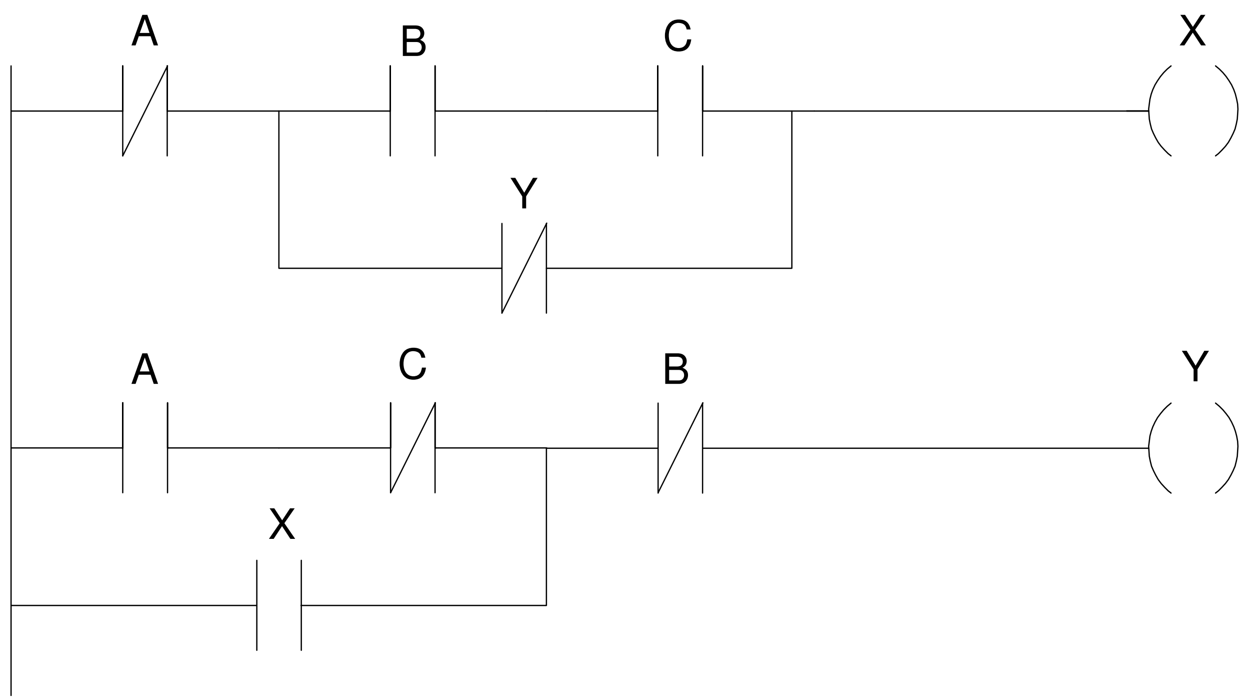
ECE 461/661 - Test #1: Name ____________________

Fall 2023

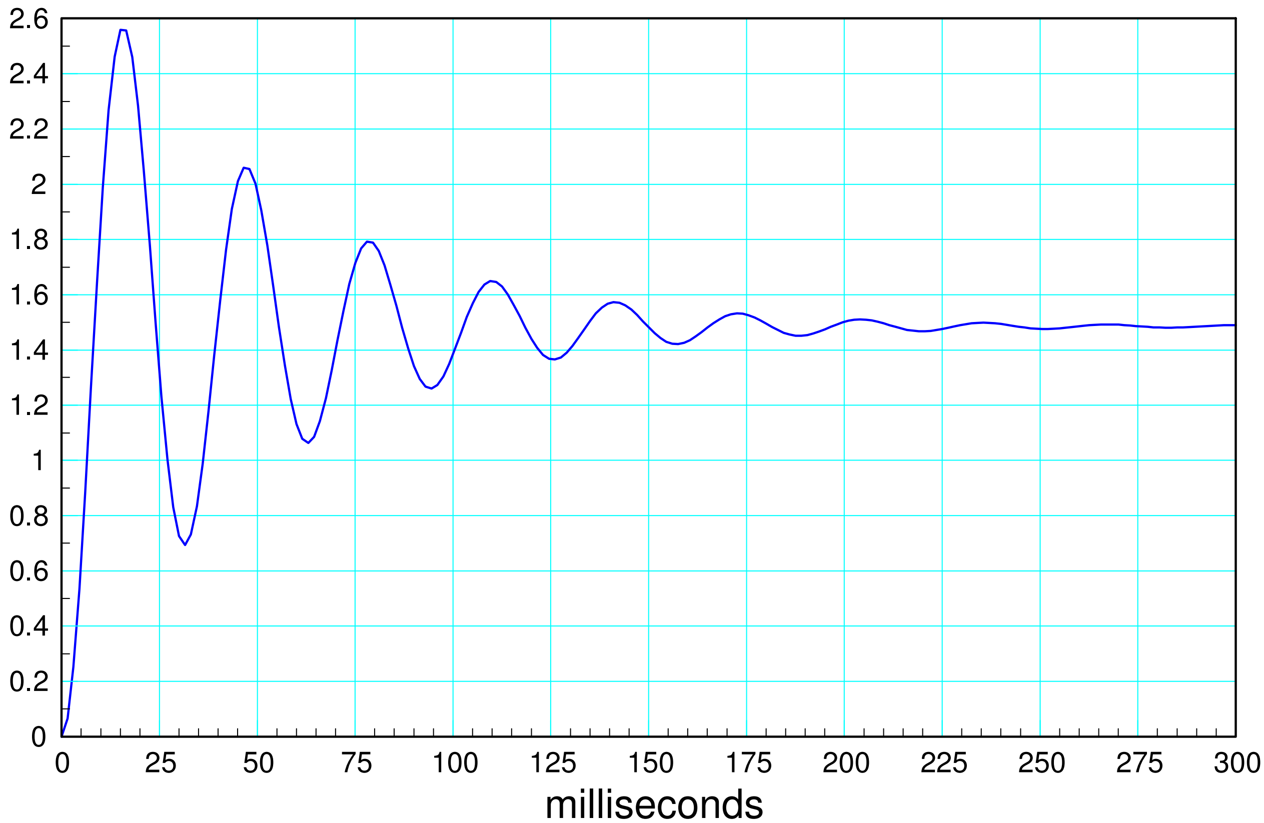
1) Determine the functions for X and Y according to the following ladder diagram. (you don't need to simplify)

$$X = \overline{A}\left(BC + \overline{Y}\right)$$

$$Y = \left(A\overline{C} + X\right)\overline{B}$$



2) Give the transfer function for a system with the following response to a unit step input:



It's a second-order system (oscillation means complex poles)

a = real(pole):

$$T_s \approx 225ms$$

$$real(pole) \approx \frac{4}{225ms} = 17.78$$

b = imag(pole)

$$b = \left(\frac{4 \text{ cycles}}{125ms}\right)2\pi = 201.1$$

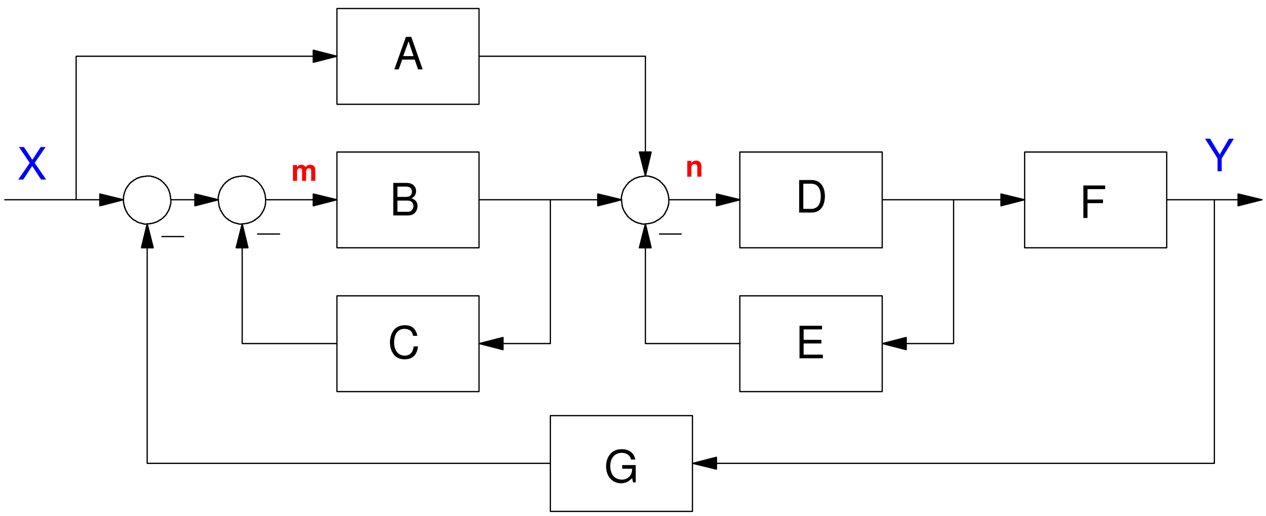
DC gain = 1.45 (ish)

$$G(s) \approx \left(\frac{k}{(s+17.78+j201.1)(s+17.78-j201.1)}\right)$$

Pick 'k' to make the DC gain 1.45

$$G(s) \approx \left(\frac{59{,}098}{(s+17.78+j201.1)(s+17.78-j201.1)}\right)$$

3) Find the transfer funciton from X to Y



Shortcut:

$$Y = \left(\frac{ADF+BDF}{1+BC+DE+BDFG}\right)X$$

Long Way:

$$m = X - CBm - GY$$

$$n = AX + Bm - EDn$$

$$Y = FDn$$

Solve for m

$$(1 + CB)m = X - GY$$

$$m = \left(\frac{X-GY}{1+CB}\right)$$

Substitute m in to the second equation, solve for n

$$n = AX + B\left(\frac{X-GY}{1+CB}\right) - EDn$$

$$(1 + ED)n = AX + B\left(\frac{X-GY}{1+CB}\right)$$

$$(1 + CB)(1 + ED)n = (1 + CB)AX + B(X - GY)$$

$$n = \frac{(A+CBA+B)X-BGY}{(1+CB)(1+ED)}$$

Plug n into the 3rd equation

$$Y = FD\left(\frac{(A+CBA+B)X-BGY}{(1+CB)(1+ED)}\right)$$

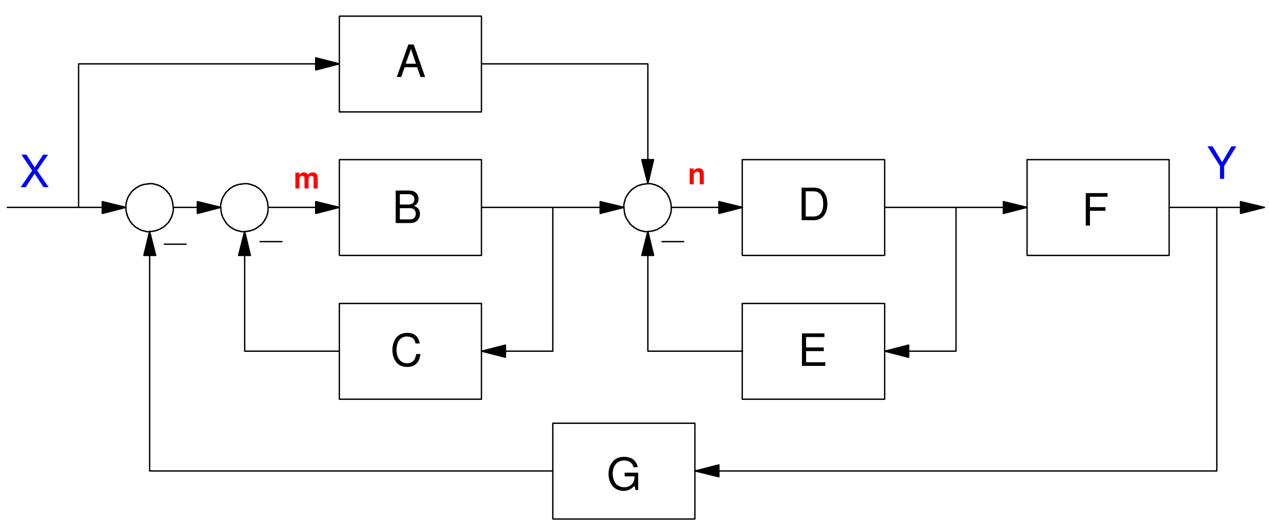
$$(1+CB)(1+ED)Y = FD((A+CBA+B)X-BGY)$$

$$(1+CB+ED+CBED+FDBG)Y = (FDA+FDCBA+FDB)X$$

$$\boxed{Y = \left(\frac{FDA+FDCBA+FDB}{1+CB+ED+CBED+FDBG}\right)X}$$

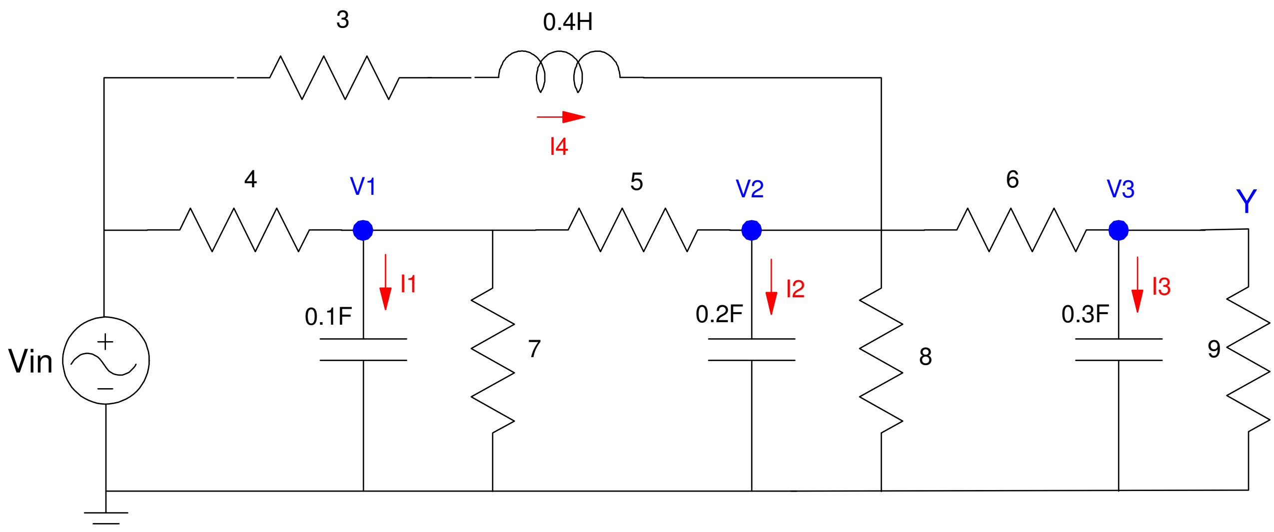
vs. shortcut (shortcut is missing a term in the numerator and the denominator)

$$Y = \left(\frac{ADF+BDF}{1+BC+DE+BDFG}\right)X$$



4) For the following RLC circuit:

- Write the dynamics of this system as four compled differential equations in terms of {Vin, V1, V2, V3, I4}
- You don't need to solve or put in state-space form (that's a different problem on the test)



$$I_1 = 0.1sV_1 = \left(\frac{V_{in}-V_1}{4}\right) - \left(\frac{V_1}{7}\right) - \left(\frac{V_1-V_2}{5}\right)$$

$$I_2 = 0.2sV_2 = \left(\frac{V_1-V_2}{5}\right) + I_4 - \left(\frac{V_2}{8}\right) - \left(\frac{V_2-V_3}{6}\right)$$

$$I_3 = 0.3sV_3 = \left(\frac{V_2-V_3}{6}\right) - \left(\frac{V_3}{9}\right)$$

$$V_4 = 0.4sI_4 = V_{in} - 3I_4 - V_2$$

5) Assume the dynamics of a mass-spring system are as follows.

$$(2s^2+4s+8)x_1-(10s+12)x_2=F$$

$$(5s^2+15s+30)x_2-(2s+6)x_1=0$$

- Give the state-space representation for the dynamics.
- Assume the output is Y = x1 - x2

Solving for the highest derivative

$$s^2x_1=(-2s-4)x_1+(5s+6)x_2+0.5F$$

$$s^2x_2=(0.4s+1.2)x_1+(-3s-6)x_2$$

$$s\begin{bmatrix} x_1 \\ x_2 \\ sx_1 \\ sx_2 \end{bmatrix}=\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -4 & 6 & -2 & 5 \\ 1.2 & -6 & 0.4 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ sx_1 \\ sx_2 \end{bmatrix}+\begin{bmatrix} 0 \\ 0 \\ 0.5 \\ 0 \end{bmatrix}F$$

$$Y=\begin{bmatrix} 1 & -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ sx_1 \\ sx_2 \end{bmatrix}+[0]F$$