Meeting Design Specs using Bode Plots

ECE 461/661 Controls Systems

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lecture notes, homework sets, and solutions

Meeting Design Specs using Bode Plots

When designing a compensator to meet design specs including:

- Steady-state error for a step input
- Phase Margin
- 0dB gain frequency

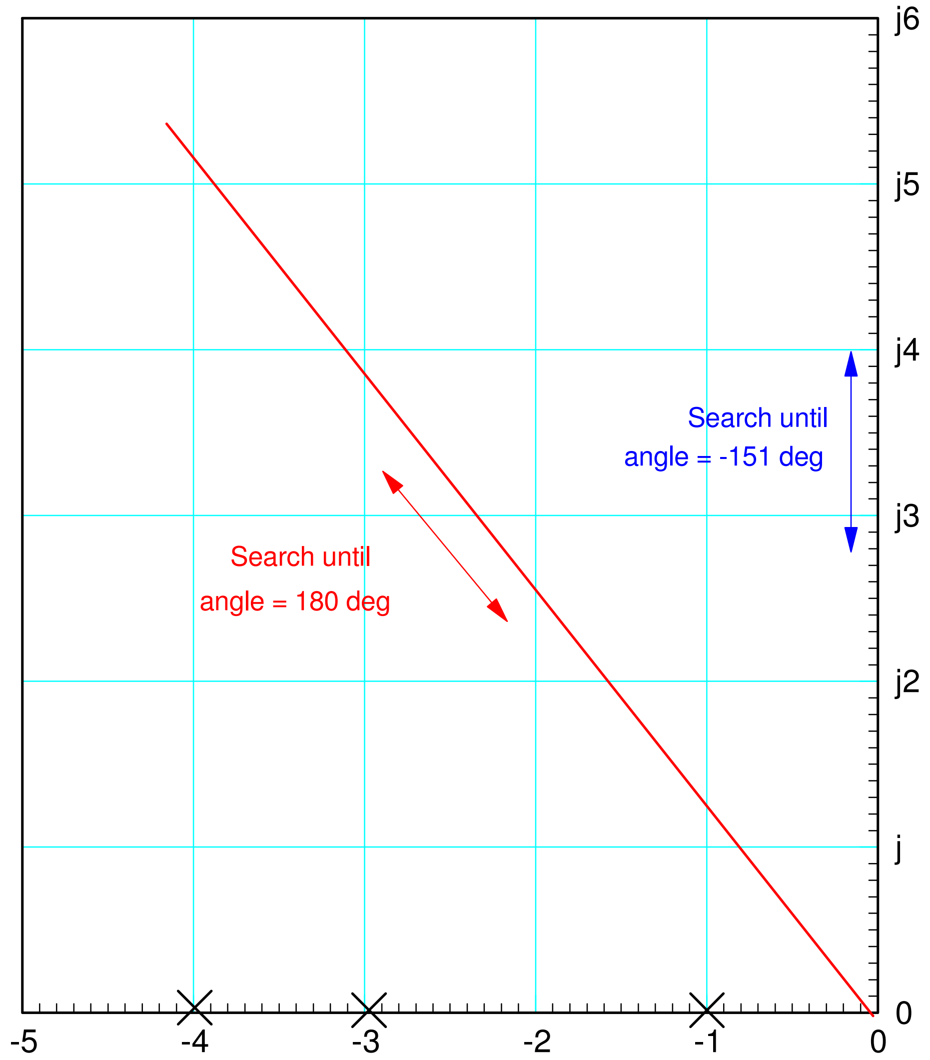
the procedure using Bode plots is about the same as it was for root-locus plots.

- Add a pole at s = 0 if needed (making the system type-1)
- Start cancelling zeros until the system is too fast
- For every zero you added, add a pole. Place these poles so that the phase adds up.

|GK| = 1

Root Locus

- Search along damping line
- Gain = 1 when the open-loop phase is 180 degrees



Bode Plots

- Search along jw axis
- Gain = 1 when the open-loop phase is ϕ

Example 1: $G(s) = \left(\frac{1000}{(s+1)(s+3)(s+6)(s+10)} \right)$

Design a compensator which result in

- No error for a step input
- 20% overshoot for a step input, and
- A settling time of 4 second

Solution: First, convert to Bode Plot terminology

- Type-1 system
- 20% overshoot means

  $\zeta = 0.4559$

  $M_m = 1.2322$

  $GK = 1\angle -132.12^0$

  Phase Margin = 47.88 degrees

- A settling time of 4 seconds means

  s = -1 + j 2

  The 0dB gain frequency is 2.00 rad/sec

In short, design K(s) so that the system is type-1 and

$$(GK)_{s=j2} = 1\angle -132.12^0$$

Start with

$$K(s) = \left(\frac{s+1}{s}\right)$$

At 4 rad/sec

$$GK = \left(\frac{1000}{s(s+3)(s+6)(s+10)}\right)_{s=j2} = 2.1508\angle -153.43^0$$

There is too much phase shift, so start cancelling zeros. If you cancel the zero at -3

$$K(s) = \left(\frac{(s+1)(s+3)}{s}\right)$$

$$GK = \left(\frac{1000}{s(s+6)(s+10)}\right)_{s=j2} = 7.7522\angle -119.75^0$$

Too far from -180 degrees (good)

Add a 2nd pole

- # poles = #zeros

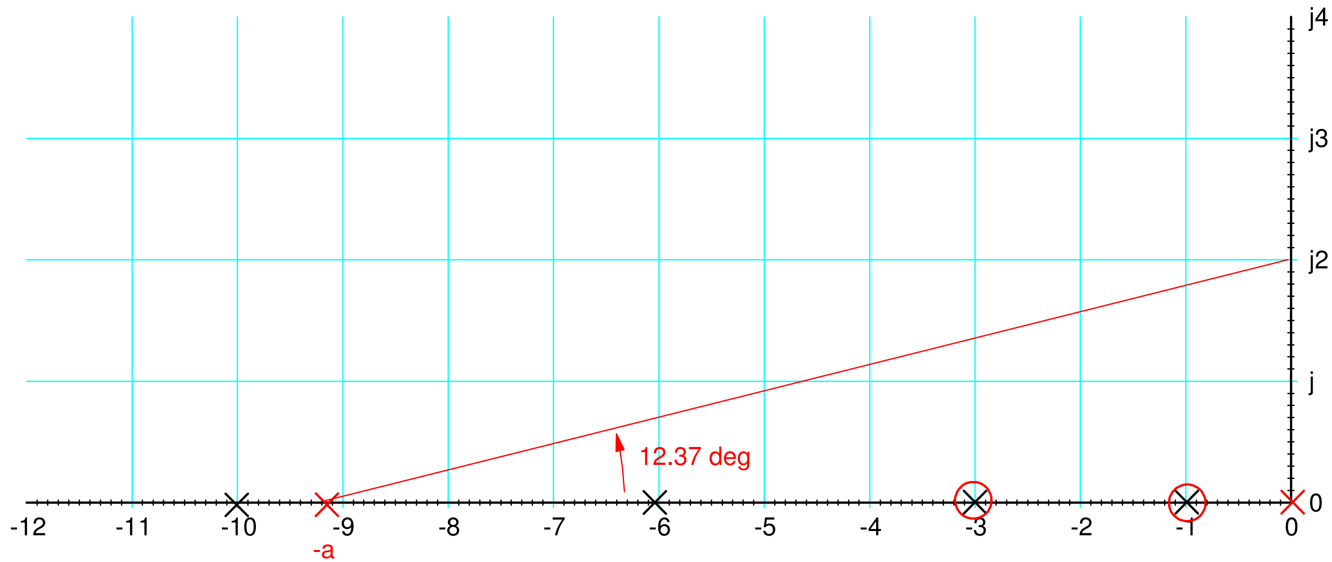
$$K(s) = \left(\frac{(s+1)(s+3)}{s(s+a)}\right)$$

Solve for 'a'

$$GK = \left(\frac{1000k}{s(s+a)(s+6)(s+10)}\right)_{s=j2} = 1\angle -132.12^0$$

$$\angle\left(\frac{1}{s+a}\right)_{s=j2} = -12.37^0$$

$$a = \frac{2}{\tan\left(12.37^0\right)} = 9.1154$$



$$K(s) = k\left(\frac{(s+1)(s+3)}{s(s+9.1154)}\right)$$

To find k,

$$GK = \left(\frac{1000}{s(s+6)(s+9.1154)(s+10)}\right)_{s=j2} = 0.8307\angle -132.12^0$$

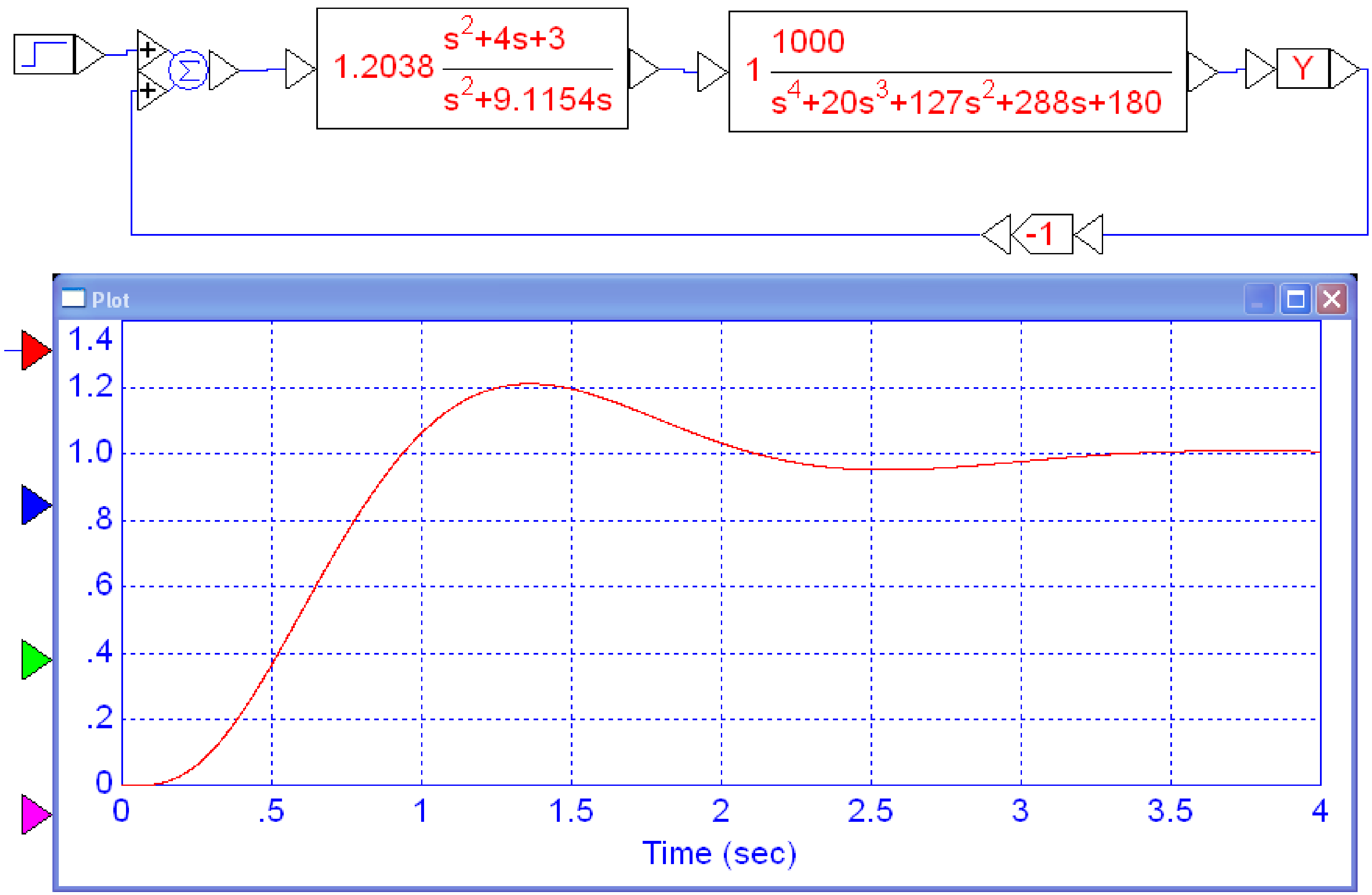
This should be 1.000, so add a gain, k,

$$k = \frac{1}{0.8307} = 1.2038$$

and

$$\boxed{K(s) = 1.2038\left(\frac{(s+1)(s+3)}{s(s+9.1154)}\right)}$$

Checking in VisSim:

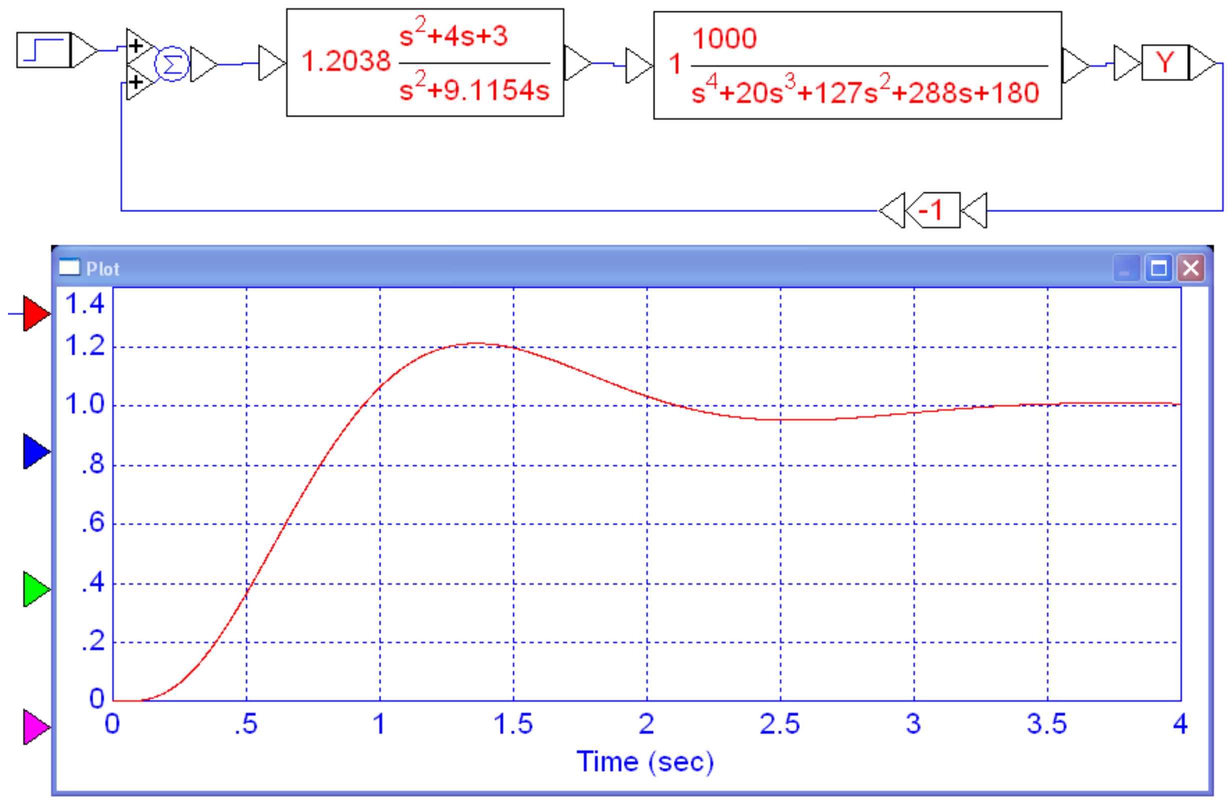


+
+
Σ
1.2038 $\frac{s^2+4s+3}{s^2+9.1154s}$
1 $\frac{1000}{s^4+20s^3+127s^2+288s+180}$
Y
-1
Plot
1.4
1.2
1.0
.8
.6
.4
.2
0
0
.5
1
1.5
2
2.5
3
3.5
4
Time (sec)

The overshoot is a little high.

- Designing to a phase margin means you intersect the M-circle rather than beging tangent
- This results in the resonance being slightly too high

Which result sin the overshoot being slightly too high.



Handout: Assume

$$G(s) = \left(\frac{100}{s(s+2)(s+5)}\right)$$

Design a compensator so that

- There is no error for a step input
- The 0dB gain frequency is 3 rad/sec, and

The phase margin is 50 degrees

Example 2: System with Delay

Add a 200ms delay

$$G(s) = \left(\frac{1000}{(s+1)(s+3)(s+6)(s+10)}\right) \cdot e^{-0.2s}$$

Solution: Again, you want to make this a type-1 system with

$$(GK)_{s=j2} = 1\angle -132.12^0$$

Start

- Add a pole at s = 0
- Add three zeros

$$K(s) = \left(\frac{(s+1)(s+3)(s+6)}{s}\right)$$

$$GK = \left(\left(\frac{1000}{s(s+10)}\right) \cdot e^{-0.2s}\right)_{s=j2} = 49.029\angle -124.22^0$$

With three zeros, you need to add three poles.

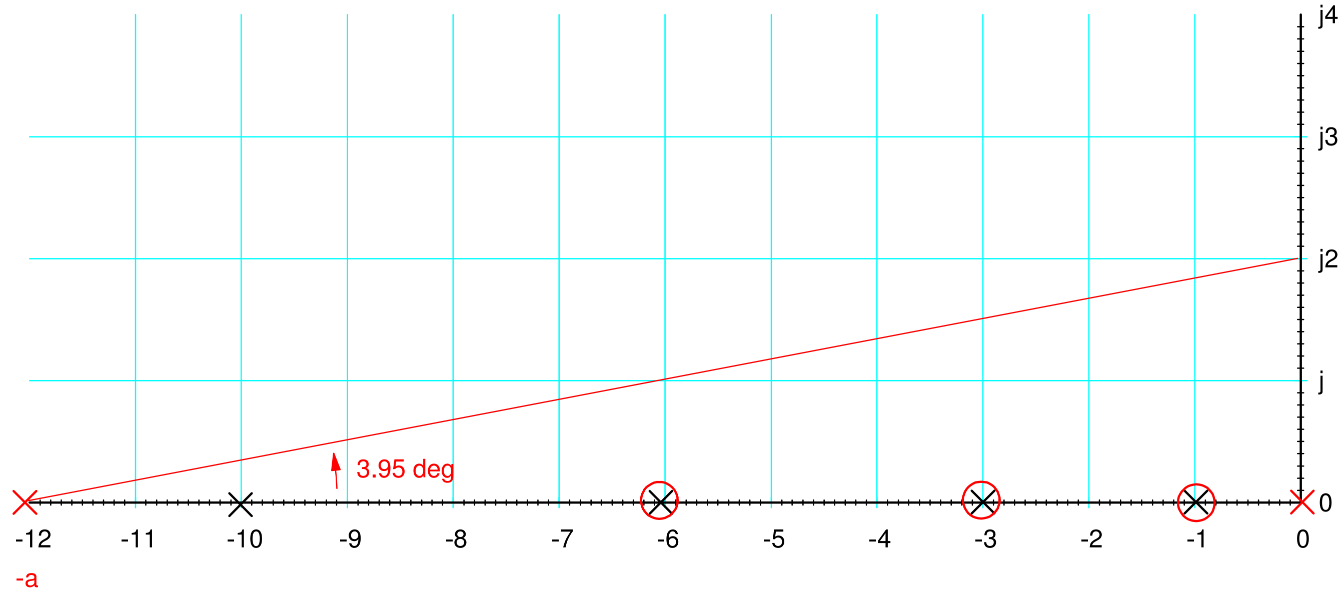
- One at s = 0
- Two at s = -a

$$K(s) = \left(\frac{(s+1)(s+3)(s+6)}{s(s+a)^2} \right)$$

$$\angle(s+a)^2 = 7.8918^0$$

$$\angle(s+a) = 3.9459^0$$

$$a = \frac{2}{\tan\left(3.94^0\right)} = 28.995$$



$$K(s) = k\left(\frac{(s+1)(s+3)(s+6)}{s(s+28.995)^2}\right)$$

To find k

$$GK = \left(\left(\frac{1000}{s(s+10)(s+28.995)^2}\right) \cdot e^{-0.2s}\right)_{s=j2} = 0.0508\angle -132.12^0$$

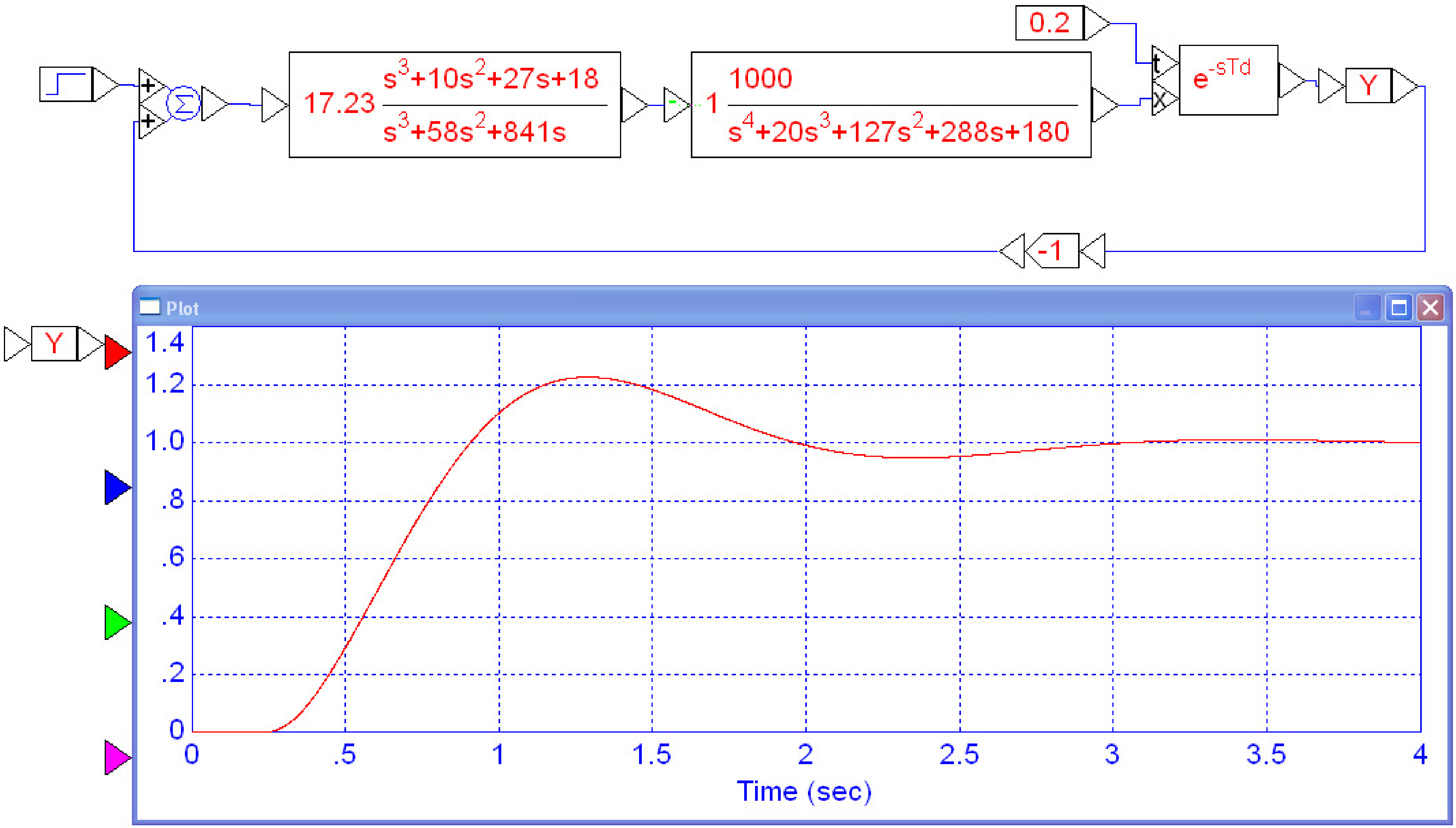
This should be one, so add a gain

$$k = \frac{1}{0.0508} = 17.2288$$

and

$$\boxed{K(s) = 17.2288\left(\frac{(s+1)(s+3)(s+6)}{s(s+28.995)^2}\right)}$$

Checking in VisSim



0.2
$17.23 \frac{s^3+10s^2+27s+18}{s^3+58s^2+841s}$
$1 \frac{1000}{s^4+20s^3+127s^2+288s+180}$
t
X
e^{-sTd}
Y
-1
Plot
Y
1.4
1.2
1.0
.8
.6
.4
.2
0
0
.5
1
1.5
2
2.5
3
3.5
4
Time (sec)

Summary

Designing K(s) so that your system meets design specs is similar to what we did with root-locus techniques.

- If the system is type-0, add a pole at s=0 to make it type-1 (usually)
- Start adding zeros to cancel poles until the phase margin is too much
- Then add poles so that

    The number of poles = the number of zeros, and

    The phase margin is correct

- Then adjust the gain so that |G*K| = 1 at the 0dB gain frequency