ECE 463/663 - Homework #9

Optimal Control with Servo Compensators & Multiple Inputs. Due Monday, April 8, 2019

1) Design a feedback controller using LQR methods for the ball and beam system so that

- It can track a contant set-point and reject a contant disturbance, and
- The step response is close to that of:

$$G_d(s) \approx \left(\frac{1}{s^2+1.5s+1}\right)$$

The desired response:

```
Gd = tf(1,[1,1.5,1]);
t = [0:0.01:10]';
yd = step(Gd,t);
```

Input the plant and the augmented system with the servo compensator

```
A = [0,0,1,0;0,0,0,1;0,-7,0,0;-3.27,0,0,0]
B = [0;0;0;0.33]

A5 = [A, zeros(4,1) ; 1,0,0,0,0]

         0         0    1.0000         0         0
         0         0         0    1.0000         0
         0   -7.0000         0         0         0
   -3.2700         0         0         0         0
    1.0000         0         0         0         0

B5r = [0*B;-1]

     0
     0
     0
     0
    -1

C5 = [1,0,0,0,0;0,1,0,0,0]
D5 = [0;0];
X0 = zeros(5,1);
R = 0*t+1;
```

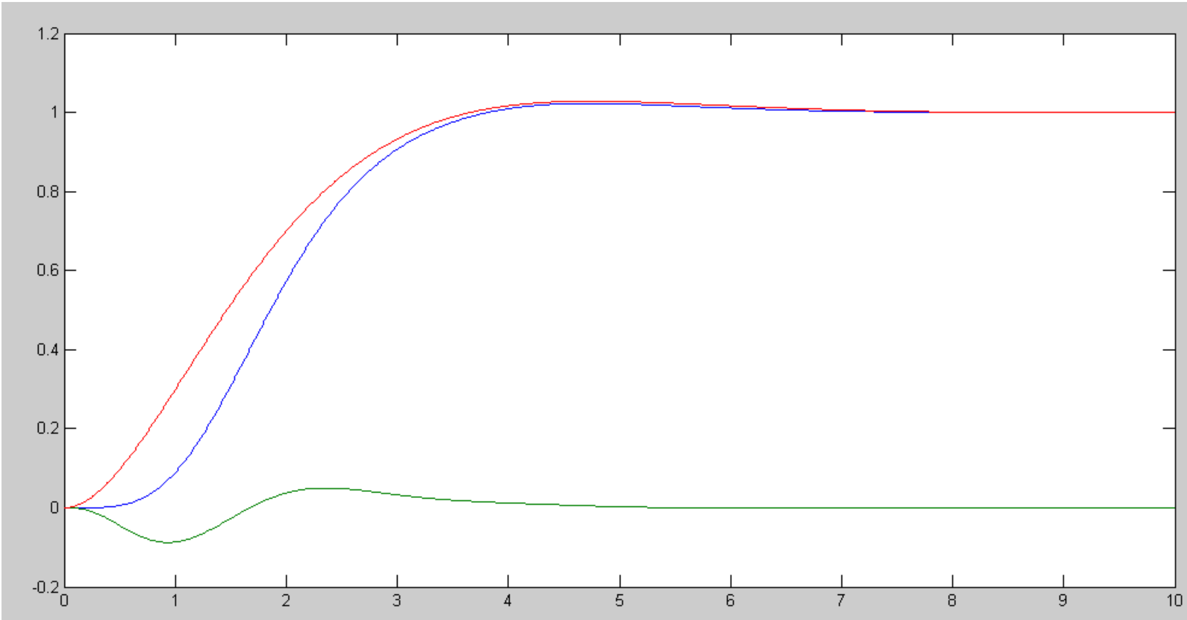
Now design the servo compensator using LQR methods until the two step responses look similar (this takes several different guesses at Q)

```
K5 = lqr(A5,B5,diag([0,0,0,0,1]),1);
y = step3(A5-B5*K5,B5r,C5,D5,t,X0,R);
plot(t,y,t,yd);
```

time passes...

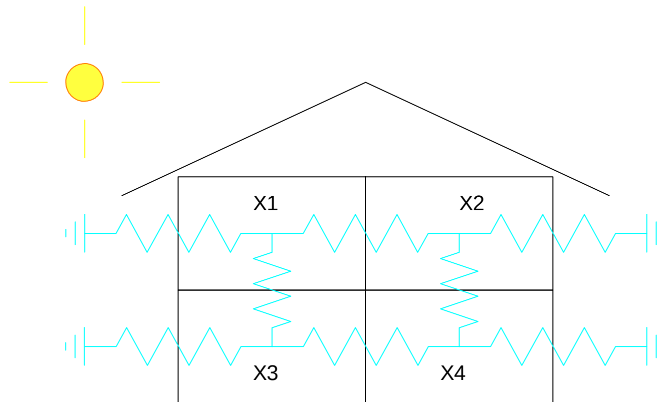
```
K5 = lqr(A5,B5,diag([0,0,200,0,400]),1);
y = step3(A5-B5*K5,B5r,C5,D5,t,X0,R);
plot(t,y,t,yd);
```

K5 = -47.5545 95.2627 -32.9746 24.0281 -20.0000



Resulting Step Response: Desired Step Response (red), Actual Step Response (blue = position, green = angle)

A 4-room house has the following dynamics:



$$s\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} = \begin{bmatrix} -0.45 & 0.2 & 0.2 & 0 \\ 0.2 & -0.45 & 0 & 0.2 \\ 0.2 & 0.1 & -0.55 & 0.2 \\ 0 & 0.2 & 0.2 & -0.55 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} d$$

where

- Xi is the temperature of each room
- Ui is the head added to each room, and
- d is the energy of the sun shining in the windows

2) Assume all heaters are output the same amount of heat:

U=U1=U2=U3=U4

and the temperature sensor is in apartment #1

$$s\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} = \begin{bmatrix} -0.45 & 0.2 & 0.2 & 0 \\ 0.2 & -0.45 & 0 & 0.2 \\ 0.2 & 0.1 & -0.55 & 0.2 \\ 0 & 0.2 & 0.2 & -0.55 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} U + \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} d$$

$$Y = X_1$$

2a) Design a feedback control law to force Y to a contant set point (i..e use a servo compensator).

Add a servo compensator:

$$sZ = Y - Ref$$

The augmented system is then

$$\begin{bmatrix} sX \\ sZ \end{bmatrix} = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix} \begin{bmatrix} X \\ Z \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} U + \begin{bmatrix} B_d \\ 0 \end{bmatrix} d + \begin{bmatrix} 0 \\ -1 \end{bmatrix} R$$

Design a full-state feebdack control law using LQR methods

Input the desired response:

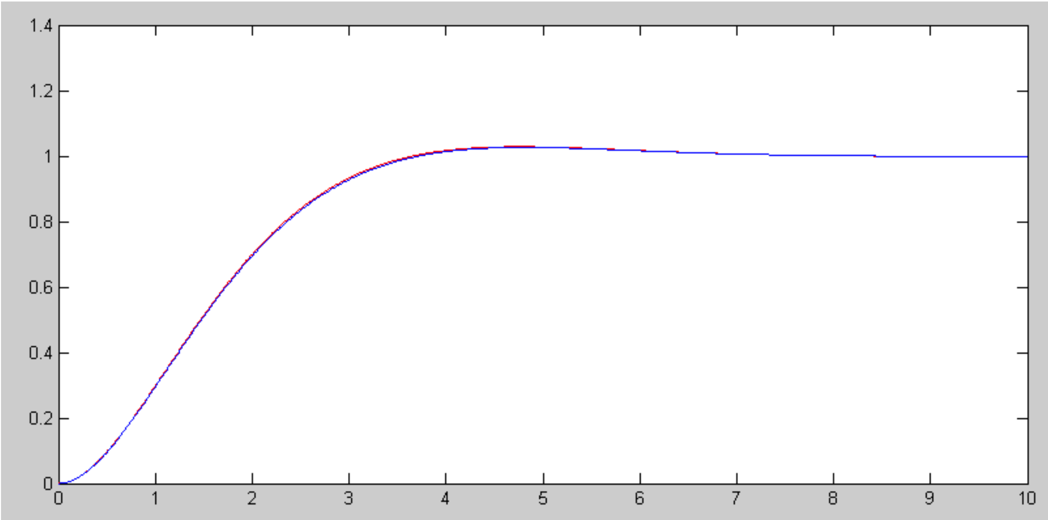
```
Gd = tf(1,[1,1.5,1]);
t = [0:0.01:10]';
yd = step(Gd,t);
```

Input the augmented system

```
A = [-0.45,0.2,0.2,0 ; 0.2,-0.45,0,0.2 ; 0.2,0.1,-0.55,0.2 ; 0,0.2,0.2,-0.55]

   -0.4500    0.2000    0.2000         0
    0.2000   -0.4500         0    0.2000
    0.2000    0.1000   -0.5500    0.2000
         0    0.2000    0.2000   -0.5500

B = [1;1;1;1]

     1
     1
     1
     1

C = [1,0,0,0];
A5 = [A, zeros(4,1) ; C, 0]

   -0.4500    0.2000    0.2000         0         0
    0.2000   -0.4500         0    0.2000         0
    0.2000    0.1000   -0.5500    0.2000         0
         0    0.2000    0.2000   -0.5500         0
    1.0000         0         0         0         0




B5 = [B ; 0]

     1
     1
     1
     1
     0

K5 = lqr(A5, B5, diag([0,0,0,0,1]),1);
C5 = [C, 0];
D5 = 0;
X0 = zeros(5,1);
R = 0*t + 1;
B5r = [0*B; -1]

     0
     0
     0
     0
    -1

y = step3(A5-B5*K5, B5r, C5, D5, t, X0, R);
plot(t,yd,'r',t,y,'b');
```

Keep interating on Q until the two step responses are close

```
K5 = lqr(A5, B5, diag([0.3,0,0,0,1]),1);
y = step3(A5-B5*K5, B5r, C5, D5, t, X0, R);
plot(t,yd,'r',t,y,'b');
```



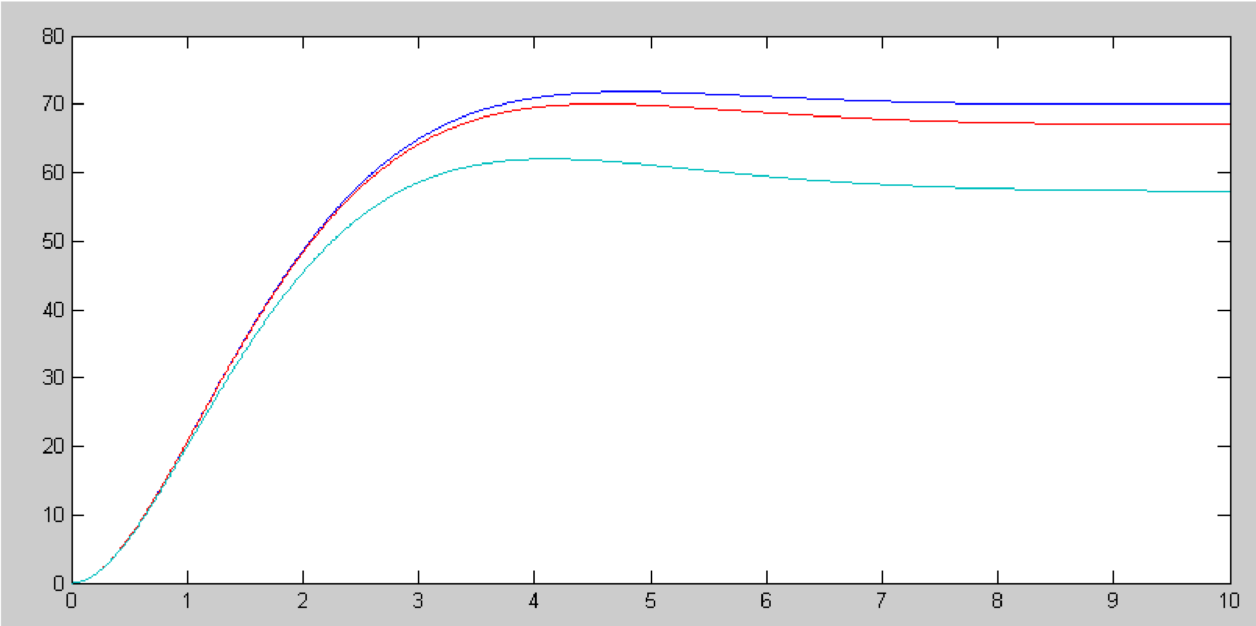
Desired Step Response (red) and Actual (blue)

This results in

```
K5 =     1.1412     0.1556     0.1424     0.0265     1.0000
                      Kx                               Kz
```

2b) Plot the step response of the system to all four states with Ref = 70F

```
>> C4 = [eye(4,4),zeros(4,1)]

C4 =

     1     0     0     0     0
     0     1     0     0     0
     0     0     1     0     0
     0     0     0     1     0

>> D4 = zeros(4,1);
>> y = step3(A5-B5*K5, B5r, C4, D4, t, X0, R*70);
>> plot(t,y);
```



The room temperatures are

```
>> -C4*inv(A5-B5*K5)*B5r * 70

   70.0000
   67.0053
   67.0053
   57.2727
```

2c) Determine the steady-state temperature of each room with

- Ref = 70F, d = 0 (night)
- Ref = 70F, d = 10 (day)

```
>> -C4*inv(A5-B5*K5)*B5r * 70   - C4*inv(A5-B5*K5)*B5d * 0

#1   70.0000
#2   67.0053
#3   67.0053
#4   57.2727


>> -C4*inv(A5-B5*K5)*B5r * 70   - C4*inv(A5-B5*K5)*B5d * 10

#1   70.0000
#2   46.6845
#3   64.8663
#4   39.0909
```

3) Assume all heaters are separate and that each room has its own thermostat:

$$s\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} = \begin{bmatrix} -0.45 & 0.2 & 0.2 & 0 \\ 0.2 & -0.45 & 0 & 0.2 \\ 0.2 & 0.1 & -0.55 & 0.2 \\ 0 & 0.2 & 0.2 & -0.55 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} d$$

3a) Design a feedback control law to force Y to a contant set point (i..e use a servo compensator).

Add a servo compensator for each input

$$sZ = \begin{bmatrix} X_1 - R_1 \\ X_2 - R_2 \\ X_3 - R_3 \\ X_4 - R_4 \end{bmatrix}$$

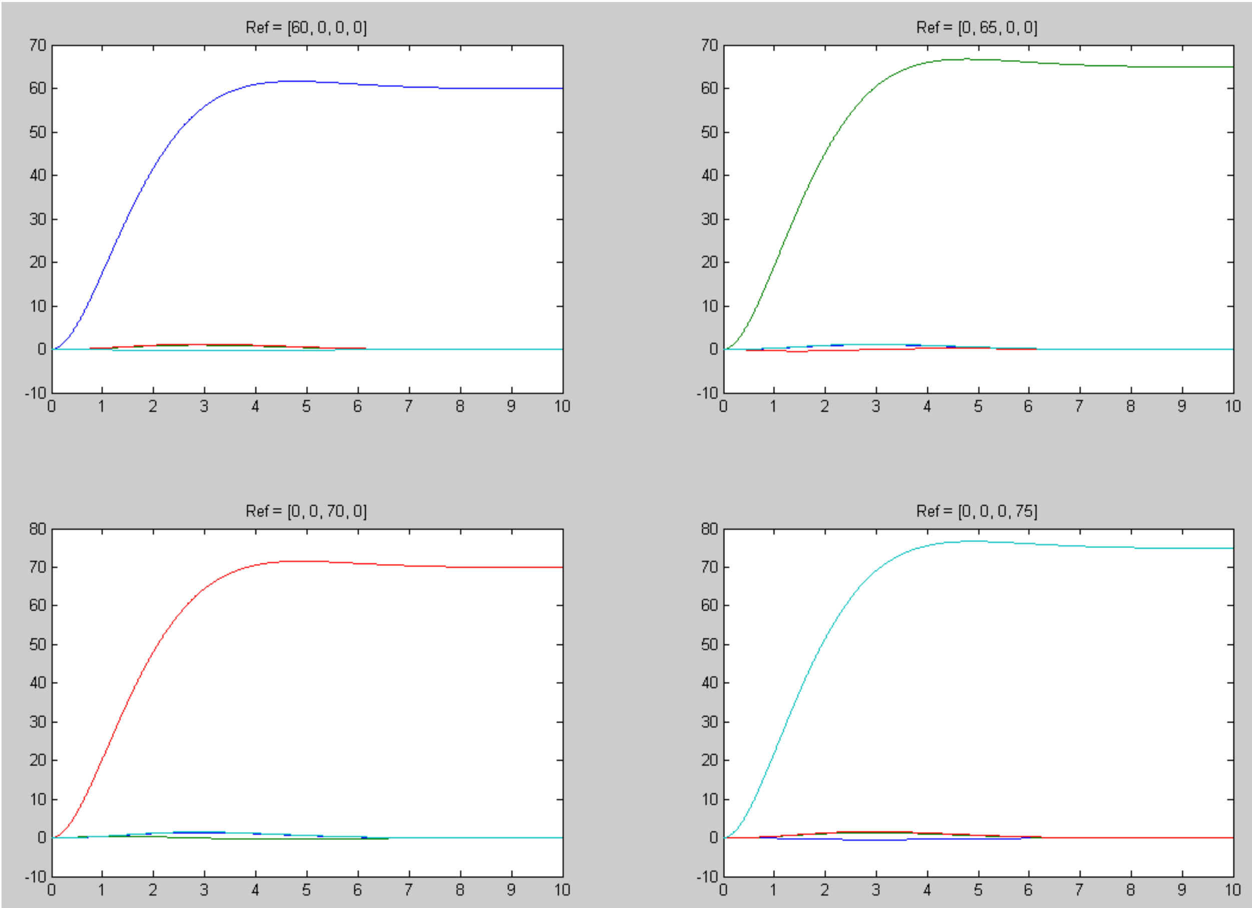
```
Az = zeros(4,4);
Bz = eye(4,4);

A8 = [A, zeros(4,4) ; Bz*C, Az]

   -0.4500    0.2000    0.2000         0         0         0         0         0
    0.2000   -0.4500         0    0.2000         0         0         0         0
    0.2000    0.1000   -0.5500    0.2000         0         0         0         0
         0    0.2000    0.2000   -0.5500         0         0         0         0
    1.0000         0         0         0         0         0         0         0
         0    1.0000         0         0         0         0         0         0
         0         0    1.0000         0         0         0         0         0
         0         0         0    1.0000         0         0         0         0

B = eye(4,4)
B8 = [B; zeros(4,4)]

B8 =

     1     0     0     0
     0     1     0     0
     0     0     1     0
     0     0     0     1
     0     0     0     0
     0     0     0     0
     0     0     0     0
     0     0     0     0


K8 = lqr(A8, B8, diag([0,0,0,0,1,1,1,1]), eye(4,4))

K8 =

    1.0583    0.1465    0.1359    0.0237    1.0000   -0.0007    0.0006   -0.0000
    0.1465    1.0610    0.0562    0.1404    0.0007    0.9995    0.0328    0.0007
    0.1359    0.0562    0.9892    0.1301   -0.0006   -0.0328    0.9995   -0.0006
    0.0237    0.1404    0.1301    0.9906    0.0000   -0.0008    0.0006    1.0000

X0 = zeros(8,1);
```

```
B8r = [zeros(4,4); -eye(4,4)]

     0     0     0     0
     0     0     0     0
     0     0     0     0
     0     0     0     0
    -1     0     0     0
     0    -1     0     0
     0     0    -1     0
     0     0     0    -1

C8 = [eye(4,4), zeros(4,4)];
D8 = zeros(4,1);
y = step3(A8-B8*K8, B8r(:,1), C8, D8, t, X0, R);
plot(t,y)
```

3b) Plot the step response of the system to all four states with d = 0 and

- Ref1 = 60F, Ref2 = Ref3 = Ref4 = 0
- Ref2 = 65F, Ref1 = Ref3 = Ref4 = 0
- Ref3 = 70F Ref1 = Ref2 = Ref4 = 0
- Ref4 = 75F Ref1 = Ref2 = Ref3 = 0

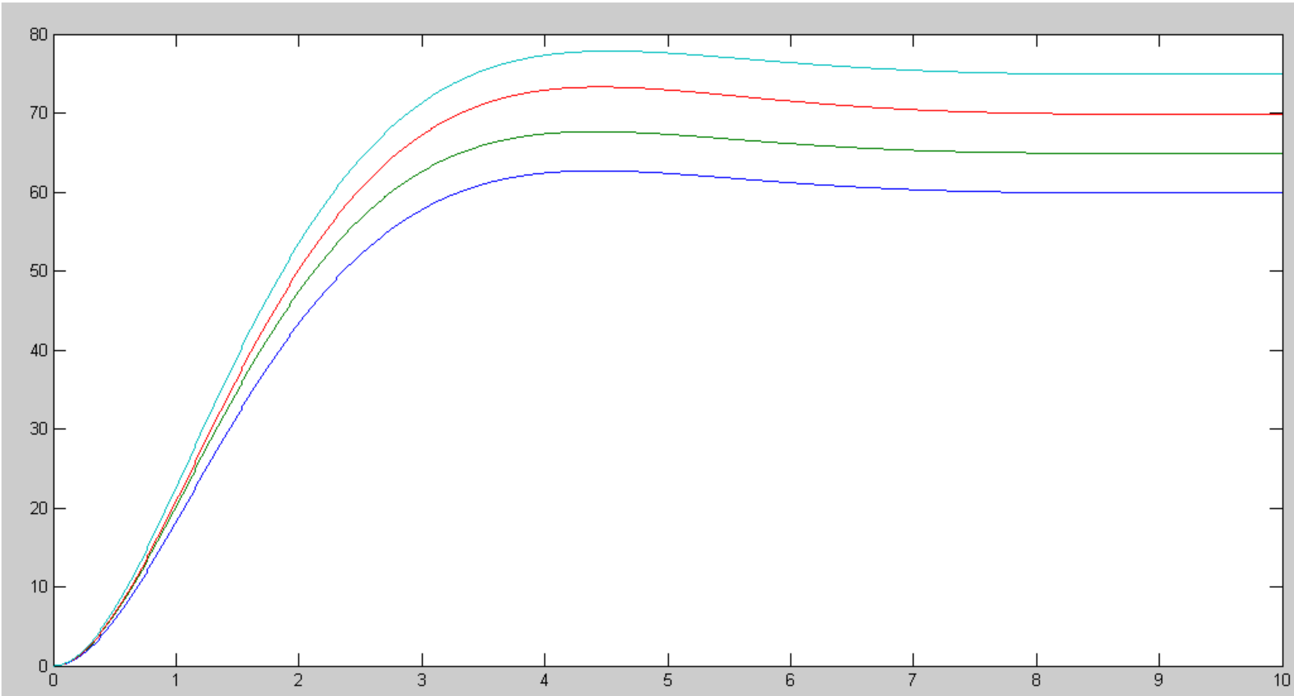
```
Bz = eye(4,4);
D8 = zeros(4,4);
R = 0*t+1;

y = step3(A8-B8*K8, B8r, C8, D8, t, X0, R*[60,0,0,0]);
subplot(221);
plot(t,y)
title('Ref = [60, 0, 0, 0]')

subplot(222);
y = step3(A8-B8*K8, B8r, C8, D8, t, X0, R*[0,65,0,0]);
plot(t,y)
title('Ref = [0, 65, 0, 0]')

subplot(223);
y = step3(A8-B8*K8, B8r, C8, D8, t, X0, R*[0,0,70,0]);
plot(t,y)
title('Ref = [0, 0, 70, 0]')

subplot(224);
y = step3(A8-B8*K8, B8r, C8, D8, t, X0, R*[0,0,0,75]);
plot(t,y)
title('Ref = [0, 0, 0, 75]')
```



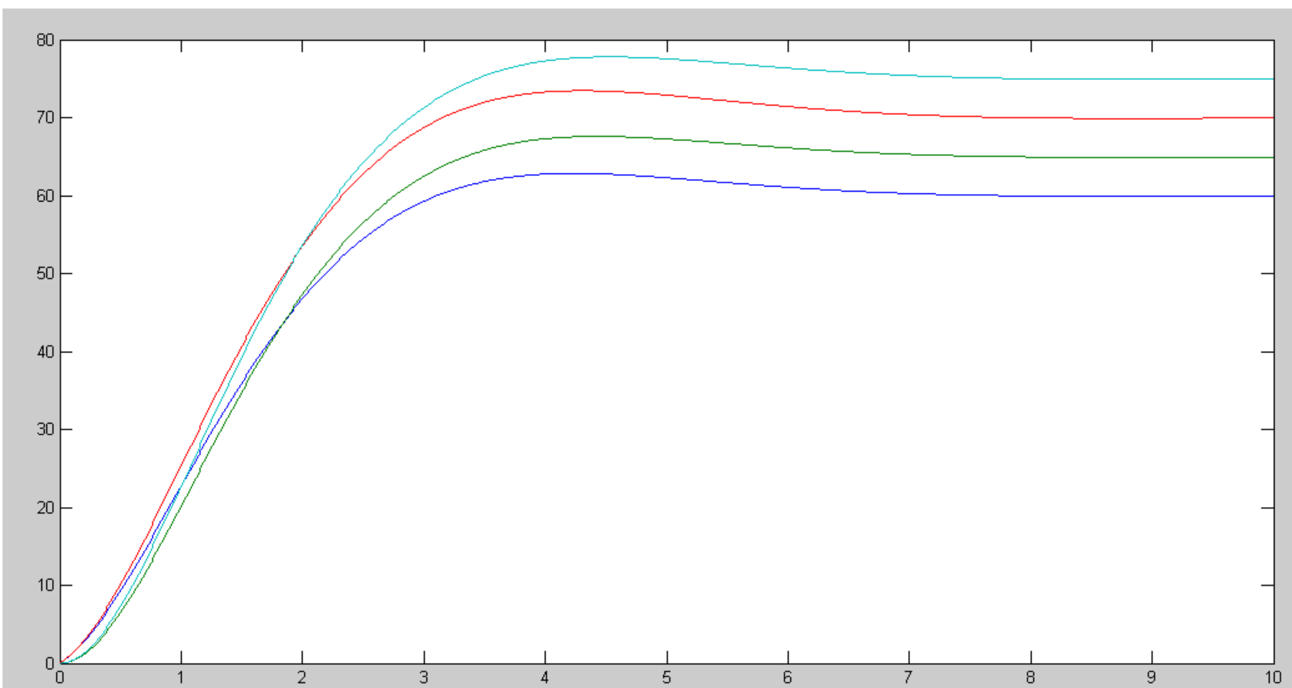
3c) Determine the steady-state temperature of each room with

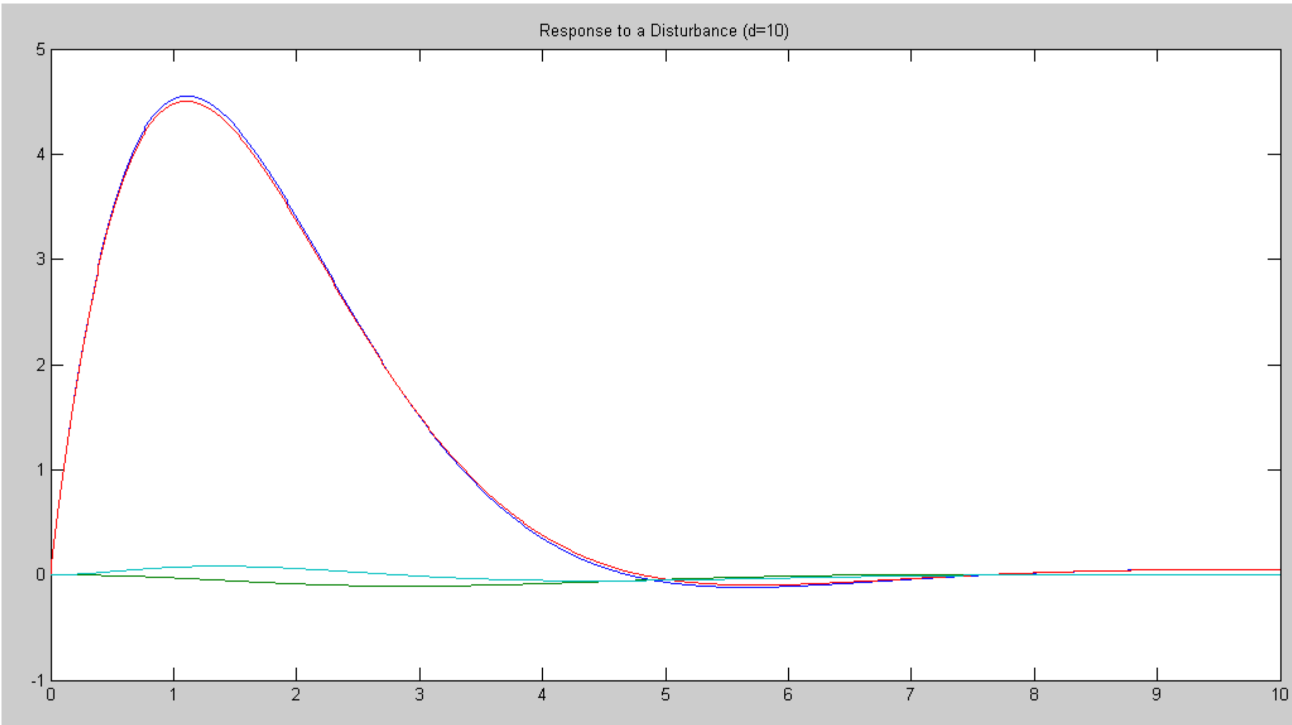
- Ref1 = 60F, Ref2 = 65F, Ref3 = 70F, Ref4 = 75F, d = 0 (night)
- Ref1 = 60F, Ref2 = 65F, Ref3 = 70F, Ref4 = 75F, d = 10 (day)

```
y = step3(A8-B8*K8, B8r, C8, D8, t, X0, R*[60,65,70,75]);

B8d = [1;0;1;0;0;0;0;0];
D8 = zeros(4,1);
yd = step3(A8-B8*K8, B8d, C8, D8, t, X0, R*10);

subplot(111);
plot(t,y)
title('Response to a Ref = {60, 65, 70, 75}, d=0)')

plot(t,y+yd)
title('Response to a Ref = {60, 65, 70, 75}, d=10)')
```



Step Response without a Disturbance (night)




Response to a Disturbance (d=10)
5
4
3
2
1
0
-1
0
1
2
3
4
5
6
7
8
9
10