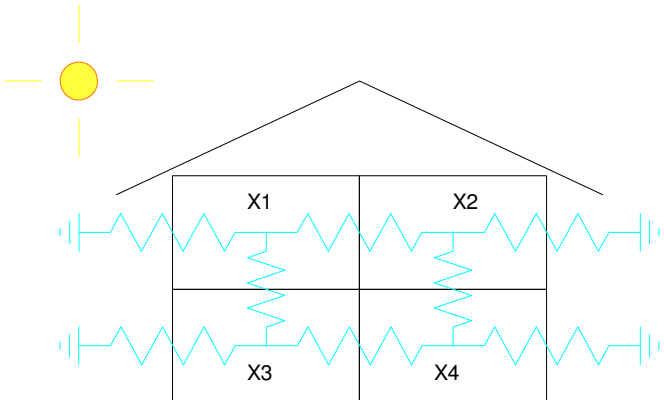
ECE 463/663 - Homework #9

Optimal Control with Servo Compensators & Multiple Inputs. Due Monday, April 8, 2019

1) Design a feedback controller using LQR methods for the ball and beam system so that
   - It can track a contant set-point and reject a contant disturbance, and
   - The step response is close to that of:

$$G_d(s) \approx \left(\frac{1}{s^2+1.5s+1}\right)$$

A 4-room house has the following dynamics:



$$s\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} = \begin{bmatrix} -0.45 & 0.2 & 0.2 & 0 \\ 0.2 & -0.45 & 0 & 0.2 \\ 0.2 & 0.1 & -0.55 & 0.2 \\ 0 & 0.2 & 0.2 & -0.55 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} d$$

where

- Xi is the temperature of each room
- Ui is the head added to each room, and
- d is the energy of the sun shining in the windows

2) Assume all heaters are output the same amount of heat:

U=U1=U2=U3=U4

and the temperature sensor is in apartment #1

$$s\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} = \begin{bmatrix} -0.45 & 0.2 & 0.2 & 0 \\ 0.2 & -0.45 & 0 & 0.2 \\ 0.2 & 0.1 & -0.55 & 0.2 \\ 0 & 0.2 & 0.2 & -0.55 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} U + \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} d$$

$$Y = X_1$$

2a) Design a feedback control law to force Y to a contant set point (i..e use a servo compensator).

2b) Plot the step response of the system to all four states with Ref = 70F

2c) Determine the steady-state temperature of each room with

- Ref = 70F, d = 0 (night)
- Ref = 70F, d = 10 (day)

3) Assume all heaters are separate and that each room has its own thermostat:

$$s\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} = \begin{bmatrix} -0.45 & 0.2 & 0.2 & 0 \\ 0.2 & -0.45 & 0 & 0.2 \\ 0.2 & 0.1 & -0.55 & 0.2 \\ 0 & 0.2 & 0.2 & -0.55 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} d$$

3a) Design a feedback control law to force Y to a contant set point (i..e use a servo compensator).

3b) Plot the step response of the system to all four states with d = 0 and

- Ref1 = 60F, Ref2 = Ref3 = Ref4 = 0
- Ref2 = 65F, Ref1 = Ref3 = Ref4 = 0
- Ref3 = 70F Ref1 = Ref2 = Ref4 = 0
- Ref4 = 75F Ref1 = Ref2 = Ref3 = 0

3c) Determine the steady-state temperature of each room with

- Ref1 = 60F, Ref2 = 65F, Ref3 = 70F, Ref4 = 75F, d = 0 (night)
- Ref1 = 60F, Ref2 = 65F, Ref3 = 70F, Ref4 = 75F, d = 10 (day)