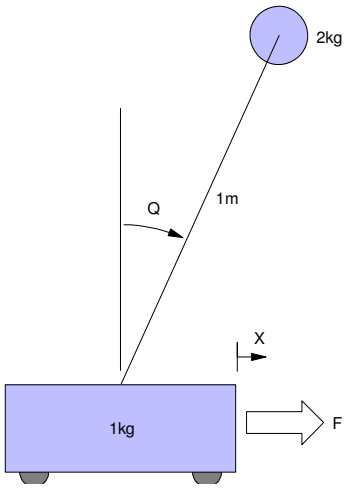
ECE 463: Homework #5

Pole Placement. Due Monday February 25th

1) Cart and Pendulum



The dynamics of a cart and pendulum with

- mass of cart = 1kg
- mass of ball = 2kg
- length = 1m

are

$$\begin{bmatrix} 3 & 2\cos\theta \\ \cos\theta & 1 \end{bmatrix}\begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 2\dot{\theta}^2\sin\theta \\ g\sin\theta \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} F$$

The linearized dynamics are:

$$s\begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -19.6 & 0 & 0 \\ -0 & 29.4 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} F$$

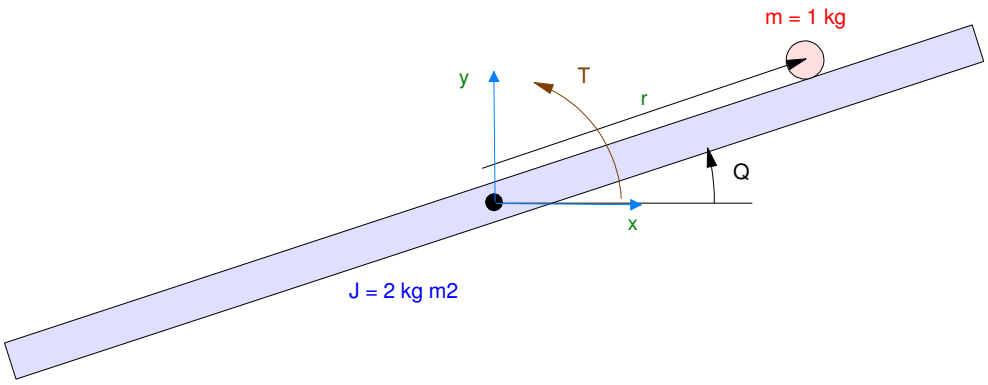
1a) Design a full-state feedback control law so that the closed-loop system has

- 10% overshoot for a step response, and
- A 2% settling time of 8 seconds

1b) Find the step respone for the linearized system in Matlab

1c) Find the step response of the nonlinear system (modify the Matlab simulation)

2) Ball and Beam



Ball and Beam from homework #4

The dynamics of a ball and beam system with

- mass of ball = 1kg
- inertia of beam = 2 kg m2

are

$$\begin{bmatrix} 1.4 & 0 \\ 0 & 2+r^2 \end{bmatrix}\begin{bmatrix} \ddot{r} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} r\dot{\theta}^2 - g\sin\theta \\ -2r\dot{r}\dot{\theta} - gr\cos\theta \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} T$$

The linearized dynamics at x = 1.00m are:

$$s\begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -7 & 0 & 0 \\ -3.27 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.33 \end{bmatrix} T$$

2a) Design a full-state feedback control law so that the closed-loop system has

- 10% overshoot for a step response, and
- A 2% settling time of 8 seconds

2b) Find the step respone for the linearized system in Matlab

2c) Find the step response of the nonlinear system (modify the Matlab simulation)