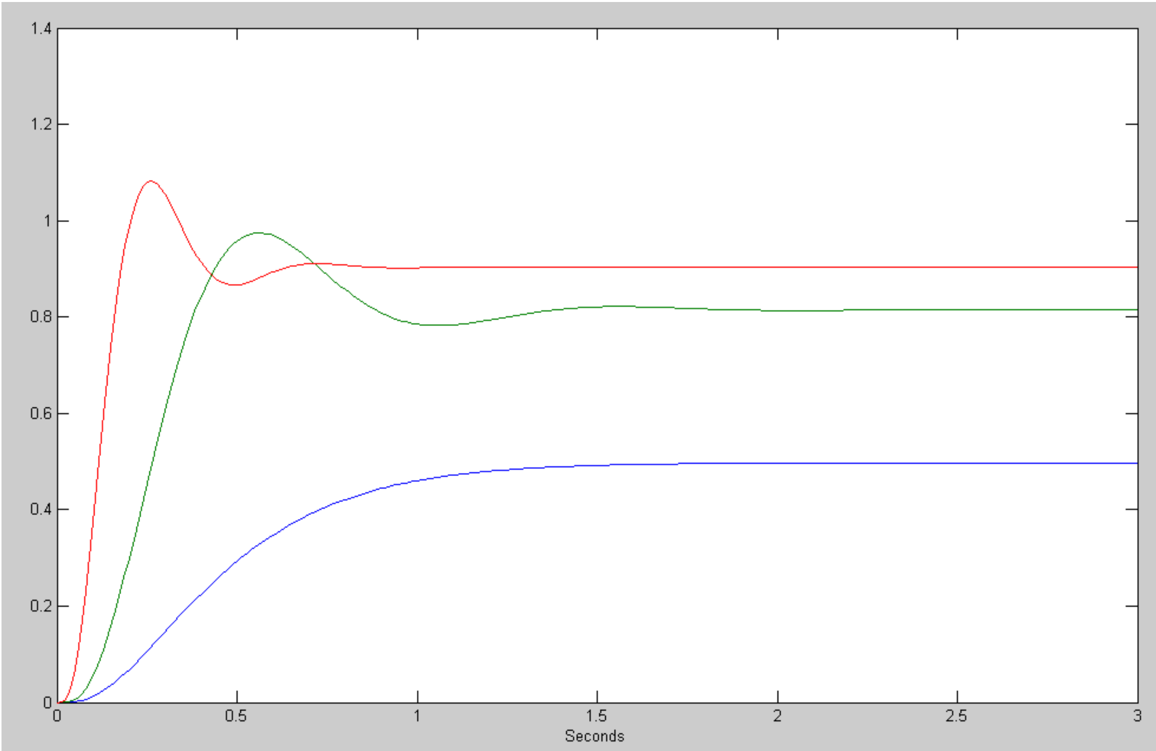
```
>> s = -6.8227 + j*13.6453;
>> evalfr(GK,s)

  -0.0083 + 0.0000i

>> k = 1/abs(ans)

k =  120.5946

>> K = zpk(-9.025,-90.25,k)

120.5946 (s+9.025)
------------------
    (s+90.25)
```

Find the closed-loop system

```
>> Gcl = minreal(G*K / (1 + G*K))

               1098737.5511
------------------------------------------
(s+61.55) (s+84.89) (s^2 + 13.65s + 232.7)

>> eig(Gcl)

  -6.8227 +13.6453i
 -61.5460
 -84.8936
  -6.8227 -13.6453i
```

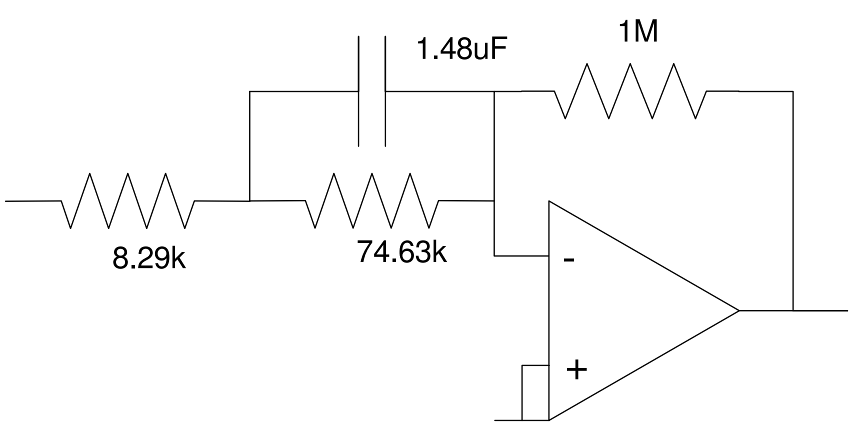
Find the settling time, Kp, E(step)

```
>> Ts = 4/6.8227

Ts =    0.5863

>> Kp = evalfr(G,0) * evalfr(K,0)

Kp =    9.3659

>> Estep = 1 / (Kp + 1)

Estep =    0.0965

>> y3 = step(Gcl,t);

>> plot(t,y1,t,y2,t,y3)
>> xlabel('Seconds')
```



Step response for breakaway point (blue), gain (green), lead (red) compensation

Op-Amp Circuit: $K(s) = \left(\frac{120.59(s+9.025)}{s+90.25} \right)$



I Compensation

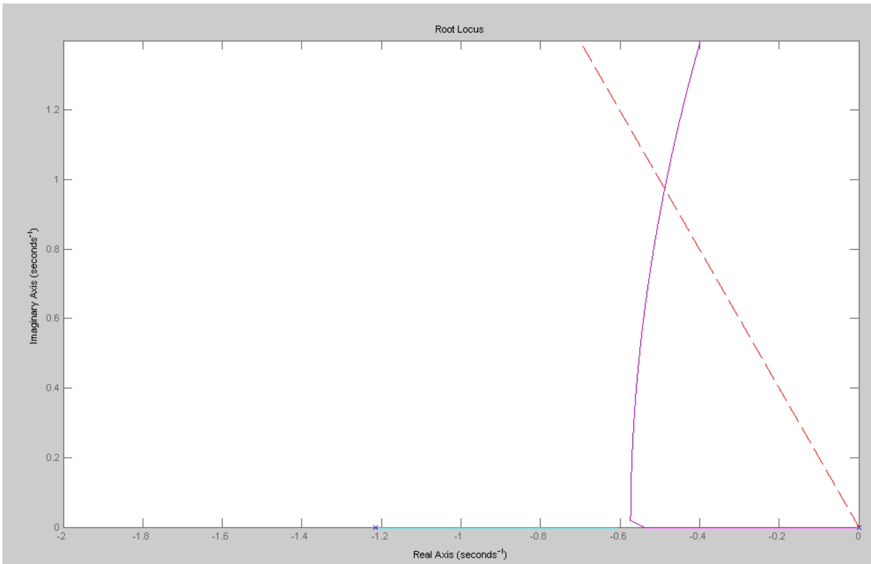
4) Design an I compensator, $K(s) = \frac{I}{s}$, which results in 20% overshoot for a step input. For this K(s), determine

- The closed-loop dominant pole(s)
- The 2% settling time,
- The error constant, Kp, and
- The steady-state error for a step input.

Check your design in Matlab or Simulink or VisSim

Give an op-amp circuit to implement K(s)

Sketch the root locus



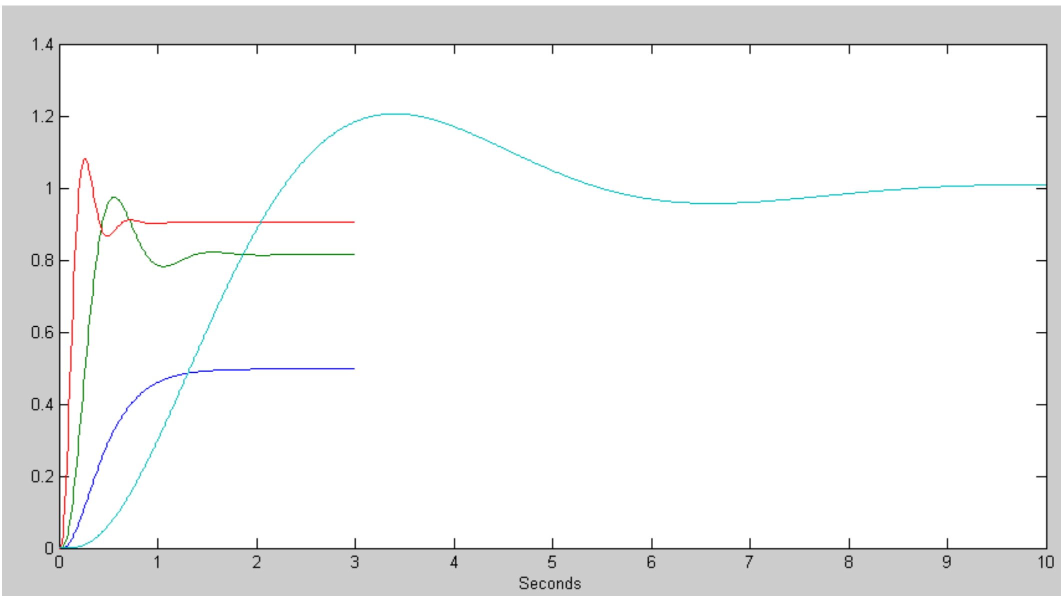
Find the spot where the root locus intersects the root locus. Pick k to make the gain 1.00 at this point

```
>> s = -0.4875 + j*0.9751;
>> evalfr(GK,s)

  -0.7703 - 0.0000i

>> k = 1/abs(ans)

k =    1.2981

>> K = zpk([],0,k)

1.2981
------
  s
```

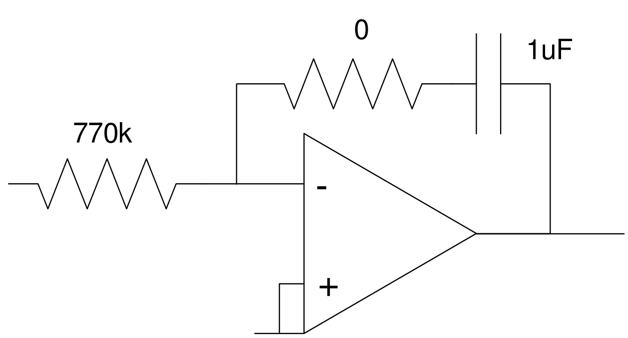
Find the response of the closed-loop system

```
>> Gcl = minreal(G*K / (1 + G*K))

                    11827.3485
-----------------------------------------------------
(s+9.327) (s+23.88) (s+44.68) (s^2 + 0.9751s + 1.189)
```

```
>> eig(Gcl)

  -0.4875 + 0.9751i
  -0.4875 - 0.9751i
  -9.3274
 -23.8792
 -44.6782

>> Ts = 4/0.4875

Ts =    8.2051

>> t4 = [0:0.01:10]';
>> y4 = step(Gcl,t);
>> plot(t,y1,t,y2,t,y3,t4,y4)
>> xlabel('Seconds')
```



Op-Amp Circuit: $K(s) = \left(\frac{1.2891}{s}\right)$



PI Compensation

5) Design a PI compensator, $K(s) = k(\frac{s+a}{s})$, which results in 20% overshoot for a step input. For this K(s), determine

- The closed-loop dominant pole(s)
- The 2% settling time,
- The error constant, Kp, and
- The steady-state error for a step input.

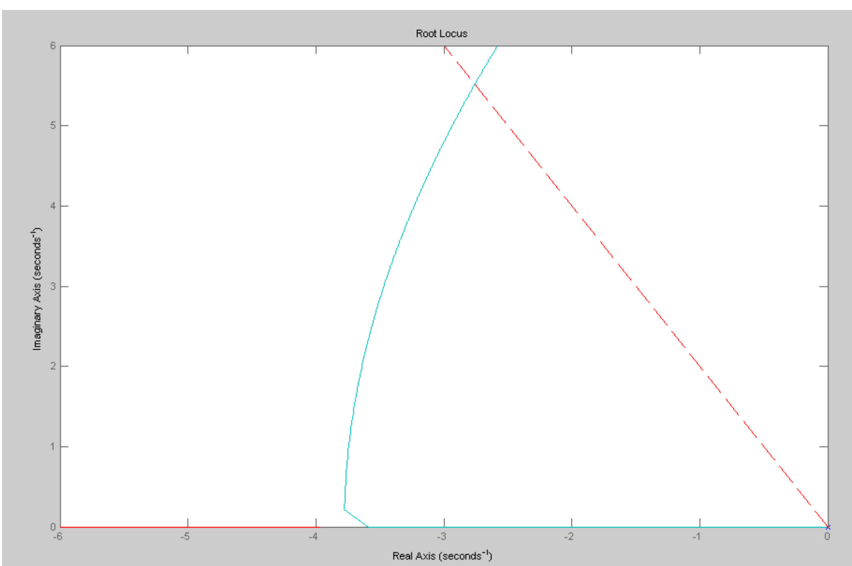
Check your design in Matlab or Simulink or VisSim

Give an op-amp circuit to implement K(s)

Let

$$K(s) = k\left(\frac{s+1.215}{s}\right)$$

```
>> GK = zpk([],[0,-9.025,-23.95,-44.67],9111)

                 9111
 --------------------------------
 s (s+9.025) (s+23.95) (s+44.67)

>> k = logspace(-2,2,2000)';
>> rlocus(GK,k)
>> hold on
>> plot([0,-10],[0,20],'r--')
>> xlim([-6,0])
>> ylim([0,6])
```



Pick 'k' to make the gain at this point:

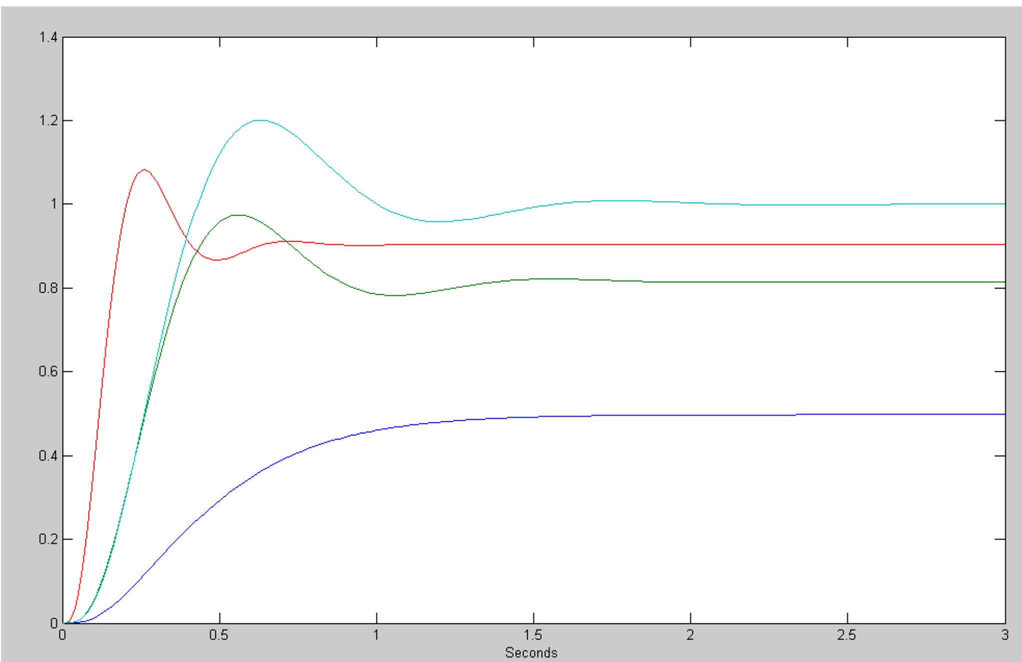
```
>> s = -2.7578 + j*5.5156;
>> evalfr(GK,s)
ans =  -0.1912 + 0.0000i

>> k = 1/abs(ans)
k =    5.2309
```

```
>> K = zpk(-1.215,0,k)
```

```
5.2309 (s+1.215)
----------------
       s
```

Plot the closed-loop step response

```
>> Gcl = minreal(G*K / (1 + G*K))

                47658.4971
------------------------------------------
(s+29.18) (s+42.95) (s^2 + 5.516s + 38.03)

>> eig(Gcl)

  -2.7578 + 5.5156i
  -2.7578 - 5.5156i
 -29.1813
 -42.9482

>> Ts = 4/2.7578

Ts =    1.4504

>> y5 = step(t,Gcl);
>> plot(t,y1,t,y2,t,y3,t,y5)
>> plot(t,y1,t,y2,t,y3,t,y5)
>> xlabel('Seconds')
```



Op-Amp Circuit: $K(s) = \left(\frac{5.23(s+1.215)}{s} \right)$