ECE 461/661 - Test #2: Name ____________________

Feedback and Root Locus - Fall 2023

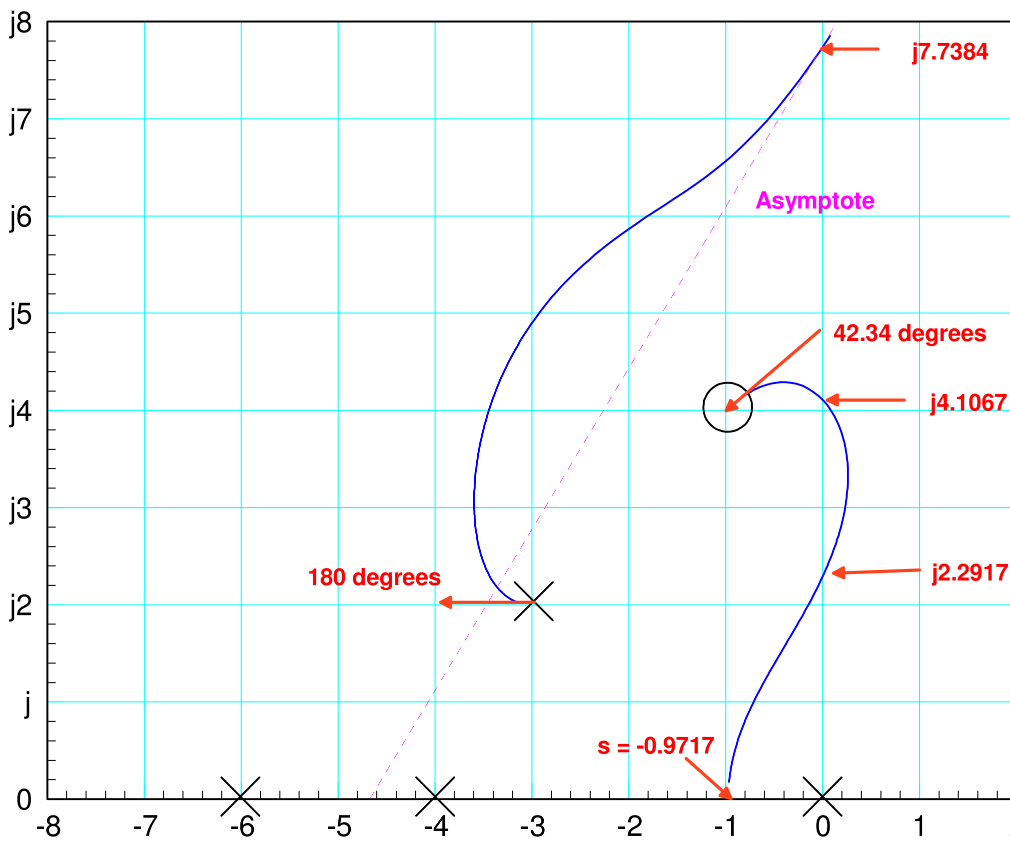
Root Locus

1) The root locus of G(s) is shown below.

$$G(s) = \left(\frac{100(s+1+j4)(s+1-j4)}{s(s+4)(s+6)(s+3+j2)(s+3-j2)} \right)$$

Determine the following

Approach Angle to the zero at -1+j4	Departure Angle from the pole at -3+j2	Real Axis Loci
42.34 degrees	**180.00 degrees**	**(0, -4), (-6, -inf)**
Breakaway Point (approx)	Asymptotes	jw Crossing(s)
-0.9717	show on graph	**j2.2917** **j4.1067** **j7.7384**



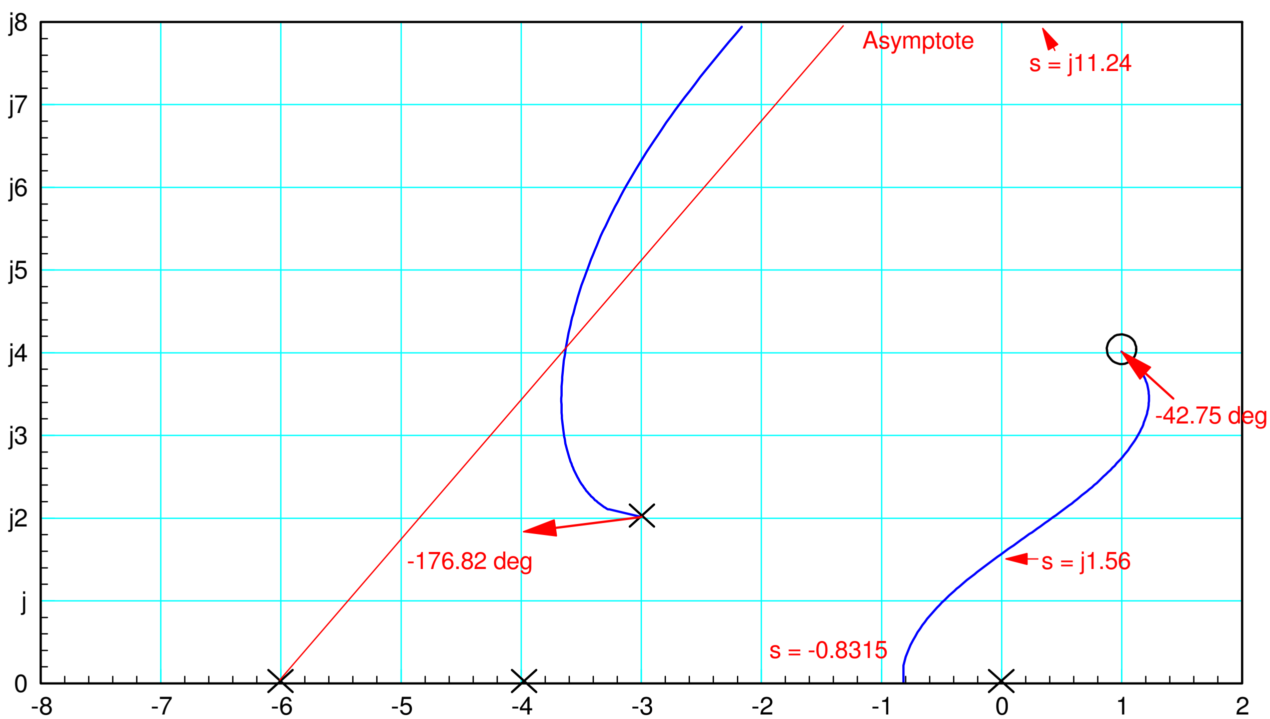
Root Locus

1) The root locus of G(s) is shown below.

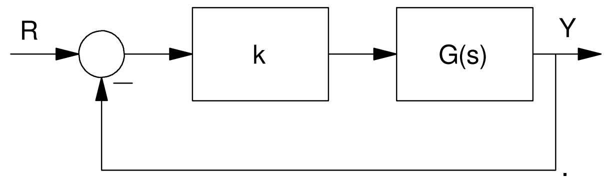
$$G(s) = \left(\frac{100(s-1+j4)(s-1-j4)}{s(s+4)(s+6)(s+3+j2)(s+3-j2)} \right)$$

Determine the following

Approach Angle to the zero at +1+j4	Departure Angle from the pole at -3+j2	Real Axis Loci
-42.75 degrees	**-176.82 degrees**	**(0, -4), (-6, -inf)**
Breakaway Point (approx)	Asymptotes	jw Crossing(s)
-0.8315	show on graph	**j1.56** **j11.24**



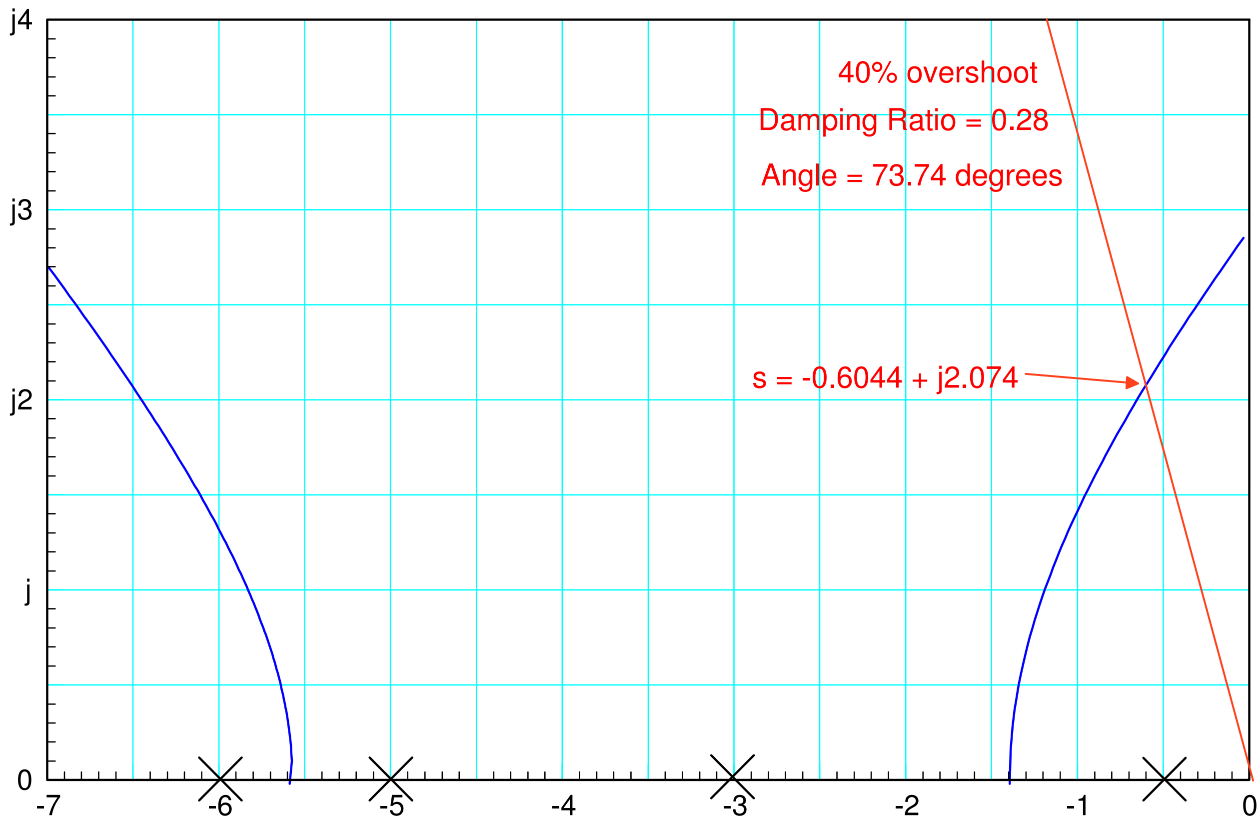
Gain Compensation



2) Determine the gain (K(s) = k) so that the feedback system has 40% overshoot for a step input. Also determine the closed-loop dominant pole(s) and error constant, Kp

$$G(s) = \left(\frac{100}{(s+0.5)(s+3)(s+5)(s+6)}\right)$$

k 40% overshoot	Closed-Loop dominant pole(s)	Kp Error Constant
1.8461	**-0.6044 + j2.074** damping ratio = 0.28 angle = 73.74 degrees	**4.1024** (2.222)(1.8461)

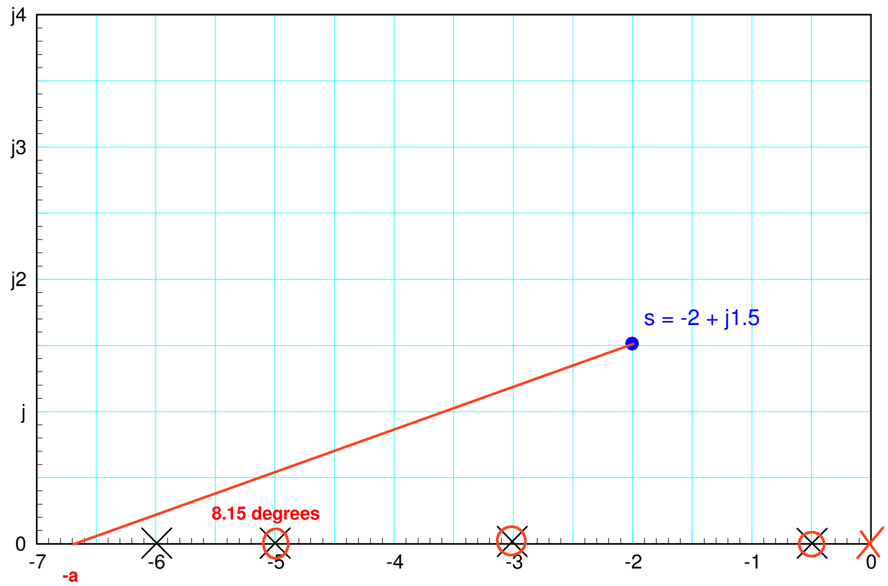


Lead/PI Compensation

3) Design a compensator, K(s), so that the closed-loop system has

- No error for a step input
- Closed-Loop dominant poles at s = -2 + j1.5, and
- Finite gain as s→ ∞ (i.e. have at least as many poles as zeros)

$$G(s) = \left(\frac{100}{(s+0.5)(s+3)(s+5)(s+6)}\right) \qquad K(s) = 11.9366\left(\frac{(s+0.5)(s+3)(s+5)}{s(s+12.4650)^2}\right)$$



$$K(s) = k\left(\frac{(s+0.5)(s+3)}{s(s+a)}\right)$$

$$\left(\frac{100}{s(s+5)(s+6)}\right)_{s=-2+j1.5} = 2.5\angle+143^0$$

$$K(s) = k\left(\frac{(s+0.5)(s+3)(s+5)}{s(s+a)^2}\right)$$

$$\left(\frac{100}{s(s+6)}\right)_{s=-2+j1.5} = 9.36\angle-163.70^0$$

$$\angle(s+a) = \tfrac{1}{2}(16.30^0) = 8.15^0$$

$$a = \frac{1.5}{\tan(8.15^0)} + 2 = 12.4650$$

$$GK = \left(\frac{100}{s(s+6)(s+12.4650)^2}\right)_{s=-2+j1.5} = 0.0838\angle180^0$$

$$k = \frac{1}{0.838} = 11.9366$$

Compensator Design (hardware)

4) Design a circuit to implement K(s)

$$K(s) = \left(\frac{30(s+3)(s+7)}{s(s+10)}\right)$$

Let

$$K(s) = \left(\frac{10(s+7)}{s+10}\right)\left(\frac{3(s+3)}{s}\right)$$