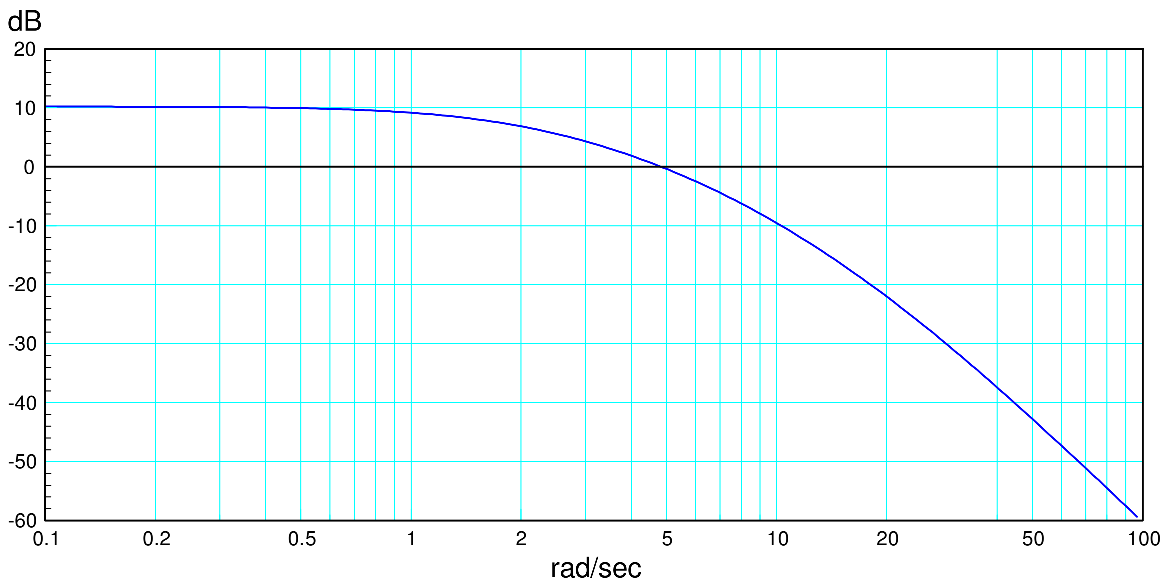
Homework #12: ECE 461/661

Bode Plots. Nichols charts and gain & lead compensation.   Due Monday, November 27th

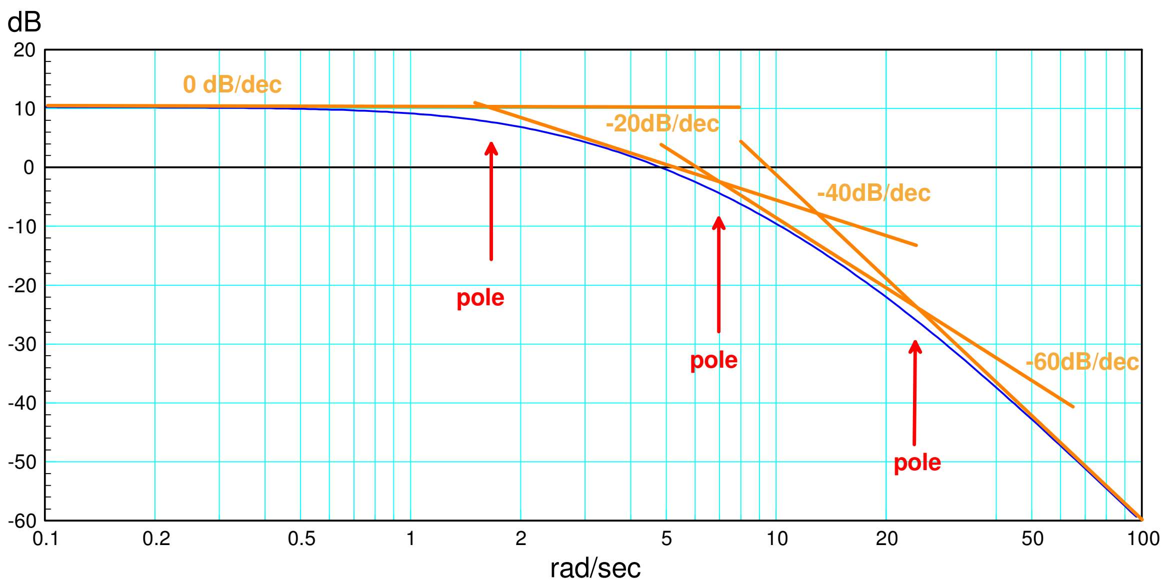
Bode Plots

1) Determine the system, G(s), with the following gain vs. frequency



Start by drawing in the asymptotes with slopes multiples of 20dB / decade

- Each corner indicates a pole



This gives

$$G(s) = \left(\frac{k}{(s+1.8)(s+7)(s+22)}\right)$$

Pick k to match the DC gain (10dB)

$$gain = 10^{dB/20}$$

$$10dB = 10^{10/20} = 3.162$$

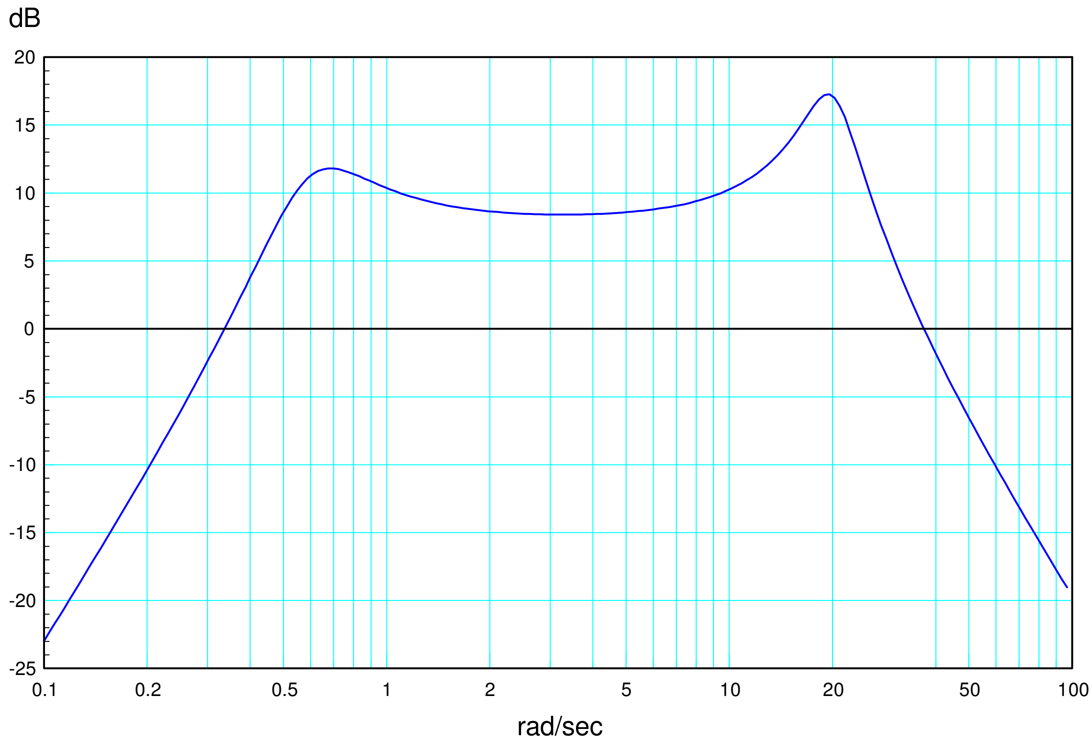
$$\left(\frac{k}{(s+1.8)(s+7)(s+22)}\right)_{s=0} = 3.162$$

$$k = 876.6$$

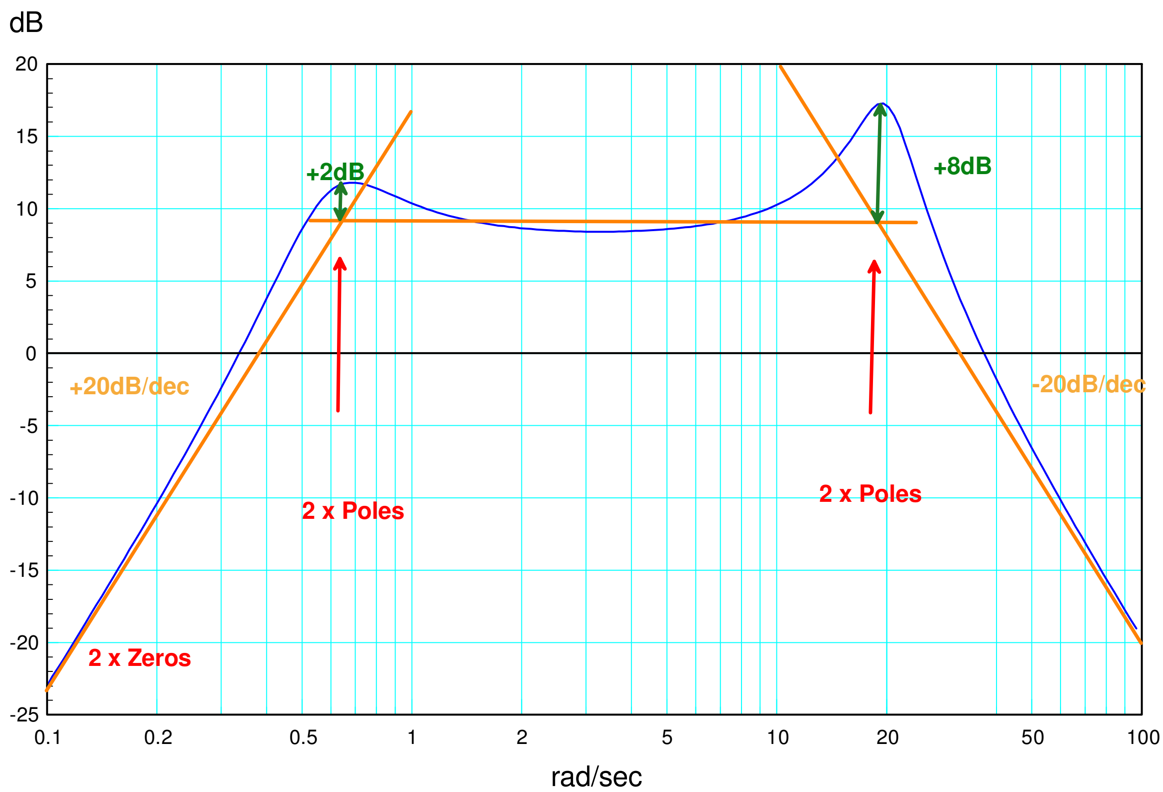
so

$$G(s) \approx \left(\frac{876.6}{(s+1.8)(s+7)(s+22)}\right)$$

2) Determine the system, G(s), with the following gain vs. frequency



First, draw in the asymptotes at multiples of 20dB/decade



Two zeros at s = 0 (left of 0.1)

Two poles at 0.61 rad/sec

$$\frac{1}{2\zeta} = +2dB = 1.259$$

$$\zeta = 0.397$$

$$\theta = 66.6^0$$

Two poles at 19 rad/sec

$$\frac{1}{2\zeta} = +8dB = 2.512$$

$$\zeta = 0.199$$

$$\theta = 78.5^0$$

So, G(s) is in the form of

$$G(s) = \left(\frac{ks^2}{\left(s+0.61\angle\pm66.6^0\right)\left(s+19\angle\pm78.5^0\right)} \right)$$

To find k, pick a frequency and match the gain. Setting the gain at 3 rad/sec = +9dB

$$\left(\frac{ks^2}{\left(s+0.61\angle\pm66.6^0\right)\left(s+19\angle\pm78.5^0\right)} \right)_{s=j3} = +9dB = 2.818$$

$$k = 963.3$$

so

$$G(s) = \left(\frac{963.3s^2}{\left(s+0.61\angle\pm66.6^0\right)\left(s+19\angle\pm78.5^0\right)} \right)$$

Nichols Charts

3) The gain vs. frequency of a system is measured

w (rad/sec)	2	3	4	5	6	10
Gain (dB)	7	3	-1	-6	-12	-17
Phase (deg)	-140	-155	-162	-175	-192	-210

Using this data

- Transfer it to a Nichols chart
- Determine the maximum gain that results in a stable system
- Determine the gain, k, that results in a maximum closed-loop gain of Mm = 1.5

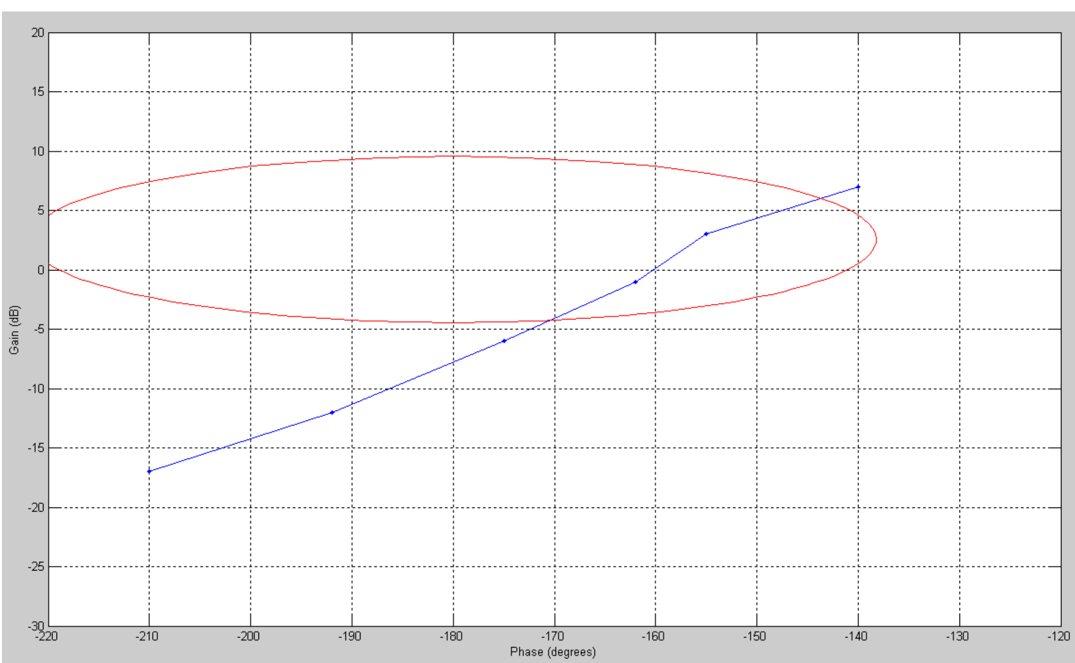
Use the function Nichols2.m

Compute G(jw) as a complex number

```
>> dB = [7,3,-1,-6,-12,-17]';
>> deg = [-140,-155,-162,-175,-192,-210]';
>> G = 10.^(dB/20) .* exp(j*deg*pi/180)

  -1.7150 - 1.4390i
  -1.2802 - 0.5970i
  -0.8476 - 0.2754i
  -0.4993 - 0.0437i
  -0.2457 + 0.0522i
  -0.1223 + 0.0706i
```

Plot on a Nichols chart along with the M-circle

```
>> Nichols2(G,1.5)
```



Adjust the gain so that

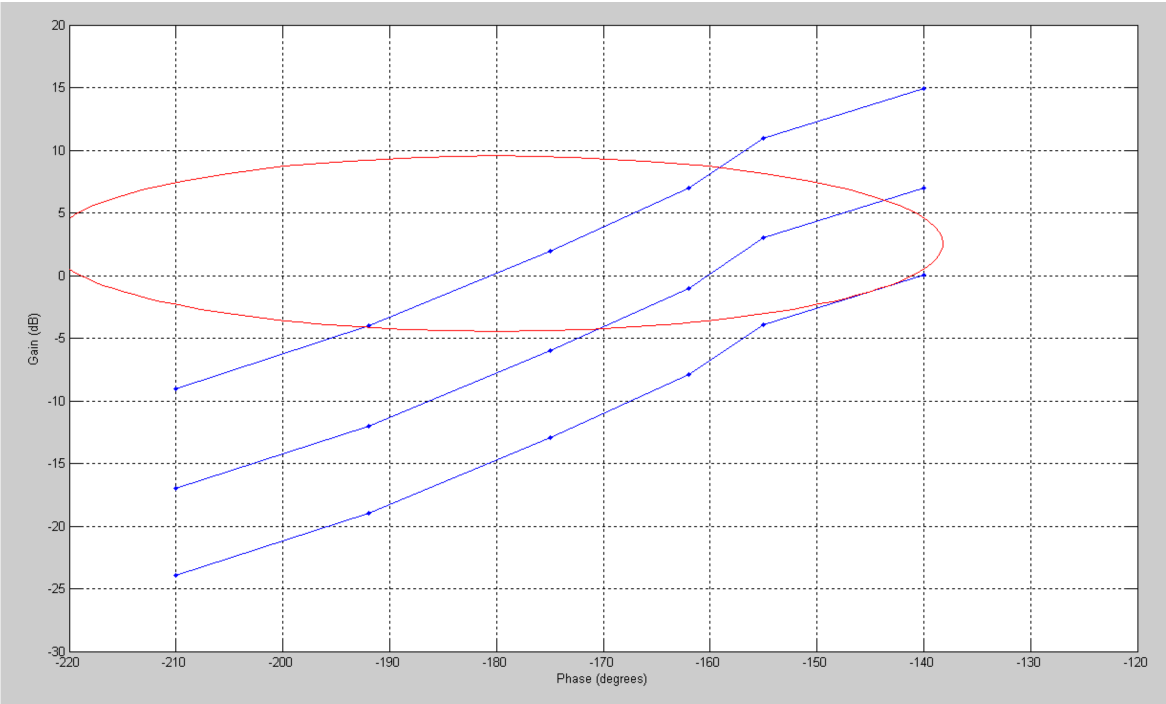
- It passes through (0dB, 180 degrees)
  - max gain for stability
- It is tangent to the M-circle

(several guesses later...)

```
>> Nichols2(G * [1,2.5,0.45],1.5);
>> grid
>>
```

Max gain for stability: k = 2.5

Gain for Mm = 1.5: k = 0.45



funciton Nichols2.m

```
function [Mol] = Nichols2(Gw, Mm)

   Gwp = unwrap(angle(Gw))*180/pi;
   Gwm = 20*log10(abs(Gw));

% M-Circle
   phase = [0:0.01:1]' * 2*pi;
   Mcl = Mm * exp(j*phase);
   Mol = Mcl ./ (1 - Mcl);
   Mp = unwrap(angle(Mol))*180/pi - 360;
   Mm = 20*log10(abs(Mol));
   plot(Gwp,Gwm,'b',Mp,Mm,'r');
   xlabel('Phase (degrees)');
   ylabel('Gain (dB)');
   xlim([-220,-120]);
   ylim([-30,20]);
   end
```

Gain and Lead Compensation

Problem 4 & 5) Assume

$$G(s) = \left(\frac{9111}{s(s+9.02)(s+23.95)(s+44.67)}\right)$$

4) Design a gain compensator that results in a 40 degree phase margin.

- Check the resulting step response in Matlab

Translation: Pick k so that at some frequency

$$G(j\omega) \cdot k = 1\angle -140^0$$

Step 1: Determine the frequency where the phase is -140 degrees

(numeric solution: search until the angle addds up)

$$G(j5.338) = 0.1475\angle -140^0$$

Step 2: Pick k to make the gain one at this frequency

$$k = \frac{1}{0.1475} = 6.7795$$

Check in Matlab. Note:

40 degree phase margin means

$$M_m \approx \left(\frac{1\angle -140^0}{1+1\angle -140^0}\right) = 1.4619$$

$$M_m = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

$$\zeta = 0.3678$$

$$OS = \exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right) = 28.86\%$$

In Matlab:

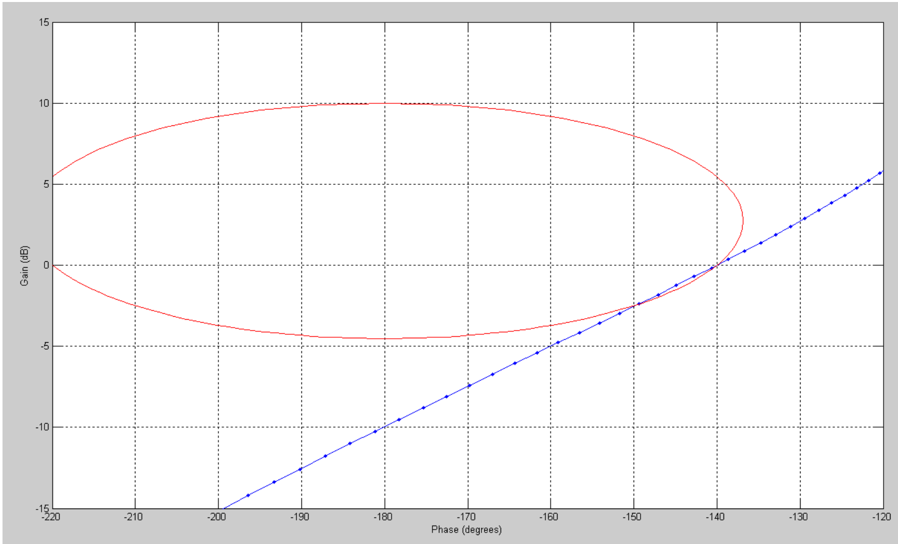
```
>> G = zpk([],[0,-9.02,-23.95,-44.67],9111)

                 9111
  ------------------------------
  s (s+9.02) (s+23.95) (s+44.67)

>> k = 6.7795;
>> Gcl = minreal(G*k / (1+G*k));
>> t = [0:0.01:10]';
>> y = step(Gcl, t);
>> max(y)

ans =     1.3019
```

The actual system has 30.19% overshoot

- Calculated was 28.86%
- Designing for phase margin is usually a little aggressive
- (you intersect the M-circle rather than being tangent to it)

This shows up on the Nichols chart (not required)

```
>> w = logspace(-2,2,200)';
>> Gw = Bode2(G,w);
>> Nichols2(Gw*6.7795, 1.4619);
>> grid
>> ylim([-15,15])
```

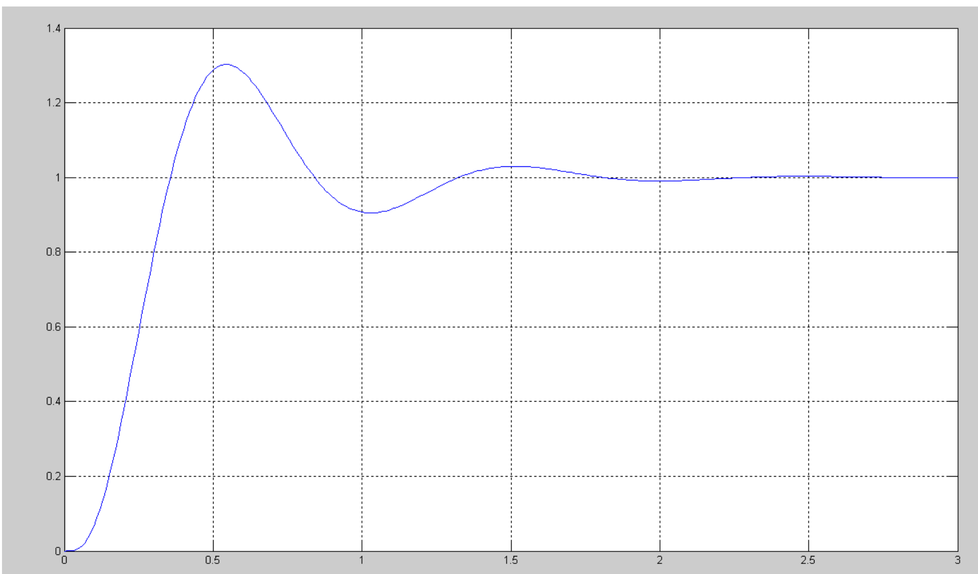


G(jw) intersects the M-circle at 0dB

(what you get when you design for a phase margin)

The step response has 30% overshoot (close to the design of 28%)

```
>> plot(t,y)
>> xlim([0,3])
>> grid
>>
```



5) Design a lead compensator that results in a 40 degree phase margin.

- Check the resulting step response in Matlab

$$G(s) = \left(\frac{9111}{s(s+9.02)(s+23.95)(s+44.67)} \right)$$

The frequency that is too close to -1 is 5.338 rad/sec (problem #4

- Pick the zero to be 1..3 times this frequency
- Pick the zero to cancel the pole at -9.02 (within this range)
- Replace it with a pole at -90

$$K(s) = k\left(\frac{s+9.02}{s+90} \right)$$

$$GK = \left(\frac{9111}{s(s+23.95)(s+44.67)(s+90)} \right)$$

Search along the jw axis until the angle adds up to -140 degrees

$$GK(j12.2028) = 0.0066\angle -140^0$$

Pick k to make the gain one

$$k = \frac{1}{0.0066} = 151.41$$

so

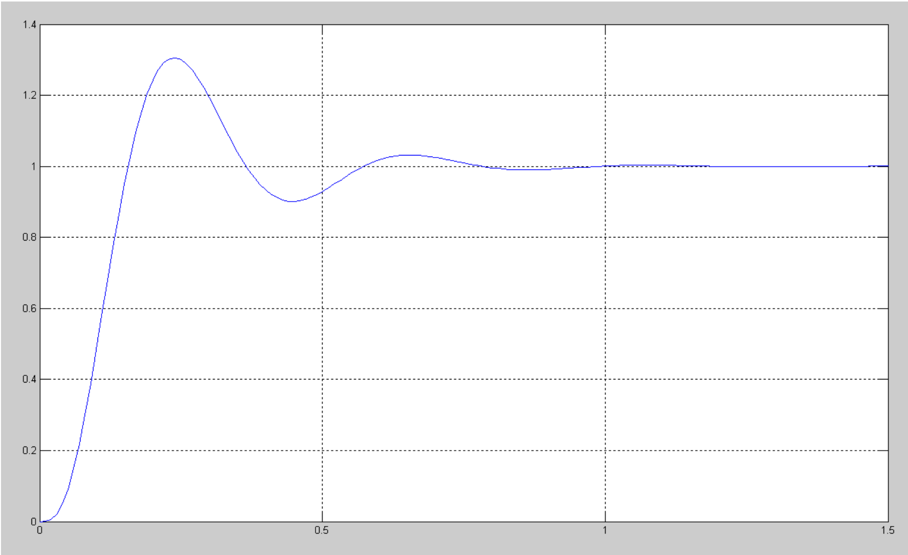
$$K(s) = 151.41\left(\frac{s+9.02}{s+90} \right)$$

Checking in Matlab:

```
>> K = zpk(-9.02,-90,151.41)

Zero/pole/gain:
151.41 (s+9.02)
---------------
     (s+90)

>> Gcl = minreal( G*K / (1 + G*K) );
>> t = [0:0.01:3]';
>> y = step(Gcl, t);
>> max(y)

     1.3056

>> grid
>>
```

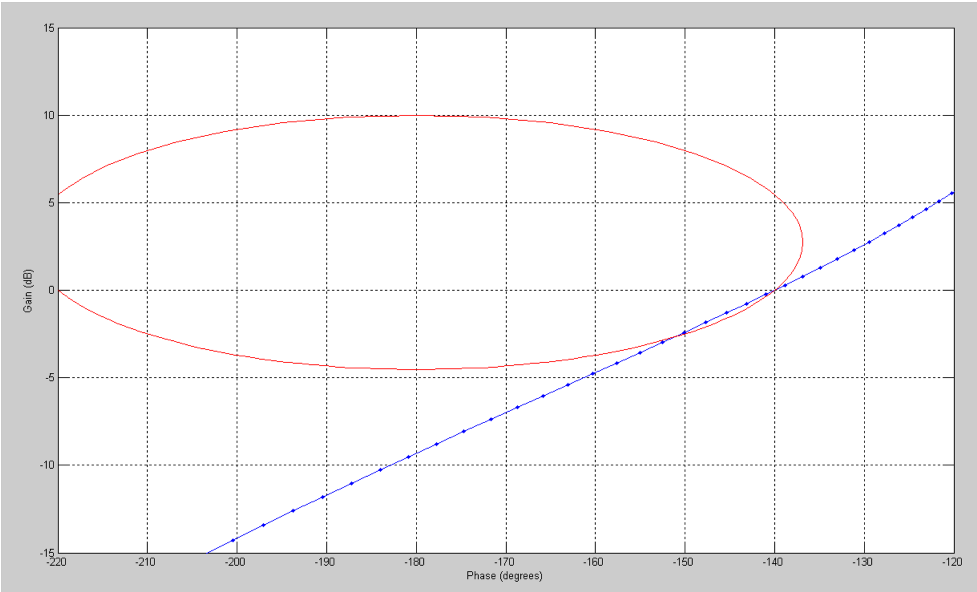
The system has 30.56% overshoot

- expected is 28%



Sidelight: The overshoot is a little high since you intersect the M-circle rather than being tangent to it

```
>> Gw = Bode2(G*K, w);
>> Nichols2(Gw, 1.4619);
>> grid
>> ylim([-15,15])
```



G(jw) intersects the M-circle at 0dB

(what you get when you design for a phase margin)

Problem 6 & 7) Assume a 200ms delay is added

$$G(s) = \left(\frac{9111}{s(s+9.02)(s+23.95)(s+44.67)}\right) e^{-0.2s}$$

6) Design a gain compensator that results in a 40 degree phase margin.

- Check the resulting step response in Matlab

Same as before: Search along the jw axis until the phase is -140 degrees

$$G(j2.3430) = 0.3876\angle -140^0$$

Pick k to make the gain one

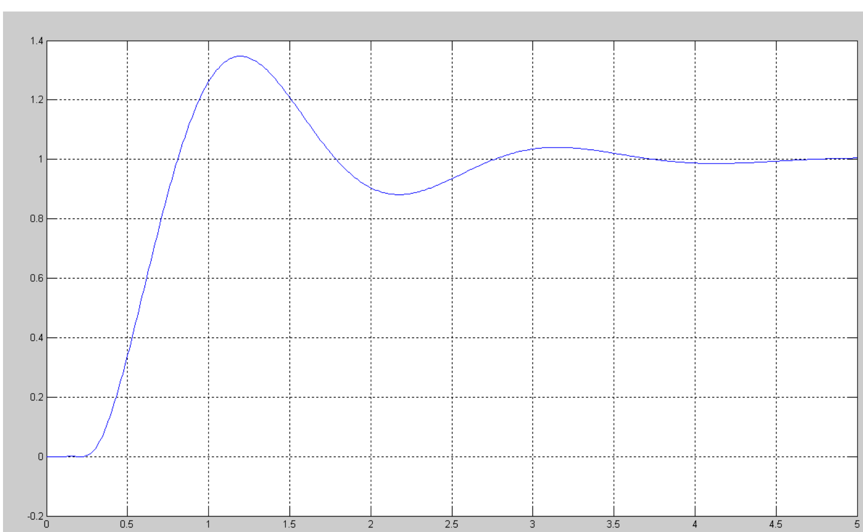
$$k = \frac{1}{0.3876} = 2.5798$$

Checking in Matlab:

Use a 4th-order Pade approximation for the delay

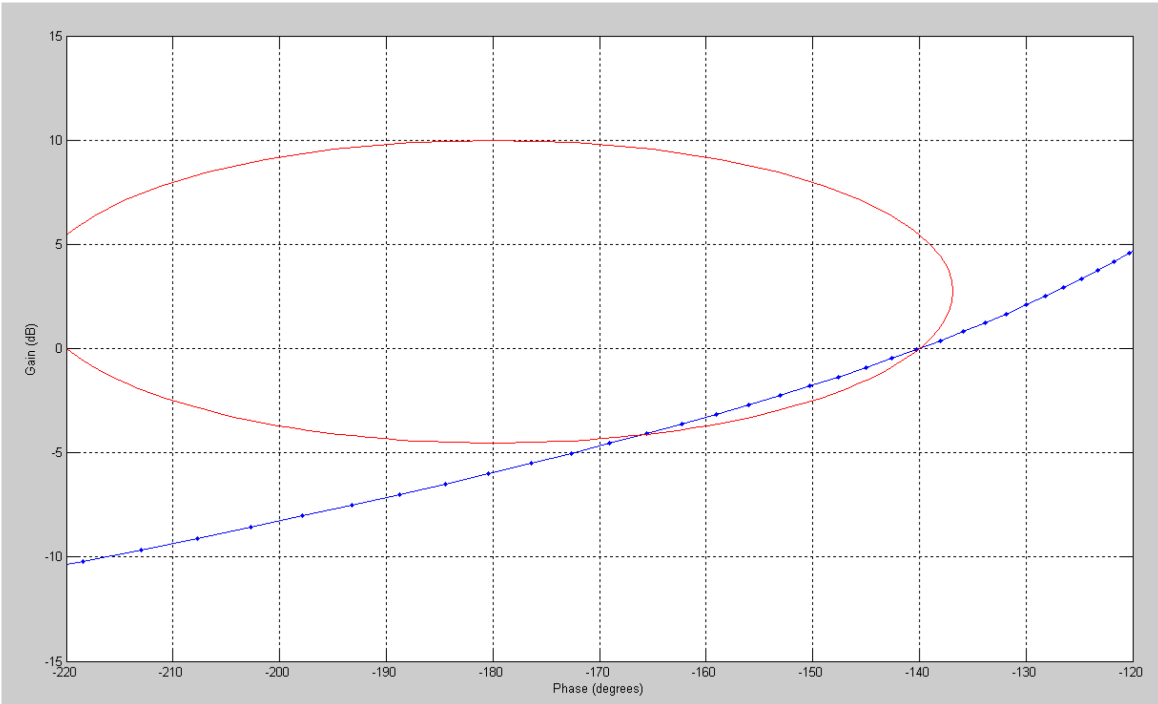
```
>> G = zpk([],[0,-9.02,-23.95,-44.67],9111)

                9111
------------------------------
s (s+9.02) (s+23.95) (s+44.67)

>> [num,den] = pade(0.2,4);
>> Delay = tf(num,den);
>> k = 2.5798;
>> Gcl = minreal(G*Delay*k / (1 + G*Delay*k));
>> t = [0:0.01:5]';
>> y = step(Gcl,t);
>> max(y)

ans =     1.3468

>> plot(t,y)
>> grid
```

The overshoot is a little high (34.68%)



Sidelight: Again, the overshoot is a little high due to intersecting with the M-circle rather than being tangent to it.

```
>> Gw = Bode2(G*Delay*k, w);
>> Nichols2(Gw, 1.4619);
>> grid
```



G(jw) intersects the M-circle at 0dB

(what you get when you design for a phase margin)

7) Design a lead compensator that results in a 40 degree phase margin.

- Check the resulting step response in Matlab

The frequency that's too close to -1 is 2.34 rad/sec

- Pick the zero to be 1..3 times this frequency

Place the zero at s = -3

$$K(s) = k\left(\frac{s+3}{s+30}\right)$$

$$GK = \left(\frac{9111}{s(s+9.02)(s+23.95)(s+44.67)}\right) e^{-0.2s}\left(\frac{k(s+3)}{s+30}\right)$$

Find the frequency where the phase of GK is -140 degrees

$$GK(j4.7035) = 0.0319\angle -140^0$$

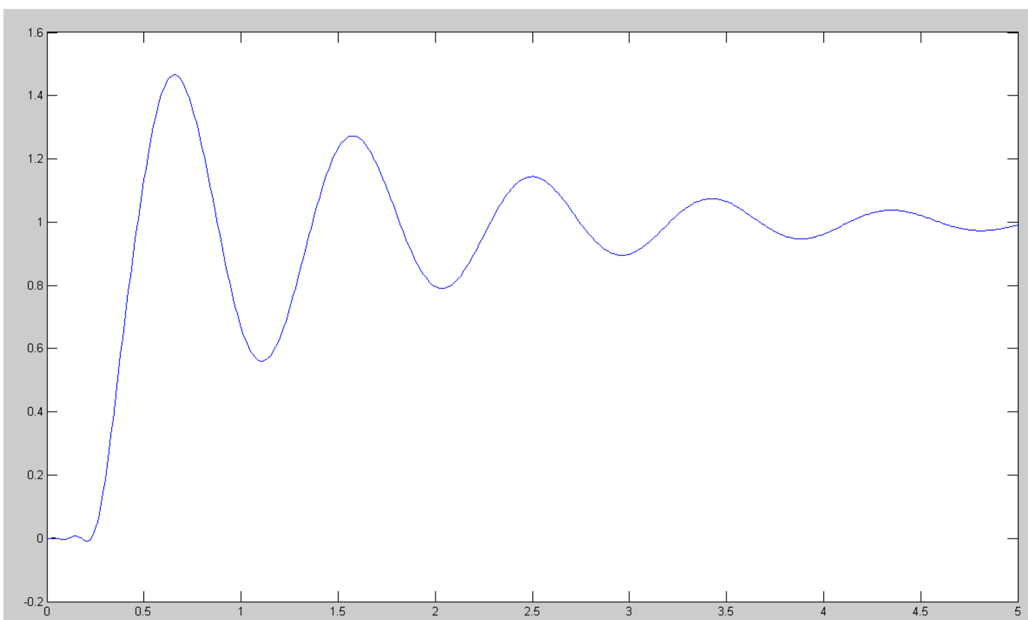
Pick k to make the gain one

$$k = \frac{1}{0.0319} = 31.3383$$

so

$$\boxed{K(s) = 31.3383\left(\frac{s+3}{s+30}\right)}$$

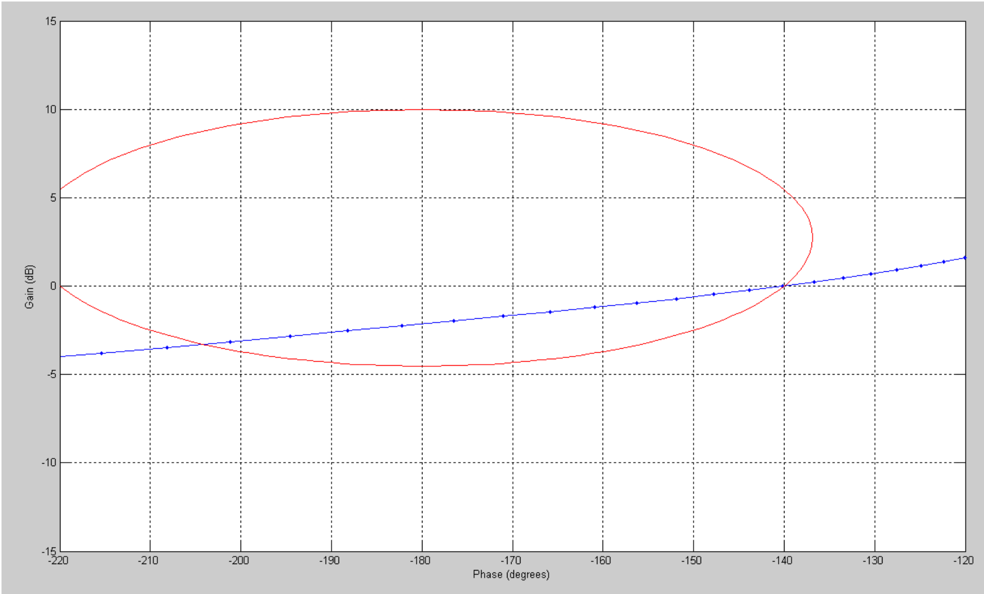
Checking in Matlab



Note: The overshoot is too much - due to intersecting the M-circle. In this case, the slope of G(jw) is more flat, resulting in the actual resonance being a lot more than it should be

```
>> Gw = Bode2(G*Delay*K, w);
>> Nichols2(Gw, 1.4619);
>> ylim([-15,15])
>> grid
>>
```

Designing for a phase margin usually is about the same as designing for Mm. In this case, it's a little too aggressive:



G(jw) intersects the M-circle at 0dB

(what you get when you design for a phase margin)