ECE 461/661 - Test #3: Name ___________________

Digital Control & Frequency Domain Techniques - Fall 2023

s to z conversion

1) Determine the discrete-time equivalent for G(s). Assume a sampling rate of T = 0.01 second

$$G(s) = \left(\frac{100(s+5)}{(s-2)(s+7+j3)(s+7-j3)} \right)$$

Convert from the s-plane to the z-plane as $z = e^{sT}$

$$s = -5 \qquad z = e^{sT} = 0.9512$$

$$s = +2 \qquad z = e^{sT} = 1.0202$$

$$s = -7 + j3 \quad z = e^{sT} = 0.9320 + j0.0280$$

$$s = -7 - j3 \quad z = e^{sT} = 0.9320 - j0.0280$$

So

$$G(z) = \left(\frac{k(z-0.9512)}{(z-1.0202)(z-0.9320+j0.0280)(z-0.9320-j0.0280)} \right)$$

The DC gain of G(s) is -4.3103. Pick k to match the DC gain

$$\left(\frac{k(z-0.9512)}{(z-1.0202)(z-0.9320+j0.0280)(z-0.9320-j0.0280)} \right)_{z=1} = -4.3103$$

$$k = 0.009658$$

resulting in

$$G(z) = \left(\frac{0.009658(z-0.9512)}{(z-1.0202)(z-0.9320+j0.0280)(z-0.9320-j0.0280)} \right)$$

Digital Compensators: K(z)

2) Assume a unity feedback system with a sampling rate of T = 0.1 second

$$G(z) = \left(\frac{0.002z}{(z-0.98)(z-0.92)(z-0.65)}\right)$$

Design a digital compensator, K(z), which results in

- No error for a step input,
- Closed-Loop Dominant poles at z = 0.9 + j0.1, and
- Is causal (the number of poles in K(z) is equal to or greater than the number of zeros)

Translation

- Add a pole at z = +1 making it a type-1 system
- Cancel zeros & add poles so that 0.9 + j0.1 is on the root locus

Let

$$K(z) = k\left(\frac{(z-0.98)(z-0.92)}{(z-1)(z-a)}\right)$$

$$GK = \left(\frac{0.002k \cdot z}{(z-1)(z-0.65)(z-a)}\right)$$

Analyze what we know:

$$\left(\frac{0.002 \cdot z}{(z-1)(z-0.65)}\right)_{z=0.9+j0.1} = 0.0476\angle -150.46^0$$

To make the angles add up to 180 degrees

$$\angle(z-a) = 29.53^0$$

$$a = 0.9 - \left(\frac{0.1}{\tan(29.53^0)}\right)$$

$$a = 0.7253$$

Checking

$$\left(\frac{0.002 \cdot z}{(z-1)(z-0.7253)(z-0.65)}\right)_{z=0.9+j0.1} = 0.2363\angle 180^0$$

so

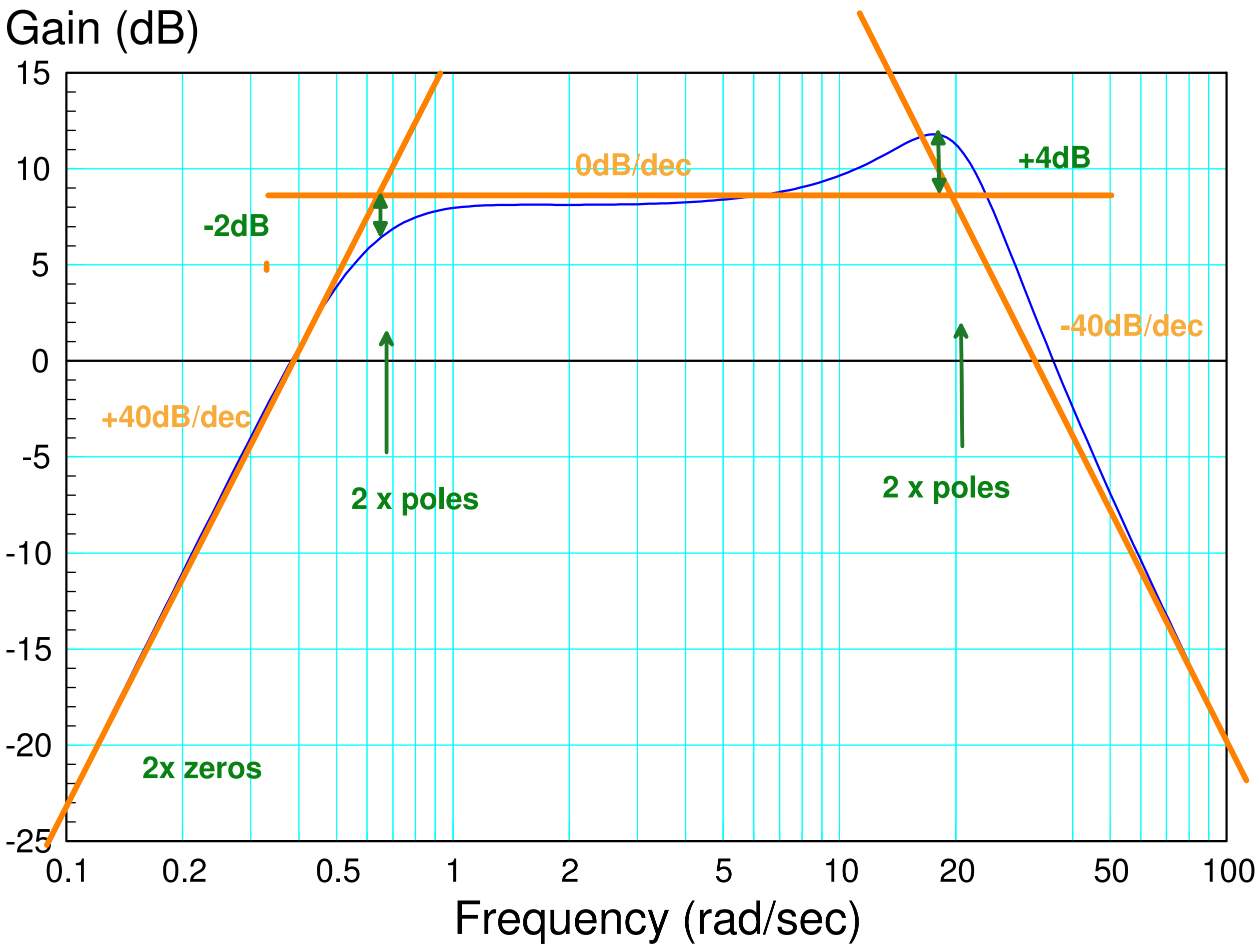
$$k = \frac{1}{0.2363} = 4.2324$$

and

$$K(z) = 4.2324\left(\frac{(z-0.98)(z-0.92)}{(z-1)(z-0.7253)}\right)$$

3) Bode Plots

Determine the system, G(s), which has the following gain vs. frequency



2 x poles at 0.7 rad/sec

$$\frac{1}{2\zeta} = -2dB = 0.7943$$

$$\zeta = 0.6295$$

$$\theta = 51.0^0$$

2 x poles at 20 rad/sec

$$\frac{1}{2\zeta} = +4dB = 1.5849$$

$$\zeta = 0.3155$$

$$\theta = 71.6^0$$

So

$$G(s) \approx \left(\frac{ks^2}{\left(s+0.7\angle\pm 51.0^0\right)\left(s+20\angle\pm 71.6^0\right)} \right)$$

To find k, mach the gain somewhere

$$G(j1) \approx 8dB = \left(\frac{ks^2}{\left(s+0.7\angle\pm51.0^0\right)\left(s+20\angle\pm71.6^0\right)} \right)_{s=j1}$$

$$2.5119 = k \cdot (0.0025\angle 58.12^0)$$

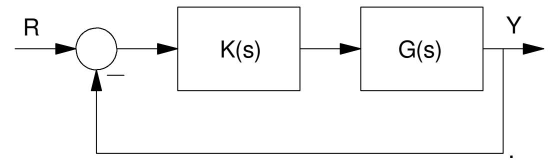
Taking the magnitude

$$k = 1020.8$$

and

$$G(s) \approx \left(\frac{1020.8 \cdot s^2}{\left(s+0.7\angle\pm51.0^0\right)\left(s+20\angle\pm71.6^0\right)} \right)$$

4) Nichols Charts

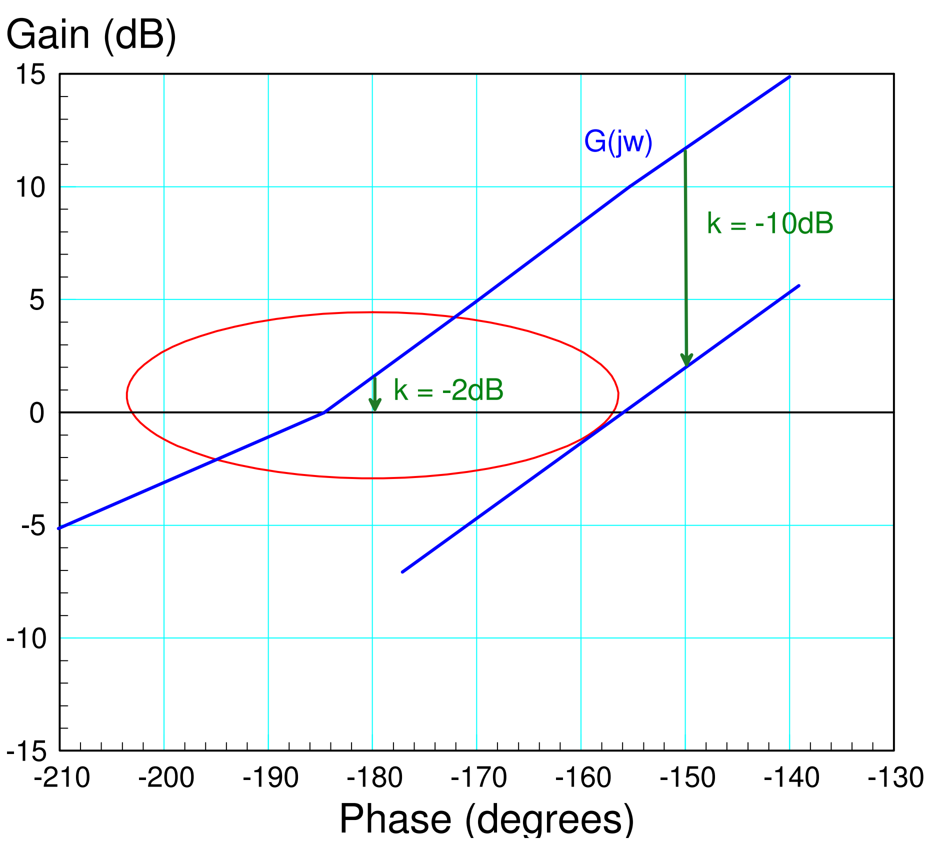


Assume a unity feedback system where the gain of G(s) is as follows:

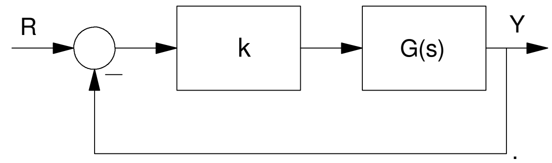
Determine

- The maximum gain, k, for stability **ans: k = -2dB**
- k that results in a resonance of Mm = 2.5 **ans: k = -10dB**

frequency (rad/sec)	3	5	7	9	15
Gain	15dB	10dB	5dB	0dB	-5dB
Phase (degrees)	-140 deg	-155 deg	-170 deg	-185 deg	-210 deg



5) Analog Compensator (Bode Plots)



Assume a unity feedback system with

$$G(s) = \left(\frac{10}{(s+0.5)(s+3)(s+9)}\right)$$

Determine a compensator, K(s), which results in

- No error for a step input
- A phase margin of 30 degrees
- A 0dB gain frequency of 3 rad/sec

Let

$$K(s) = k\left(\frac{(s+0.5)(s+3)}{s(s+a)}\right)$$

$$GK = \left(\frac{10k}{s(s+9)(s+a)}\right)$$

What we know is

$$GK(j3) = 1\angle -150^0$$

Anayze what we know

$$\left(\frac{10}{s(s+9)}\right)_{s=j3} = 0.3514\angle -108.43^0$$

For the angle to add up to -150 degrees

$$\angle(s+a) = 41.5651^0$$

$$a = \left(\frac{3}{\tan\left(41.5651^0\right)}\right) = 3.3831$$

Analyze what we know now:

$$\left(\frac{10}{s(s+3.3931)(s+9)}\right)_{s=j3} = 0.07760\angle -150.0^0$$

meaning

$$k = \frac{1}{0.07760} = 12.89$$

and

$$K(s) = 12.89\left(\frac{(s+0.5)(s+3)}{s(s+3.3931}\right)$$