# Homework #9: ECE 461/661

Meeting Specs, Delays, Unstable Systems. Due Monday, October 23rd
20 points per problem

## Meeting Design Specs

1) Assume

$$G(s) = \left( \frac{9111}{(s+1.21)(s+9.02)(s+23.95)(s+44.67)} \right)$$

Design a compensator, K(s), For the 4th-order model that results in

- No error for a step input
- A 2% settling time of 2 seconds, and
- 20% overshoot for the step response

Check your design in Matlab or Simulink or VisSim

Give an op-amp circuit to implement K(s)

Translate:

- Make the system type-1 (no error for a step input)
- Place the closed-loop dominant pole at s = -2 + j4
- (2 second settling time, 20% overshoot)

Start with the form of K(s)

- Add a pole at s = 0 to make the system type-
- Add two zeros to cancel the poles at {-1.21, -9.02}
- Add a pole at s = -a so that s = -2 + j4 is on the root locus

$$K(s) = k\left( \frac{(s+1.21)(s+9.02)}{s(s+a)} \right)$$

$$GK = \left( \frac{9111k}{s(s+23.95)(s+44.67))(s+a)} \right)$$

Pick 'a' and 'k' so that

$$GK(s = -2 + j4) = 1\angle 180^0$$

Evaluate what we know:

$$\left( \frac{9111}{s(s+23.95)(s+44.67)} \right)_{s=-2+j4} = 2.1306\angle -132.2483^0$$

The angles are off by 47.75 degrees. Add a pole so that the angle to s = -2 + j4 is 47.75 degrees

$$a = \frac{4}{\tan\left(47.7517^0\right)} + 2$$

$$a = 5.6331$$

meaning

$$K(s) = k\left(\frac{(s+1.21)(s+9.02)}{s(s+5.6331)}\right)$$

To find k:

$$GK = \left(\frac{9111}{s(s+23.95)(s+44.67))(s+5.6331)}\right)_{s=-2+j4} = 0.3943\angle 180^0$$

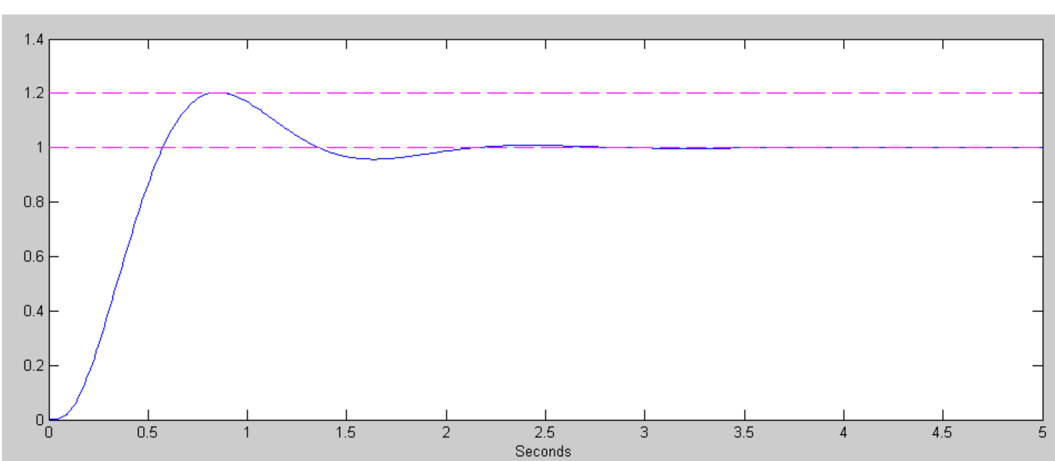
$$k = \frac{1}{0.3943} = 2.5362$$

and

$$\boxed{K(s) = 2.5362\left(\frac{(s+1.21)(s+9.02)}{s(s+5.6331)}\right)}$$

Checking in Matlab

```
>> G = zpk([],[-1.21,-9.02,-23.95,-44.67],9111)

                9111
-------------------------------------
(s+1.21) (s+9.02) (s+23.95) (s+44.67)

>> K = zpk([-1.21,-9.02],[0,-5.6331],2.5362)

2.5362 (s+1.21) (s+9.02)
------------------------
      s (s+5.633)

>> Gcl = minreal(G*K / (1+G*K));
>> eig(Gcl)

  -2.0000 + 4.0000i
  -2.0000 - 4.0000i
 -26.2667
 -43.9864

>> t = [0:0.01:5]';
>> y = step(Gcl,t);
>> plot(t,y)
>> plot(t,y,t,0*t+1,'m--',t,0*t+1.2,'m--')
>> xlabel('Seconds');
```



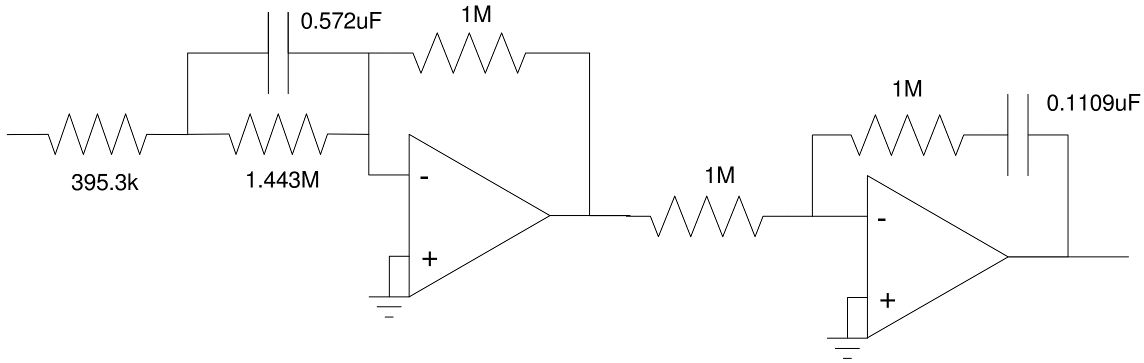
Op-Amp Circuit:

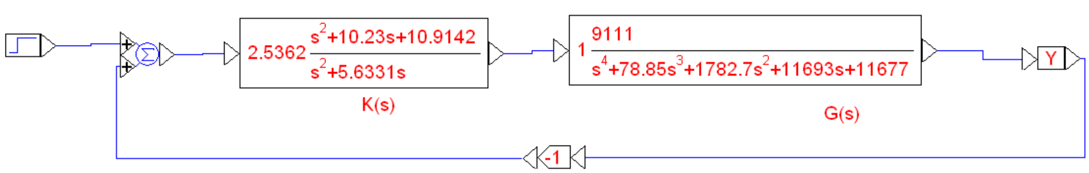
$$K(s) = 2.5362\left(\frac{(s+1.21)(s+9.02)}{s(s+5.6331)}\right)$$

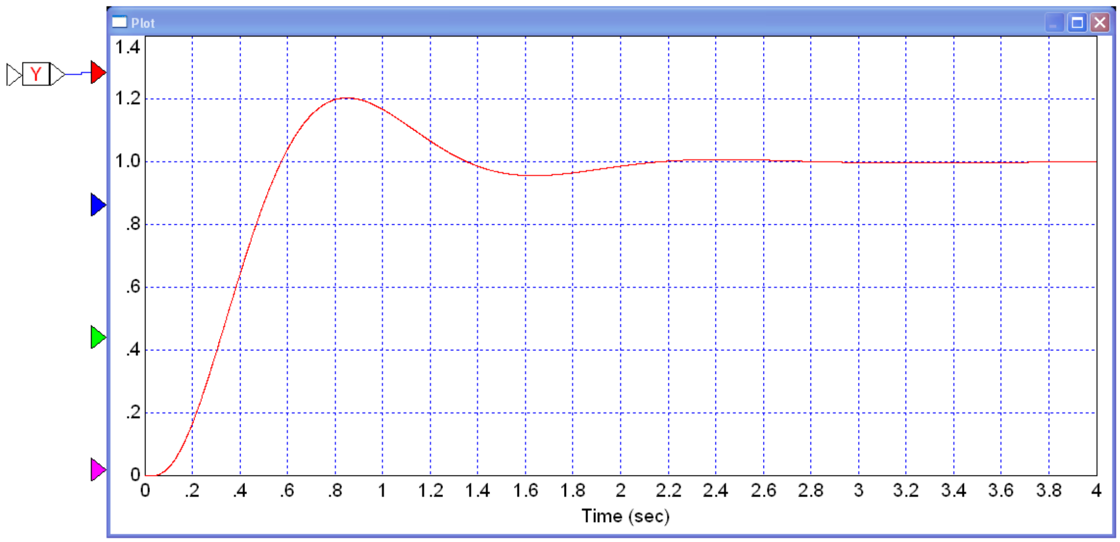
Rewrite as

$$K(s) = \left(\frac{2.5362(s+1.21)}{(s+5.6331)}\right) \cdot \left(\frac{s+9.02}{s}\right)$$

This is a Lead + PI compensator



## Systems with Delays

2) Assume a 100ms delay is added to the system

$$G(s) = \left(\frac{9111}{(s+1.21)(s+9.02)(s+23.95)(s+44.67)}\right)e^{-0.1s}$$

Design a compensator, K(s), For the 4th-order model that results in

- No error for a step input
- A 2% settling time of 2 seconds, and
- 20% overshoot for the step response

Check your design in Matlab or Simulink or VisSim

Give an op-amp circuit to implement K(s)

Same procedure: Add K(s) of the form

$$K(s) = k\left(\frac{(s+1.21)(s+9.02)}{s(s+a)}\right)$$

$$GK = \left(\frac{9111k}{s(s+23.95)(s+44.67)(s+a)}\right)e^{-0.1s}$$

so that s = -2 + j4 is on the root locus. Evaludate what we know:

$$\left(\left(\frac{9111}{s(s+23.95)(s+44.67)}\right)e^{-0.1s}\right)_{s=-2+j4} = 2.6023\angle-155.1666^0$$

Pick 'a' so that the angles add up to 180 degrees at s = -2 + j4

$$a = \frac{4}{\tan(24.8334^0)} + 2$$

$$a = 10.6436$$

meaning

$$K(s) = k\left(\frac{(s+1.21)(s+9.02)}{s(s+10.6436)}\right)$$

Pick k so that the gain of GK = -1

$$\left(\left(\frac{9111}{s(s+23.95)(s+44.67)(s+10.6436)}\right)e^{-0.1s}\right)_{s=-2+j4} = 0.2732\angle180^0$$

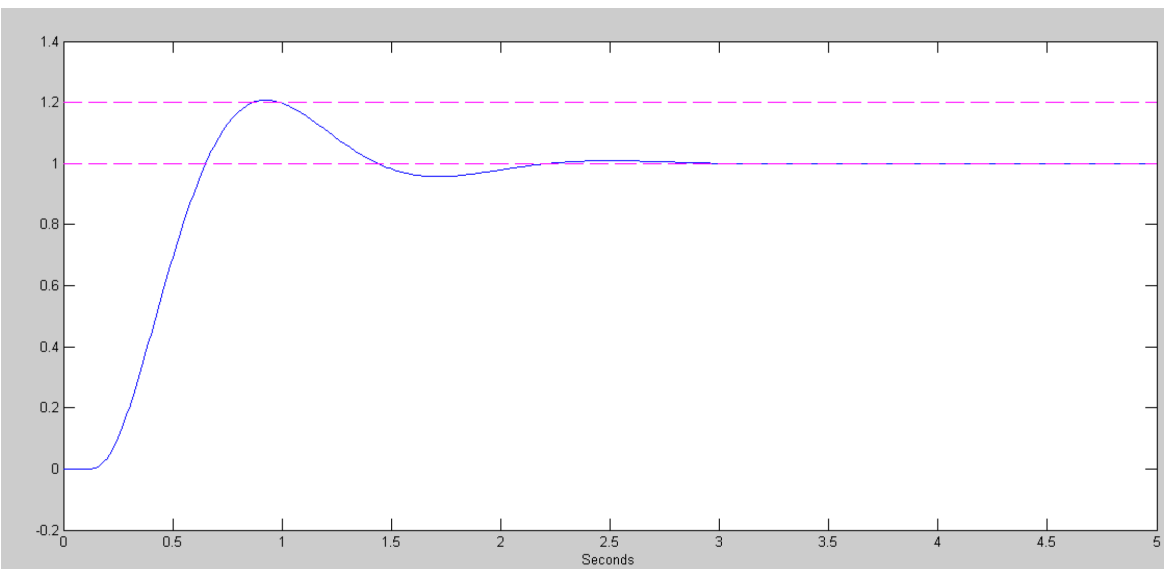
$$k = \frac{1}{0.2732} = 3.6599$$

and

$$\boxed{K(s) = 3.6599\left(\frac{(s+1.21)(s+9.02)}{s(s+10.6436)}\right)}$$

Checking in Matlab

```
>> G = zpk([],[-1.21,-9.02,-23.95,-44.67],9111)

                 9111
-------------------------------------
(s+1.21) (s+9.02) (s+23.95) (s+44.67)

>> K = zpk([-1.21,-9.02],[0,-10.6436],3.6599)

3.6599 (s+1.21) (s+9.02)
------------------------
      s (s+10.64)

>> [num,den] = Pade(0.1,6);
>> Delay = tf(num,den);
>> Gcl = minreal(G*Delay*K / (1 + G*Delay*K));
>> eig(Gcl)

  -2.0000 + 4.0000i
  -2.0000 - 4.0000i
 -27.3908 +20.6794i
 -27.3908 -20.6794i
 -99.7828 +27.5621i
 -70.1580 +58.6448i
 -50.3002 +89.3277i
 -99.7828 -27.5621i
 -70.1580 -58.6448i
 -50.3002 -89.3277i

>> t = [0:0.01:5]';
>> y = step(Gcl,t);
>> plot(t,y,t,0*t+1,'m--',t,0*t+1.2,'m--')
>> xlabel('Seconds');
```

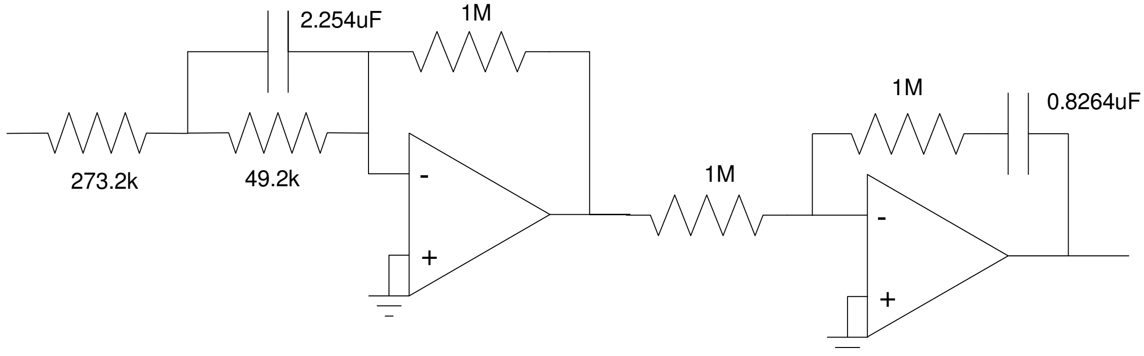


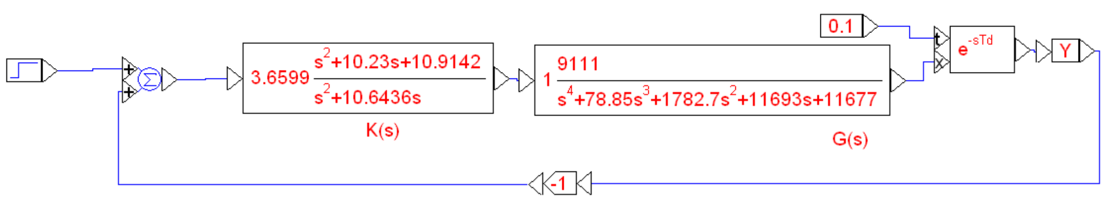
Give an op-amp circuit to implement K(s)

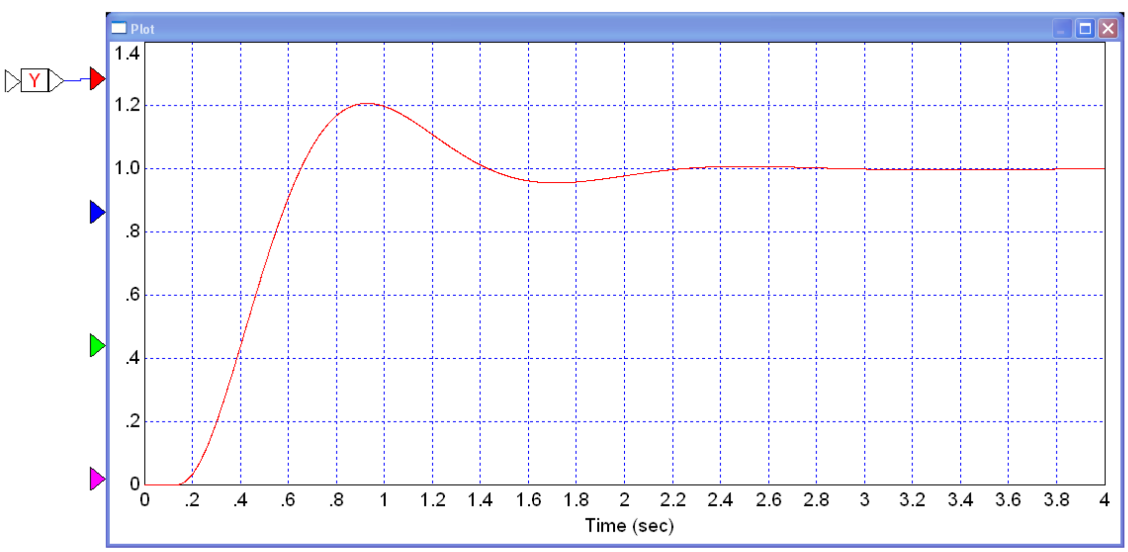
$$K(s) = 3.6599\left(\frac{(s+1.21)(s+9.02)}{s(s+10.6436)}\right)$$

Rewrite as

$$K(s) = \left(\frac{3.6599(s+9.02)}{(s+10.6436)}\right)\left(\frac{s+1.21}{s}\right)$$



## Unstable Systems

3) Assume the slow pole was unstable

$$G(s) = \left(\frac{9111}{(s-1.21)(s+9.02)(s+23.95)(s+44.67)}\right)$$

Design a compensator, K(s), For the 4th-order model that results in

- No error for a step input
- A 2% settling time of 2 seconds, and
- 20% overshoot for the step response

Check your design in Matlab or Simulink or VisSim

Step 1: Stabilze the system. Don't worry about the requirements.

A gain compensator works. Place the closed-loop dominant pole at s = -1 (nice round number)

$$\left(\frac{9111}{(s-1.21)(s+9.02)(s+23.95)(s+44.67)}\right)_{s=-1} = -0.5129$$

$$k = \frac{1}{0.5129} = 1.9497$$

The closed-loop system is then

$$G_2 = \left(\frac{Gk}{1+Gk}\right) = \left(\frac{17763.7167}{(s+1)(s+5.305)(s+26.01)(s+44.12)}\right)$$

```
>> G = zpk([],[1.21,-9.02,-23.95,-44.67],9111)

Zero/pole/gain:
                9111
-------------------------------------
(s-1.21) (s+9.02) (s+23.95) (s+44.67)

>> k1 = 1.9497;
>> G2 = minreal(G*k1 / (1+G*k1))

Zero/pole/gain:
            17763.7167
-----------------------------------
(s+1) (s+5.305) (s+26.01) (s+44.12)
```

Step 2: Now that we have a stable system, add K2(s) to meet the requirements

$$K_2 = k\left(\frac{(s+1)(s+5.305)}{s(s+a)}\right)$$

Pick 'a' so that the angles add up to 180 degrees at s = -2 + j4

```
>> G2K2 = zpk([],[0,-26.01,-44.12],17763.7167)

     17763.7167
---------------------
s (s+26.01) (s+44.12)
```

```
>> s = -2 + j*4;
>> x = evalfr(G2K2, s)

x =  -2.5531 - 2.8910i

>> angle(x)*180/pi

ans = -131.4484

>> 180 + ans

ans =   48.5516
```

$$a = \frac{4}{\tan\left(48.5516^0\right)} + 2$$

$$a = 5.5325$$

meaning

$$K_2 = k\left(\frac{(s+1)(s+5.305)}{s(s+5.5325)}\right)$$

Find 'k' so that the gain is one at s = -2 + j4

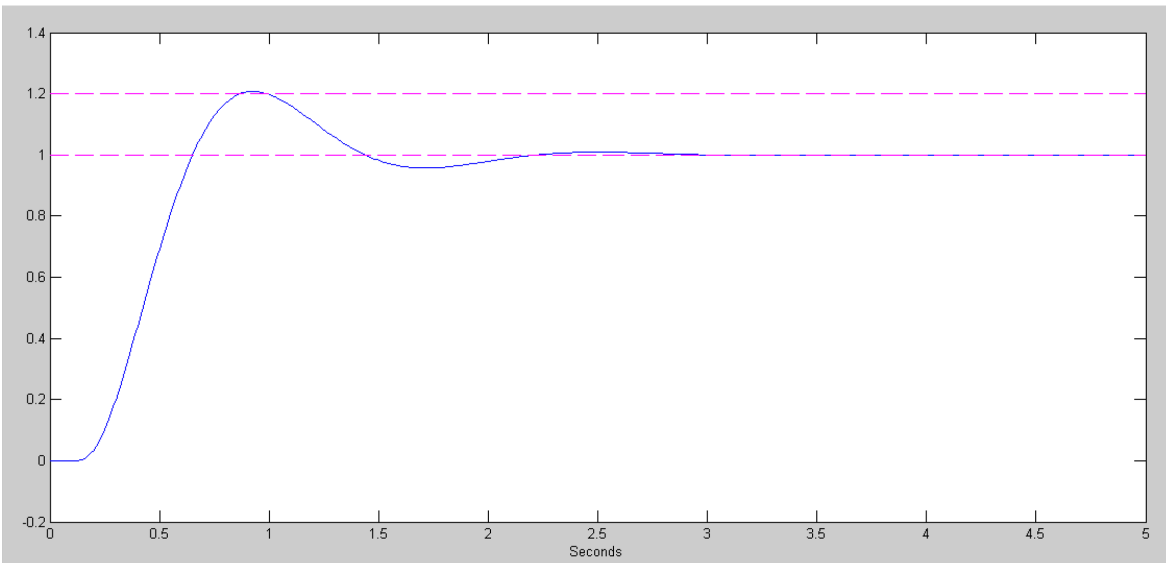
```
>> G2

             17763.7167
-----------------------------------
(s+1) (s+5.305) (s+26.01) (s+44.12)

>> K2 = zpk([-1,-5.305],[0,-5.5325],1)

(s+1) (s+5.305)
---------------
  s (s+5.532)

>> evalfr(G2*K2, -2+j*4)

ans =  -0.7228 - 0.0000i

>> k = 1/abs(ans)

k =    1.3834
```

So now we have K2:

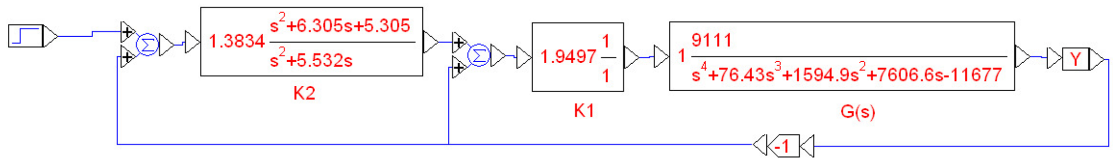
```
>> K2 = zpk([-1,-5.305],[0,-5.5325],k)

1.3834 (s+1) (s+5.305)
----------------------
     s (s+5.532)
```

The net closed-loop system is:

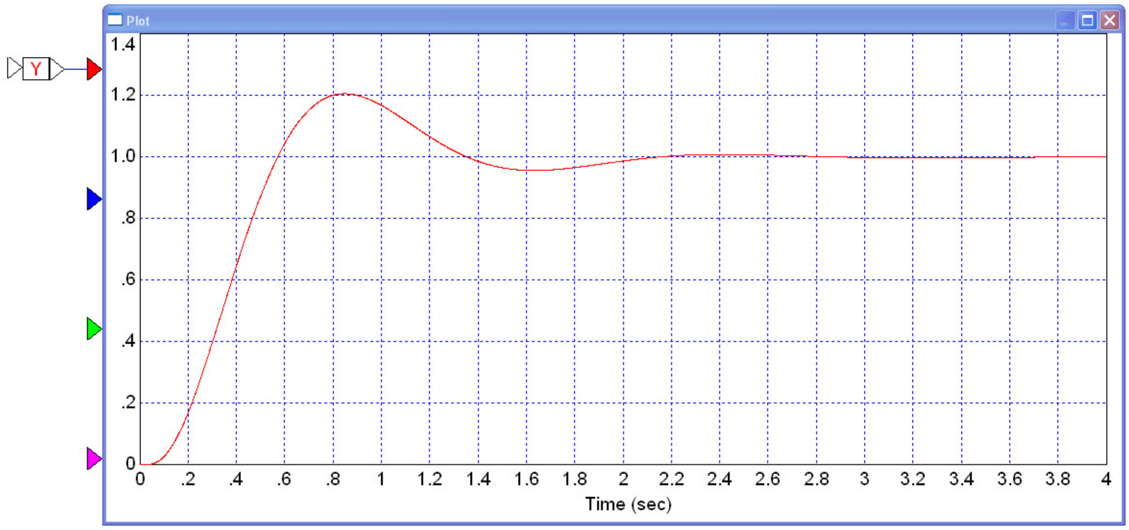
```
>> G3 = minreal(G2*K2 / (1 + G2*K2));

>> eig(G3)

   -1.0000
   -1.0000
   -2.0000 + 4.0000i
   -5.3050
   -2.0000 - 4.0000i
  -28.4157
  -43.2417

>> t = [0:0.01:5]';
>> y = step(Gcl,t);
>> plot(t,y,t,0*t+1,'m--',t,0*t+1.2,'m--')
>> xlabel('Seconds');
```




This actually shows off better in VisSim



1.3834 s^2+6.305s+5.305 / s^2+5.532s
K2
1.9497 1/1
K1
1 9111 / s^4+76.43s^3+1594.9s^2+7606.6s-11677
G(s)
Y
-1

Y
Plot
1.4
1.2
1.0
.8
.6
.4
.2
0
0 .2 .4 .6 .8 1 1.2 1.4 1.6 1.8 2 2.2 2.4 2.6 2.8 3 3.2 3.4 3.6 3.8 4
Time (sec)