ECE 463/663 Handout #4

Eigenvalues and Eigenvectors

The eivenvalues and eigenvectors for a system are

$$\lambda = \{-2, -5\}$$

$$\Lambda = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}$$

1) Determine the initial condition which causes system to decay as fast as possible.

2) Determine x(t) given that it's initial condition is

$$X(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Solution:

You know that the answer is of the form

$$x(t) = a_1 \Lambda_1 e^{\lambda_1 t} + a_2 \Lambda_2 e^{\lambda_2 t}$$

$$x(t) = a_1 e^{-2t} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + a_2 e^{-5t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

1) To decay as fast as possible, you want to excite only the fast mode

$$x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

2) At t = 0

$$x(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = a_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + a_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

meaning

a2 = 1

a1 = -1

so

$$x(t) = -e^{-2t} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + e^{-5t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad t > 0$$

$$x(t) = \begin{bmatrix} e^{-5t} \\ e^{-5t} - e^{-2t} \end{bmatrix} \qquad t > 0$$