ECE 463/663 - Homework #8

Calculus of Variations. LQG Control. Due Monday, April 1th, 2019

Soap Film

1) Calculate the shape of a soap film connecting two rings around the X axis:

- Y(0) = 5
- Y(3) = 4

2) Calculate the shape of a soap film connecting two rings around the X axis: (trick question...)

- Y(0) = 5
- Y(10) = 4

3) Calculate the shape of a soap film connecting two rings around the X axis:

- Y(0) = 5
- Y(2) = free

Hanging Chain

4) Calculate the shape of a hanging chain subject to the following constraints

- Length of chain = 10 meters
- Left Endpoint: (-3,3)
- Right Endpoint: (+3,0)

Ricatti Equation

5) Find the function, x(t), which minimizes the following funcional

$$J = \int_0^2 (x^2 + 5\dot{x}^2)dt$$

x(0) = 2

x(2) = 0

6) Find the function, x(t), which minimizes the following funcional

$$J = \int_0^2 (x^2 + 5u^2)dt$$

$$\dot{x} = -0.1x + u$$

x(0) = 2

x(2) = 0

LQG Control

7) Cart and Pendulum (HW #5): Design a full-state feedback control law of the form

$$U = K_r R - K_x X$$

for the cart and pendulum system from homework #5 using LQG control so that

- The DC gain is 1.00
- The 2% settling time is 8 seconds, and
- There is less than 10% overshoot for a step input.

Compare your results with homework #5

- Where are the closed-loop poles with pole placement and with LQG control?
- Are the feedback gains larger or smaller with LQG control?
- Which one works better?

8) Ball and Beam (HW #5): Design a full-state feedback control law of the form

$$U = K_r R - K_x X$$

for the ball and beam system from homework #6 using LQG control so that

- The DC gain is 1.00
- The 2% settling time is 8 seconds, and
- There is less than 10% overshoot for a step input.

Compare your results with homework #5

- Where are the closed-loop poles with pole placement and with LQG control?
- Are the feedback gains larger or smaller with LQG control?
- Which one works better?

Note: With LQG control it's pretty much trial and error.

- Let R = 1
- Increasing the weighting on y^2 ($Q = C^T C$) speeds up the system
- Incresing the weighting on $(dy/dt)^2$ ($Q = (CA)^T(CA)$) adds friction / reduces oscillation
- Repeat until the step response or eigenvalues meet the requirements