PID Compensators Using Root Locus

ECE 461/661 Controls Systems

Jake Glower - Lecture #24

Please visit Bison Academy for corresponding
lecture notes, homework sets, and solutions

PID Compensators

Very common type of compensator

$$K(s) = P + \frac{I}{s} + Ds$$

I:

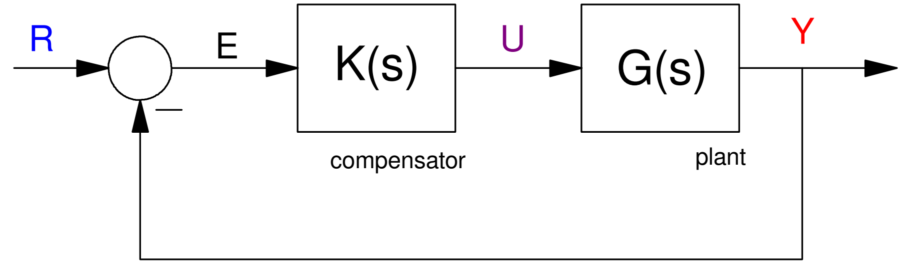
- Add a pole at s = 0
- Makes a type-0 system type-1

PI:

- Add a pole at s = 0
- Add a zero to cancel slow poles

PID

- Add a pole at s = 0
- Add two zeros to cancel two slow poles



Why PID?

It works

- Works well for systems with real, stable poles
- That describes a *lot* of what you'll see

People can tune them

- One knob is easy to tune (adjust k for 20% overshoot)
- Two knobs can be tuned by iterating back and forth
- Three knobs are much harder to tune: requires training

J.D Cowan: *In your years of designing feedback controllers, what is the most important thing you've learned for designing feedback systems?*

Bode: *Well, I'll tell you. If you want your control system to work, you have to make sure the gain increases as you turn your know to the right.*

Lecture by J.D. Cowan at Ohio State in ECE 751 Controls Systems, c. 1982.

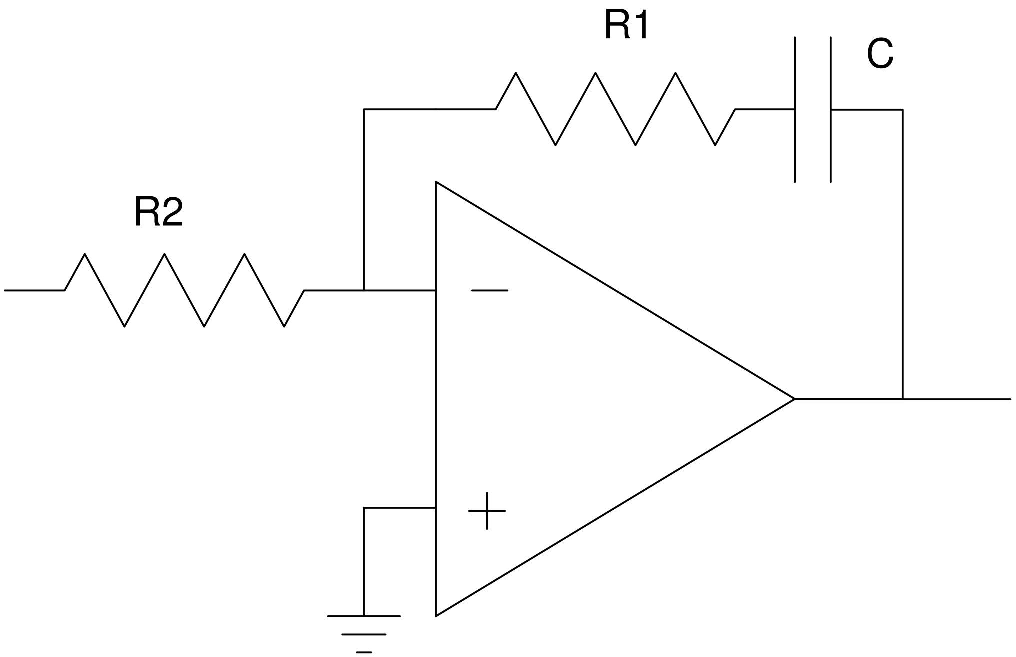
PI Circuit:

$$K(s) = -k\left(\frac{s+a}{s}\right)$$

where

$$a = \left(\frac{1}{R_1 C}\right)$$

$$k = \frac{R_1}{R_2}$$

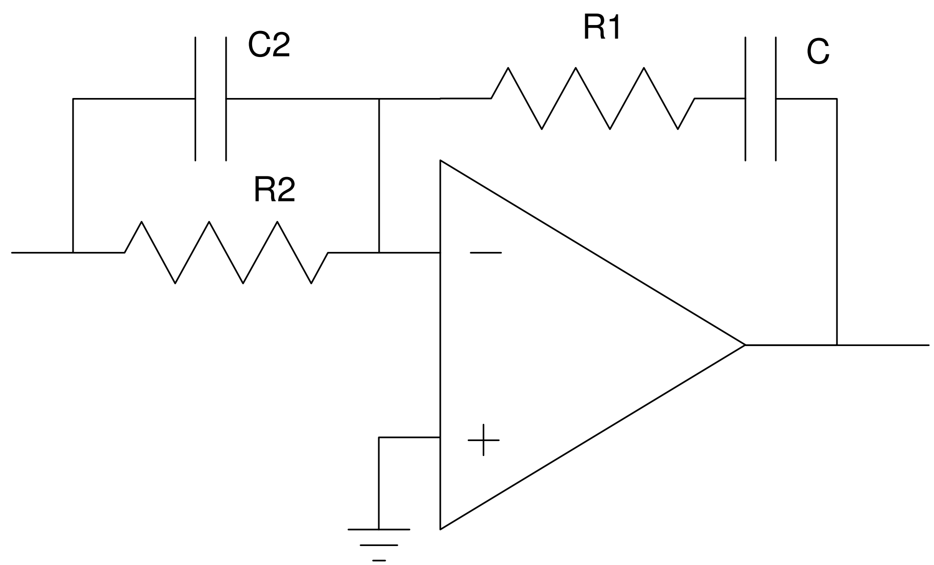


Note:

- P: C = 0
- I: $R_1 = 0$

PID Circuit

$$K(s) = -\left(\frac{(R_1C_1s+1)(R_2C_2s+1)}{R_2C_1s}\right)$$



I Compensation:

$$G(s) = \left(\frac{361.2378}{(s+15.65)(s+10.1)(s+5.439)(s+2.081)(s+0.3234)}\right)$$

$$K(s) = \frac{I}{s} = k\left(\frac{1}{s}\right)$$

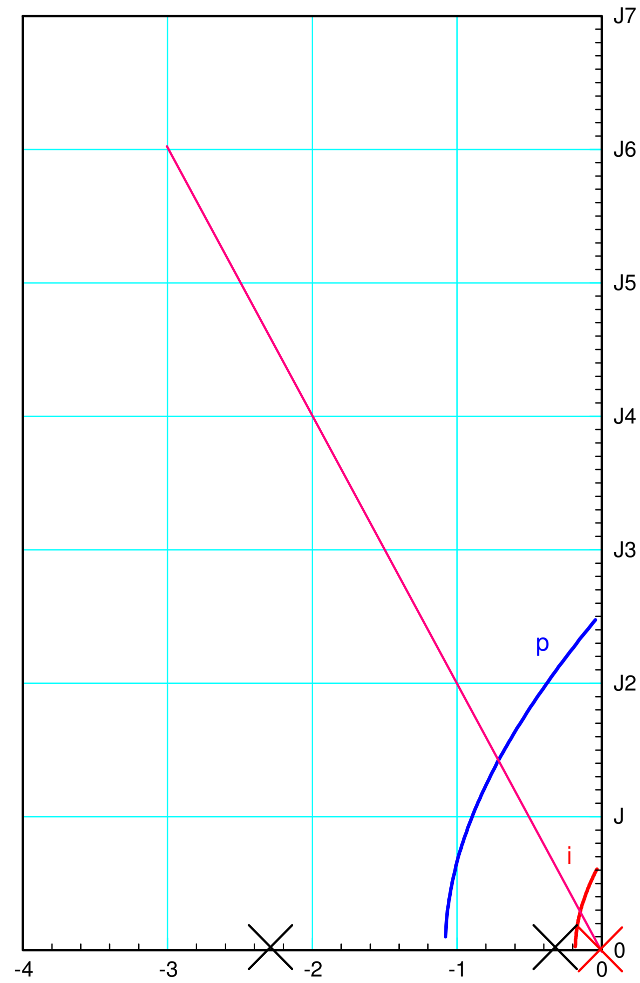
$$GK = \left(\frac{361.2378k}{(s+15.65)(s+10.1)(s+5.439)(s+2.081)(s+0.3234)s}\right)$$

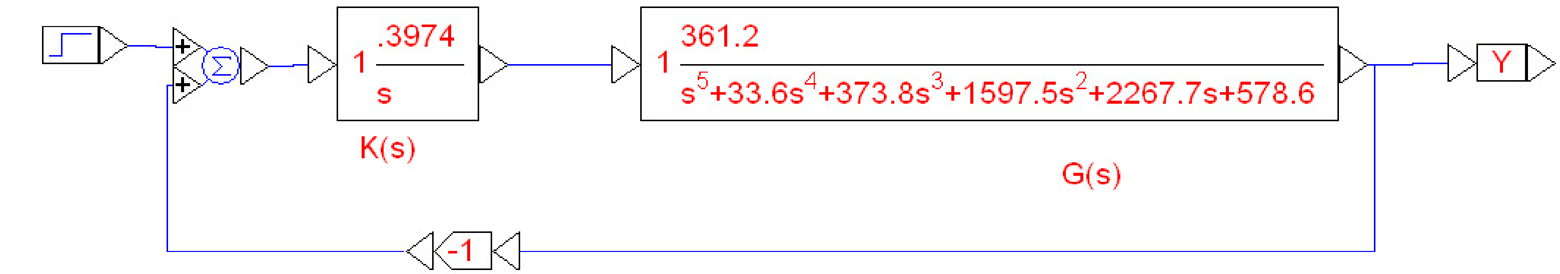
Result:

- s = -0.1249 + j0.2483
- $K(s) = \left(\frac{0.3974219}{s}\right)$
- $K_v = (s \cdot GK)_{s\to 0} = 0.2481$
- $U(t=0) = (K(s))_{s\to\infty} = 0$

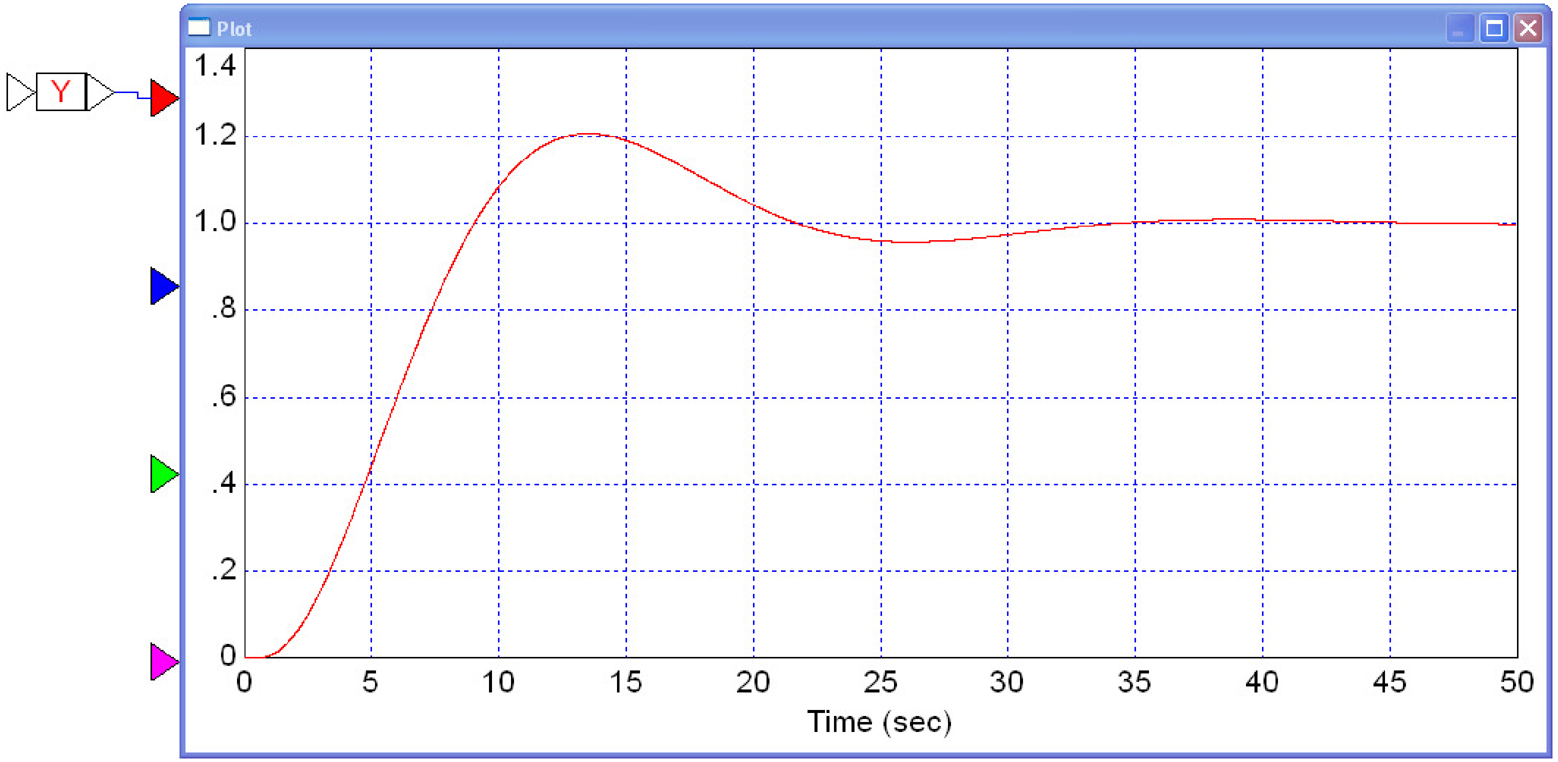
I Compensatos

- Make the system type-1
- Slow down the system





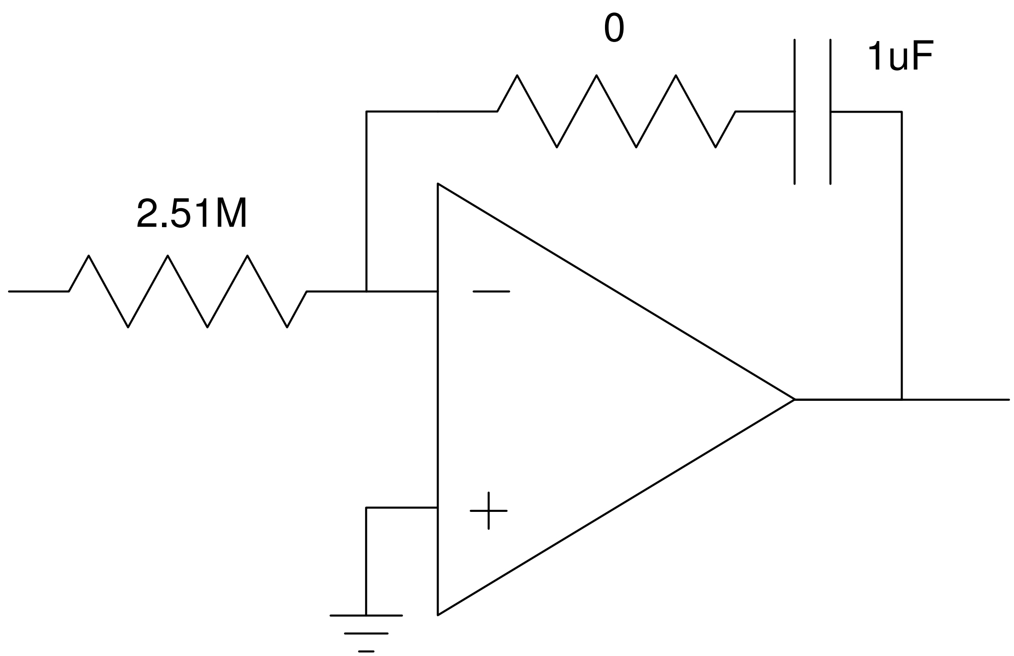
.3974
1
s
K(s)
361.2
1
s^5+33.6s^4+373.8s^3+1597.5s^2+2267.7s+578.6
G(s)
Y
-1



Plot
Y
1.4
1.2
1.0
.8
.6
.4
.2
0
0
5
10
15
20
25
30
35
40
45
50
Time (sec)

I Circuit:

$$K(s) = \left(\frac{0.3974219}{s}\right)$$



Circuit to implement an I compensator

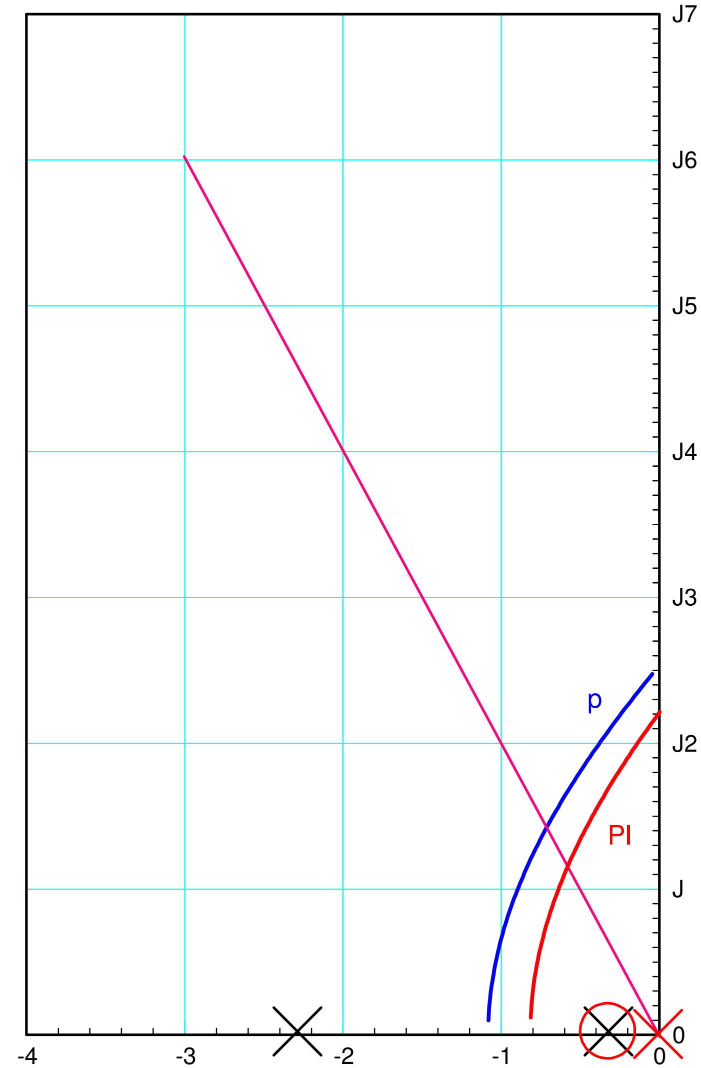
PI Compensation

- Add a pole at s = 0
- Add a zero to cancel one pole

$$K(s) = P + \frac{I}{s} = \left(\frac{Ps+I}{s}\right) = k\left(\frac{s+a}{s}\right)$$

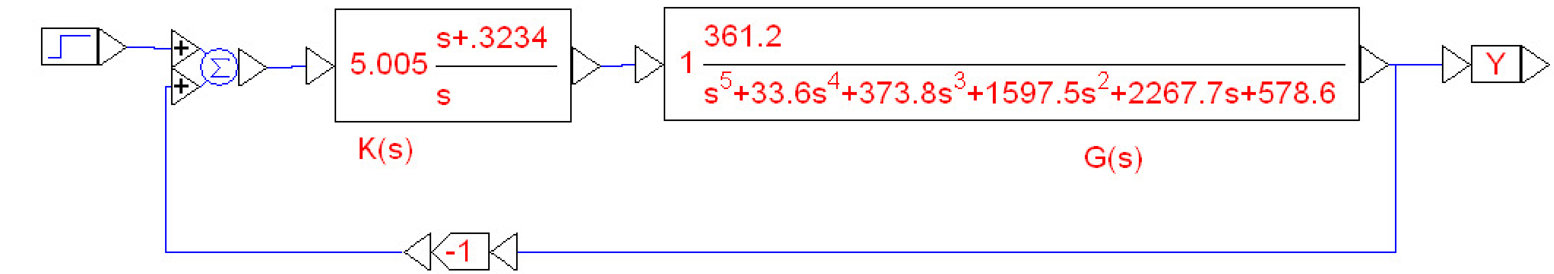
$$K(s) = k\left(\frac{s+0.3234}{s}\right)$$

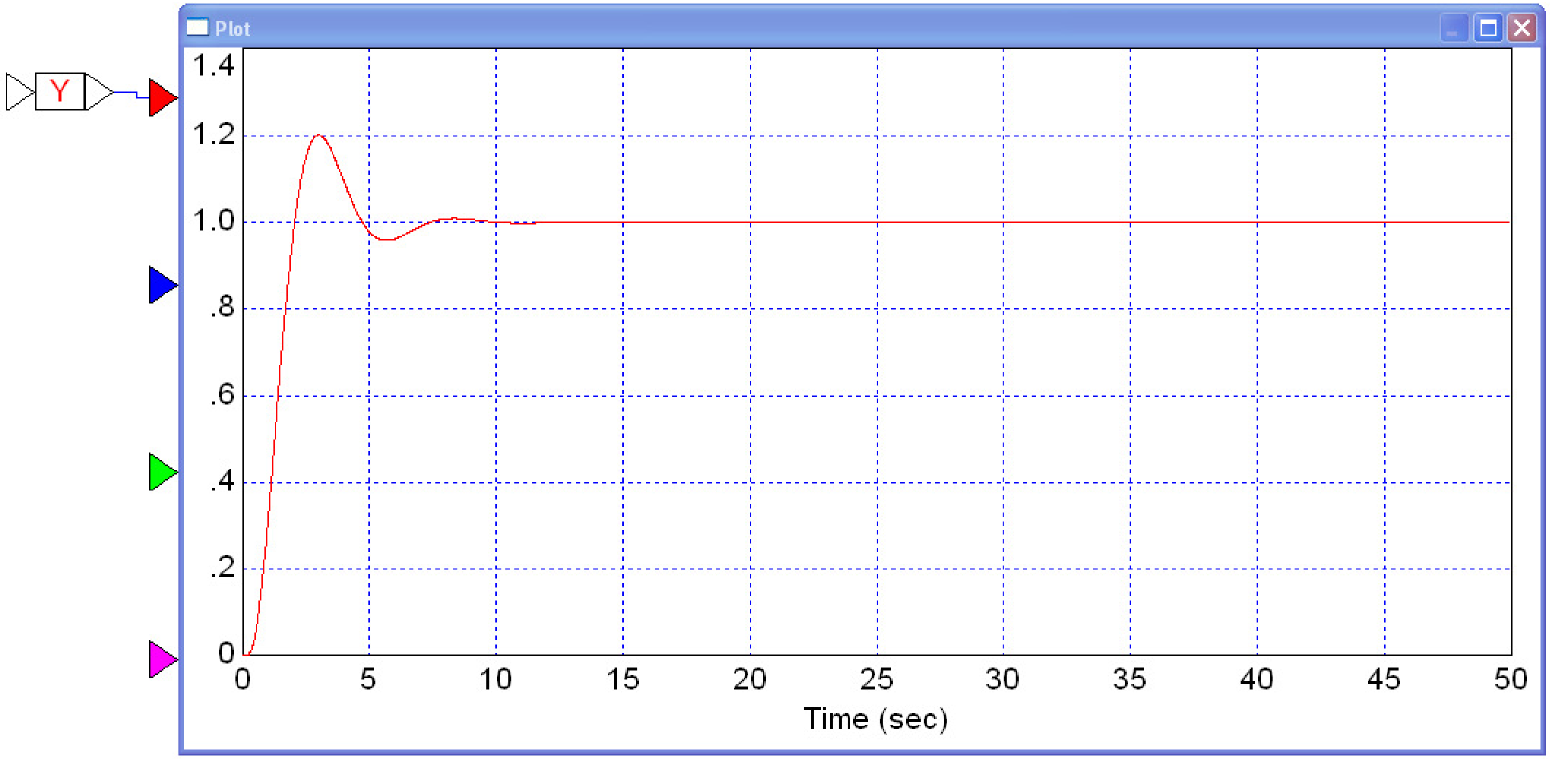
$$GK = \left(\frac{361.2378k}{(s+15.65)(s+10.1)(s+5.439)(s+2.081)s}\right)$$



Result:

- s = -0.5886 + j1.1772
- $K(s) = 5.0050\left(\frac{s+0.3234}{s}\right)$
- $K_v = \lim_{s \to 0} (s \cdot G \cdot K) = 1.0106$
- U(0) = 5.0050


5.005 (s+.3234)/s
K(s)
361.2
1
s^5+33.6s^4+373.8s^3+1597.5s^2+2267.7s+578.6
G(s)
Y
-1


Plot
Y
1.4
1.2
1.0
.8
.6
.4
.2
0
0 5 10 15 20 25 30 35 40 45 50
Time (sec)

PI Circuit

$$K(s) = 5.0050\left(\frac{s+0.3234}{s}\right)$$

3 degrees of freedom, 2 constraints.

R1 = 1M

The zero specifies R1

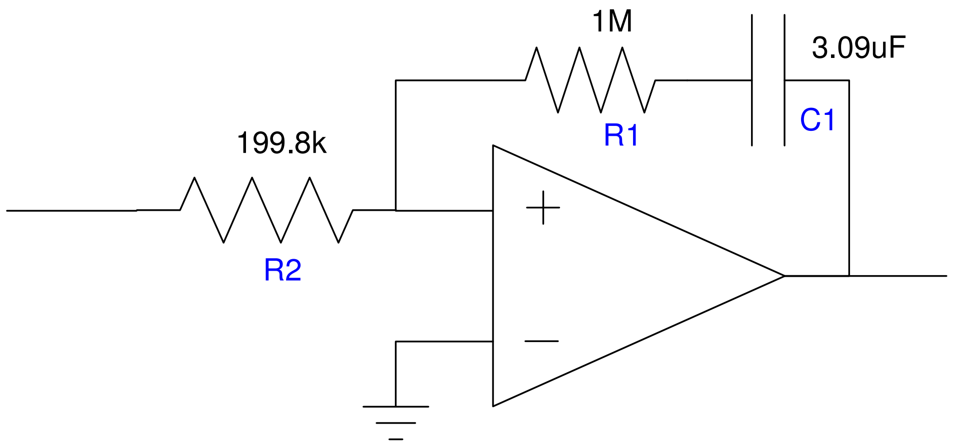
$$\frac{1}{R_1 C_1} = 0.3234$$

$$C_1 = 3.09uF$$

The gain specifies R2

$$K(s \to \infty) = 5.0050 = \frac{R_1}{R_2}$$

$$R_2 = 199.8k$$



PID

$$K(s) = P + \frac{I}{s} + Ds = \left(\frac{Ds^2+Ps+I}{s}\right) = D\left(\frac{s^2+\frac{P}{D}s+\frac{I}{D}}{s}\right)$$

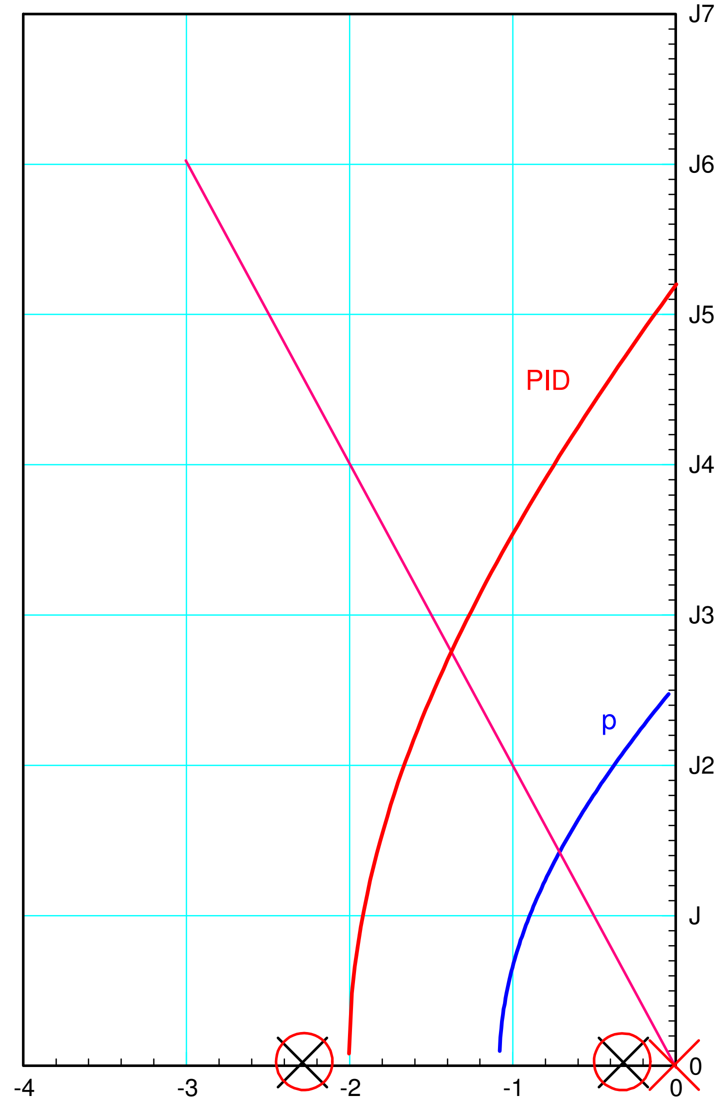
- Add a pole at s = 0, add two zeros

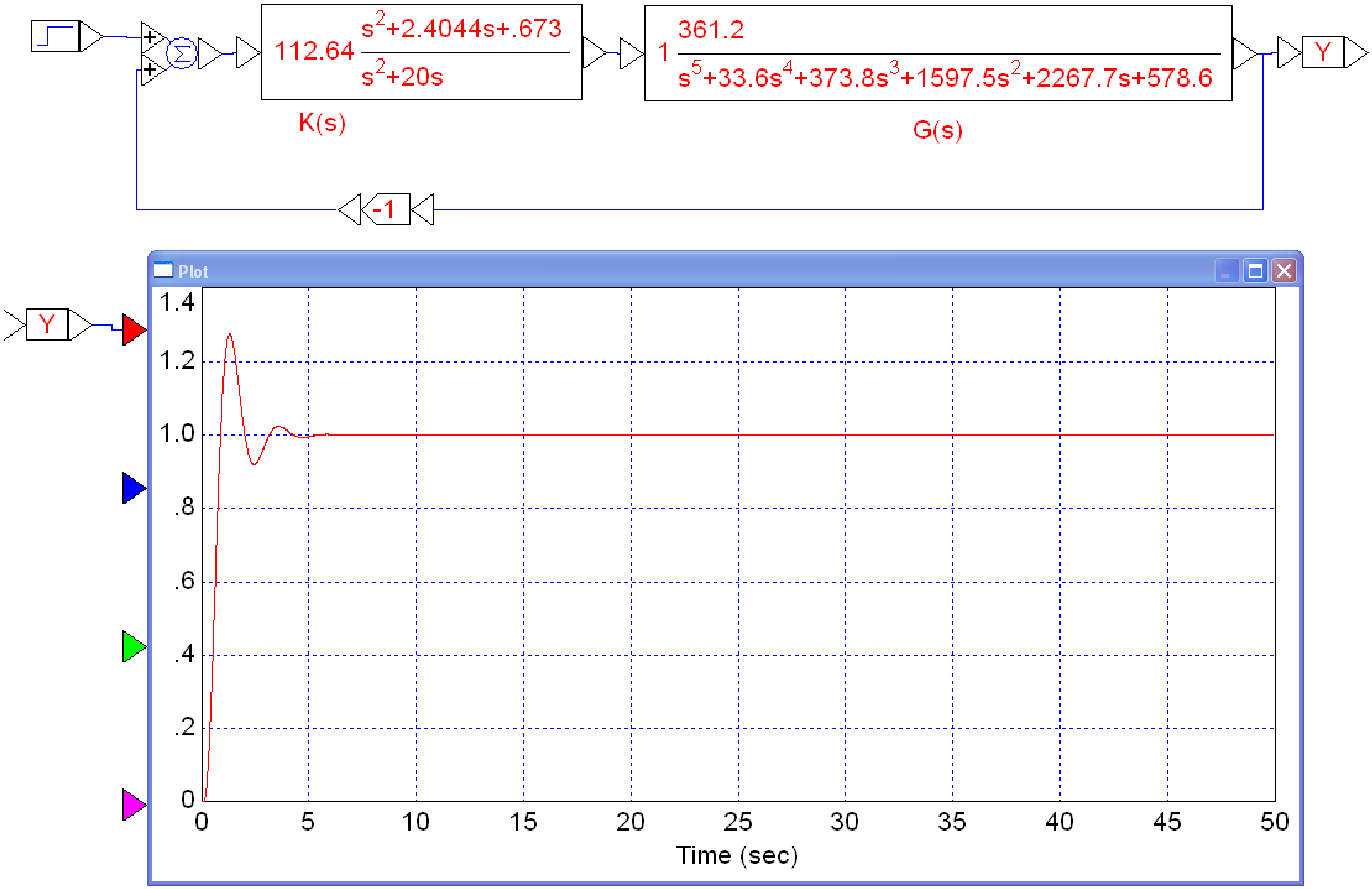
$$K(s) = k\left(\frac{(s+0.3234)(s+2.081)}{s}\right)$$

$$GK = \left(\frac{361.2378k}{(s+15.65)(s+10.1)(s+5.439)s}\right)$$

Result

- s = -1.3947 + j2.7895
- $K(s) = 5.6321\left(\frac{(s+0.3234)(s+2.081)}{s}\right)$
- $K(s) \approx 5.6321\left(\frac{(s+0.3234)(s+2.081)}{s}\right)\left(\frac{20}{s+20}\right)$





112.64 (s^2+2.4044s+.673)/(s^2+20s)
K(s)
1 361.2/(s^5+33.6s^4+373.8s^3+1597.5s^2+2267.7s+578.6)
G(s)
Y
-1
Plot
1.4
1.2
1.0
.8
.6
.4
.2
0
0
5
10
15
20
25
30
35
40
45
50
Time (sec)

P vs. I vs. PI vs. PID

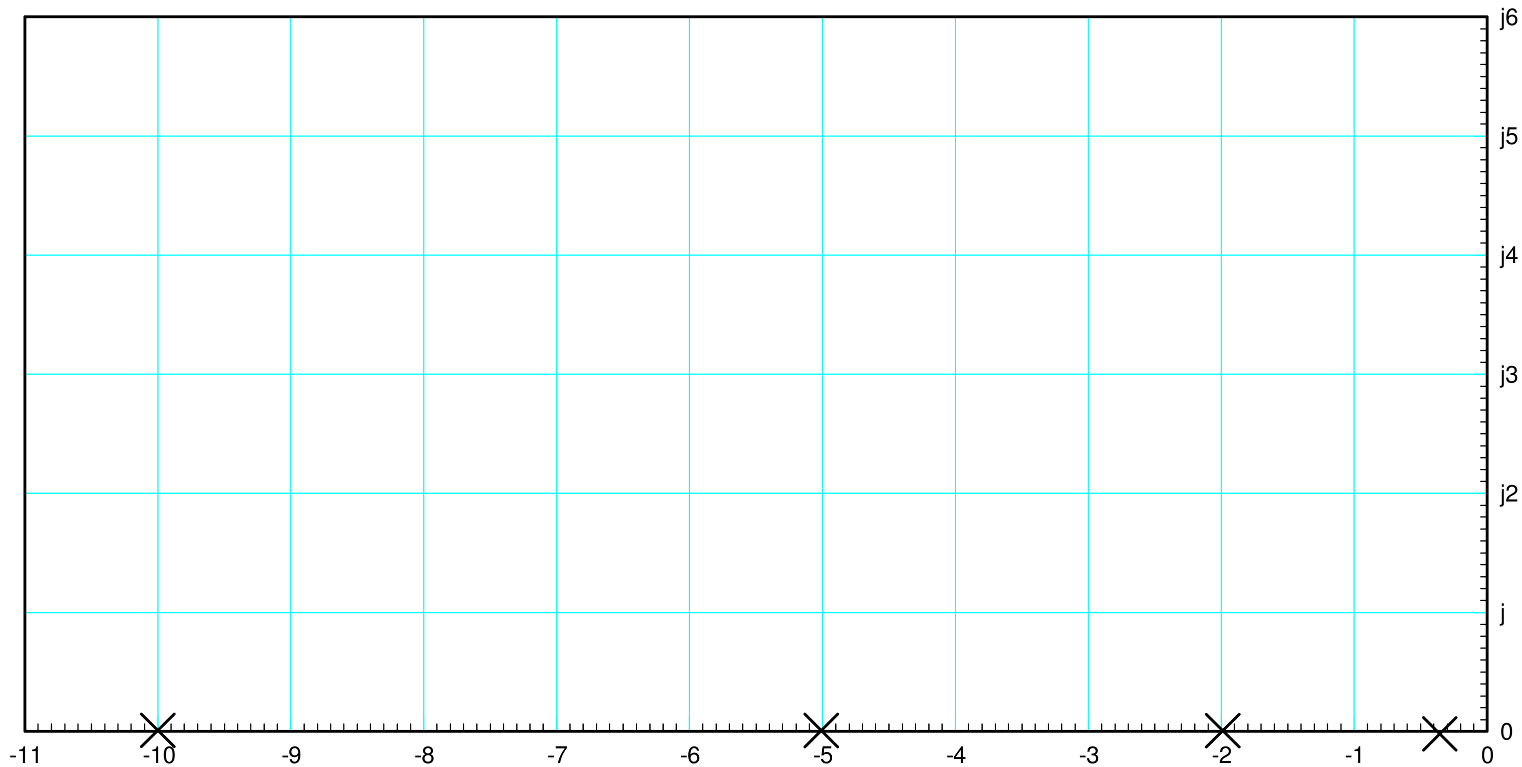
PID Controllers				
K(s)	Closed-Loop Dominant Pole(s)	U at t=0 K(s) as s -> infinity	Kv	$T_{2\%}$ seconds
5.5117	-0.6942 + j1.3884	5.51	0	5.76
$\left(\frac{0.3974219}{s}\right)$	-0.1249 + j0.2483	0	0.2481	32.02
$5.0050\left(\frac{s+0.3234}{s}\right)$	-0.5886 + j1.1772	5.00	1.0106	6.79
$5.6321\left(\frac{(s+0.3234)(s+2.081)}{s}\right)$	-1.3947 + j2.7895	112.64	2.3665	2.87

Handout

Design a PID compensator for the following system

$$G(s) = \left(\frac{200}{(s+0.3)(s+2)(s+5)(s+10)} \right)$$

so that the damping ratio is 0.707 (45 degrees)



Summary

PID Compensators are very common

- P: gain compensation
- I: integration
  - Add a pole at s = 0
- PI:
  - Add a pole at s = 0
  - Add a zero to cancel the slowest stable pole
- PID:
  - Add a pole at s = 0
  - Add two zeros to cancel the two slowest poles
  - Usually PID is actually PI + Lead to reduce the noise (# poles >= # zeros)

Each can be implemented with a single op-amp circuit