Homework #11: ECE 461/661

Digital PID Control. Due Monday, November 13th

PID Control

Assume T = 0.1 seconds:

$$G(s) = \left(\frac{9111}{(s+1.21)(s+9.02)(s+23.95)(s+44.67)}\right)$$

1) Design a digital I controller

$$K(z) = k\left(\frac{z}{z-1}\right)$$

that results in 20% overshoot in the step response.

Simulate the step response of the closed-loop system (VisSim or Simulink preferred with K(z)*G(s))

Using numerical methods, find the point on the damping line where the phase is 180 degrees

$$G(s) \cdot zoh \cdot K(z) = 1\angle 180^0$$

$$\angle\left(\left(\frac{9111}{(s+1.21)(s+9.02)(s+23.95)(s+44.67)}\right) \cdot \exp\left(\frac{-sT}{2}\right) \cdot \left(\frac{z}{z-1}\right)\right)_{s=\alpha(-1+j2)} = 180^0$$

Doing a numerical search

s = -0.4861 + j0.9721

z = 0.9481 + j0.0925

$$G(s) \cdot zoh \cdot K(z) = 7.7605\angle 180^0$$

Pick k to make the gain equal to one

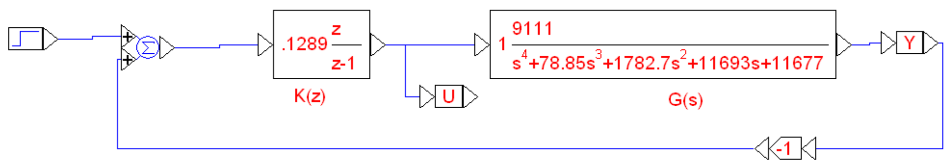
$$k = \frac{1}{7.7605} = 0.1289$$

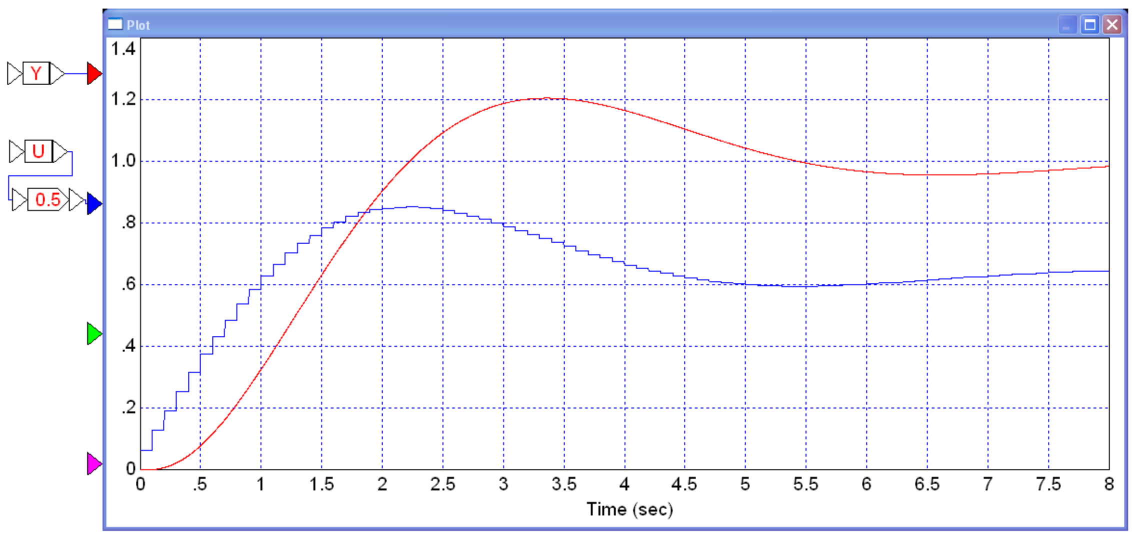
In Matlab

```
>> G = zpk([],[-1.21,-9.02,-23.95,-44.67],9111);
>> s = -0.3 + j*0.6;
>> T = 0.1;
>> z = exp(s*T);
>> evalfr(G,s) * exp(-s*T/2) * (z / (z-1))

ans = -12.4186 - 5.4982i

>> s = s * 1.1;
>> z = exp(s*T);
>> evalfr(G,s) * exp(-s*T/2) * (z / (z-1))

ans = -11.5681 - 4.1600i

>> s = s * 1.1;
>> z = exp(s*T);
>> evalfr(G,s) * exp(-s*T/2) * (z / (z-1))

ans = -10.6826 - 2.9326i
```

time passes.....

```
>> s = s * 0.999;
>> z = exp(s*T);
>> evalfr(G,s) * exp(-s*T/2) * (z / (z-1))

ans =  -7.7622 - 0.0013i

>> k = 1/abs(ans)

k =     0.1288


s =  -0.4860 + 0.9720i
z =   0.9481 + 0.0924i
```





2) Assume T = 0.1 seconds and

$$G(s) = \left(\frac{9111}{(s+1.21)(s+9.02)(s+23.95)(s+44.67)}\right)$$

Design a digital PI controller

$$K(z) = k\left(\frac{z-a}{z-1}\right)$$

that results in 20% overshoot in the step response.

Simulate the step response of the closed-loop system (VisSim or Simulink preferred with K(z)*G(s))

Step 1: Pick 'a' to cancel a pole

s = -1.21

$$z = e^{sT} = 0.8860$$

$$K(z) = k\left(\frac{z-0.8860}{z-1}\right)$$

Step 2: Search along the damping line until the angles add up to 180 degrees

$$G(s) \cdot zoh \cdot K(z) = 1\angle 180^0$$

$$\left(\left(\frac{9111}{(s+1.21)(s+9.02)(s+23.95)(s+44.67)}\right) \cdot \exp\left(\frac{-sT}{2}\right) \cdot \left(\frac{z-0.8860}{z-1}\right)\right)_s = 180^0$$

In Matlab

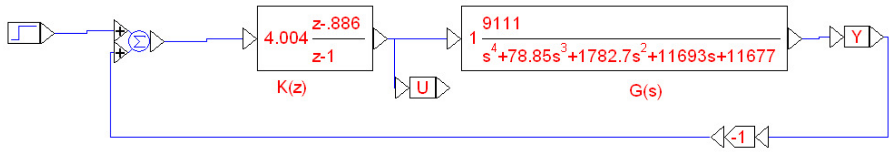
```
>> G = zpk([],[-1.21,-9.02,-23.95,-44.67],9111);
>> s = -2 + j*4;
>> z = exp(s*T);
>> evalfr(G,s) * exp(-s*T/2) * ( (z-0.9960) / (z-1))

ans =  -0.2955 - 0.1167i

>> s = 1.1*s;
>> z = exp(s*T);
>> evalfr(G,s) * exp(-s*T/2) * ( (z-0.9960) / (z-1))

ans =  -0.2828 - 0.0712i

>> s = 1.1*s;
>> z = exp(s*T);
>> evalfr(G,s) * exp(-s*T/2) * ( (z-0.9960) / (z-1))

ans =  -0.2653 - 0.0291i
```

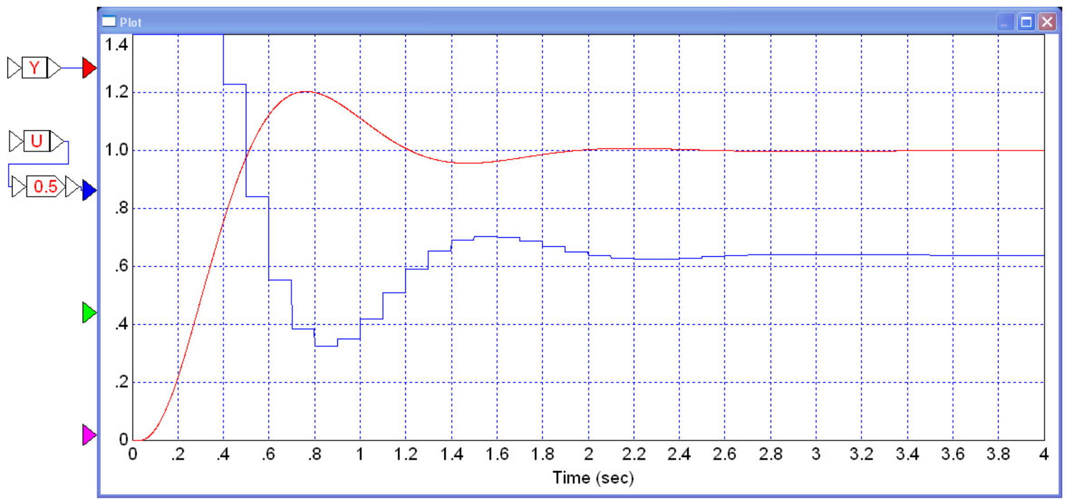
time passes...

```
>> s = 1.01*s;
>> z = exp(s*T);
>> evalfr(G,s) * exp(-s*T/2) * ( (z-0.9960) / (z-1))

ans =  -0.2497 - 0.0010i

>> k = 1/abs(ans)

k =    4.0047
s =  -2.5946 + 5.1891i
z =   0.6699 + 0.3826i
```

so

$$K(z) = 4.004\left(\frac{z-0.8860}{z-1}\right)$$





3) Assume T = 0.1 seconds and

$$G(s) = \left(\frac{9111}{(s+1.21)(s+9.02)(s+23.95)(s+44.67)}\right)$$

Design a digital PID controller

$$K(z) = k\left(\frac{(z-a)(z-b)}{z(z-1)}\right)$$

that results in 20% overshoot in the step response.

Simulate the step response of the closed-loop system (VisSim or Simulink preferred with K(z)*G(s))

Step 1: Pick the zeros to cancel the poles at {-1.21, -9.02}

$$s = -1.21 \qquad z = e^{sT} = 0.8860$$

$$s = -9.02 \qquad z = e^{sT} = 0.4058$$

$$K(z) = k\left(\frac{(z-0.8860)(z-0.4058)}{z(z-1)}\right)$$

Step 2: Find the point on the damping line where the angles add up to 180 degrees

$$G(s) \cdot zoh \cdot K(z) = 1\angle 180^0$$

$$\left(\left(\frac{9111}{(s+1.21)(s+9.02)(s+23.95)(s+44.67)}\right) \cdot \exp\left(\frac{-sT}{2}\right) \cdot \left(\frac{(z-0.8860)(z-0.4058)}{z(z-1)}\right)\right)_s = 180^0$$

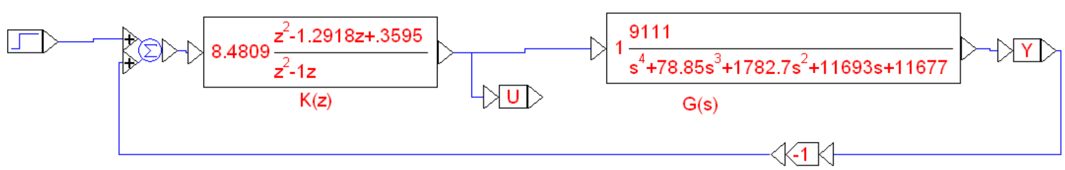
Searching in Matlab

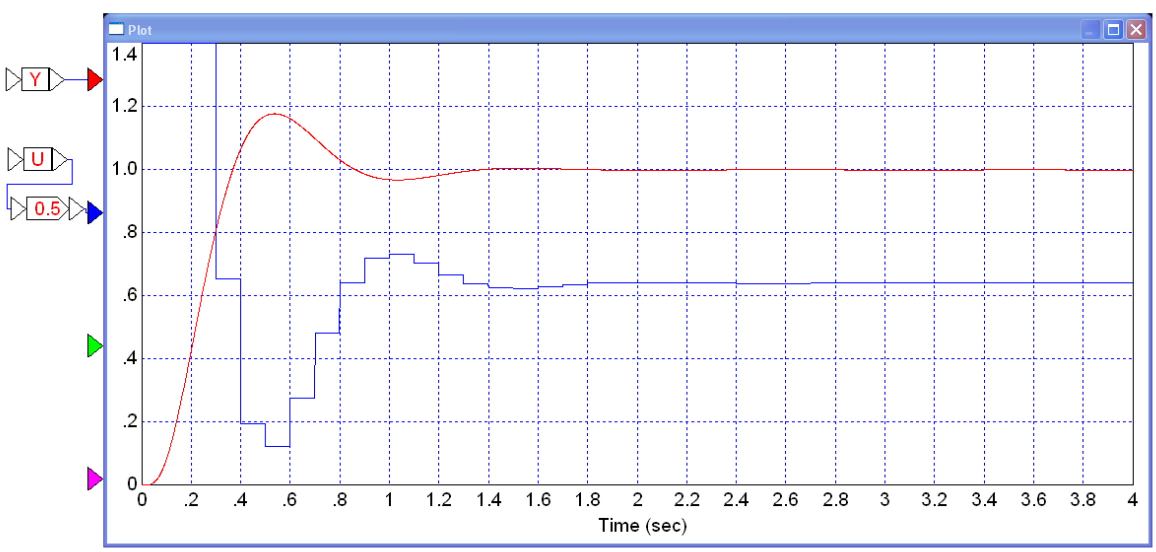
```
>> G = zpk([],[-1.21,-9.02,-23.95,-44.67],9111);
>> T = 0.1;
>> s = -4 + j*8;
>> z = exp(s*T);
>> evalfr(G,s) * exp(-s*T/2) * ( (z-0.9960)*(z-0.4058) / (z*(z-1)))

ans =  -0.1117 + 0.0124i

>> s = s * 1.1;
>> z = exp(s*T);
>> evalfr(G,s) * exp(-s*T/2) * ( (z-0.9960)*(z-0.4058) / (z*(z-1)))

ans =  -0.1023 + 0.0277i

>> s = s * 0.9;
>> z = exp(s*T);
>> evalfr(G,s) * exp(-s*T/2) * ( (z-0.9960)*(z-0.4058) / (z*(z-1)))

ans =  -0.1126 + 0.0107i

>> s = s * 0.9;
>> z = exp(s*T);
>> evalfr(G,s) * exp(-s*T/2) * ( (z-0.9960)*(z-0.4058) / (z*(z-1)))

ans =  -0.1208 - 0.0072i

time passes....
```

```
>> s = s * 1.01;
>> z = exp(s*T);
>> evalfr(G,s) * exp(-s*T/2) * ( (z-0.9960)*(z-0.4058) / (z*(z-1)))

ans =  -0.1179 - 0.0003i

>> k = 1/abs(ans)

k =    8.4809

s =  -3.7087 + 7.4174i

z =   0.5088 + 0.4662i
```

so

$$K(z) = 8.4809\left(\frac{(z-0.8860)(z-0.4058)}{z(z-1)}\right)$$





Meeting Design Specs

4) Assume a sampling rate of T = 0.1 seconds and

$$G(s) = \left(\frac{9111}{(s+1.21)(s+9.02)(s+23.95)(s+44.67)}\right)$$

Design a digital controller that results in

- No error for a step input
- 20% overshoot for the step response, and
- A 2% settling time of 2 seconds

Simulate the step response of the closed-loop system (VisSim or Simulink preferred with K(z)*G(s))

Translate:

- Make this a type-1 system
- Place the closed-loop dominant poles at
  - s = -2 + j4
  - z = 0.7541 + j0.3188

Let

$$K(s) = k\left(\frac{(s+1.21)(s+9.02)}{s(s+a)}\right)$$

With T = 0.1

$$K(z) = k\left(\frac{(z-0.886)(z-0.4058)}{(z-1)(z-a)}\right)$$

Evaluate what we know

$$G(s) \cdot \exp\left(\frac{-sT}{2}\right) \cdot K(z) = 180^0$$

```
>> G = zpk([],[-1.21,-9.02,-23.95,-44.67],9111);
>> s = -2 + j*4;
>> T = 0.1;
>> z = exp(s*T)

z =   0.7541 + 0.3188i

>> GK = evalfr(G,s) * exp(-s*T/2) * (z-0.886)*(z-0.4058)/(z-1)

GK =  -0.0843 - 0.0981i

>> angle(GK)*180/pi

ans = -130.6852
```

To make the angles add up to 180 degrees, the pole at z = -a needs to add 49.3148 degrees

$$\angle(z-a) = 49.3148^0$$

$$a = 0.7541 - \frac{0.3188}{\tan\left(49.3148^0\right)}$$

```
>> a = real(z) - imag(z)/tan(pi + angle(GK))
a =     0.4800
```

Checking

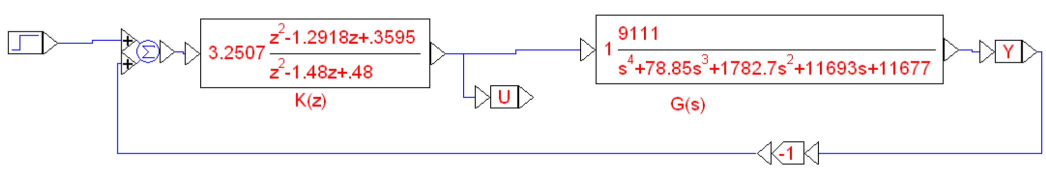
```
>> GK = evalfr(G,s) * exp(-s*T/2) * (z-0.886)*(z-0.4058)/((z-1)*(z-0.4800))

GK =  -0.3076 - 0.0000i
```

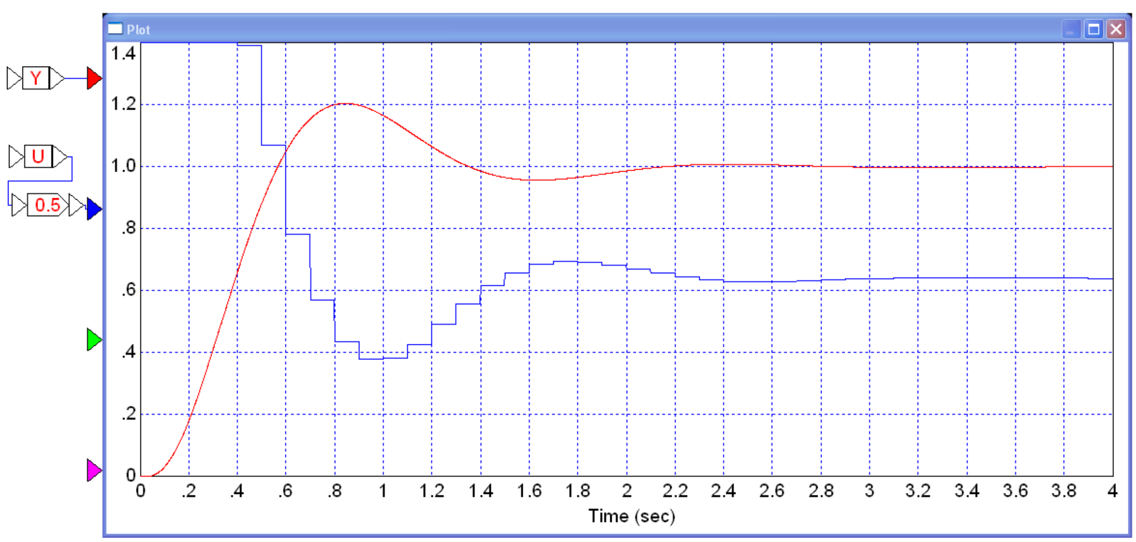
The complex part is zero, so 'a' is correct. Finding k

```
>> k = 1/abs(GK)

k =     3.2507
```

and

$$K(z) = 3.2507\left(\frac{(z-0.886)(z-0.4058)}{(z-1)(z-0.48)}\right)$$





5) Assume

$$G(s) = \left(\frac{9111}{(s+1.21)(s+9.02)(s+23.95)(s+44.67)}\right)$$

Design a digital controller with T = 0.2 seconds that results in

- No error for a step input
- 20% overshoot for the step response, and
- A 2% settling time of 2 seconds

Simulate the step response of the closed-loop system (VisSim or Simulink preferred with K(z)*G(s))

Note: Changing the sampling rate is a big deal: it means a complete redesign of K(z)

Grumble slightly - changing T means a complete redesign.

Let

$$K(s) = k\left(\frac{(s+1.21)(s+9.02)}{s(s+a)}\right)$$

With T = 0.2

$$K(z) = k\left(\frac{(z-0.7851)(z-0.1646)}{(z-1)(z-a)}\right)$$

Place the closed-loop dominant pole at

s = -2 + j4

z = 0.4670 + j0.4809

Evaluate what we know

```
>> T = 0.2;
>> s = -2 + j*4;
>> z = exp(s*T)

z =   0.4670 + 0.4809i

>> GK = evalfr(G,s) * exp(-s*T/2) * (z-0.7851)*(z-0.1646)/((z-1))

GK =  -0.0949 - 0.1303i

>> angle(GK)*180/pi

ans = -126.0733

>> 180 + ans

ans =   53.9267
```

The pole at z = a needs to subtract 53.9267 degrees for the angles to add up.

```
>> a = real(z) - imag(z)/tan(pi + angle(GK))

a =    0.1167
```

Checking:

```
>> GK = evalfr(G,s)*exp(-s*T/2)*(z-0.7851)*(z-0.1646)/((z-1)*(z-0.1167))

GK =  -0.2709 - 0.0000i
```

Check - the imaginary part is zero. Pick 'k' to make the gain one

```
>> k = 1/abs(GK)

k =    3.6912
```

giving

$$K(z) = 3.6912\left(\frac{(z-0.7851)(z-0.1646)}{(z-1)(z-0.1167)}\right)$$