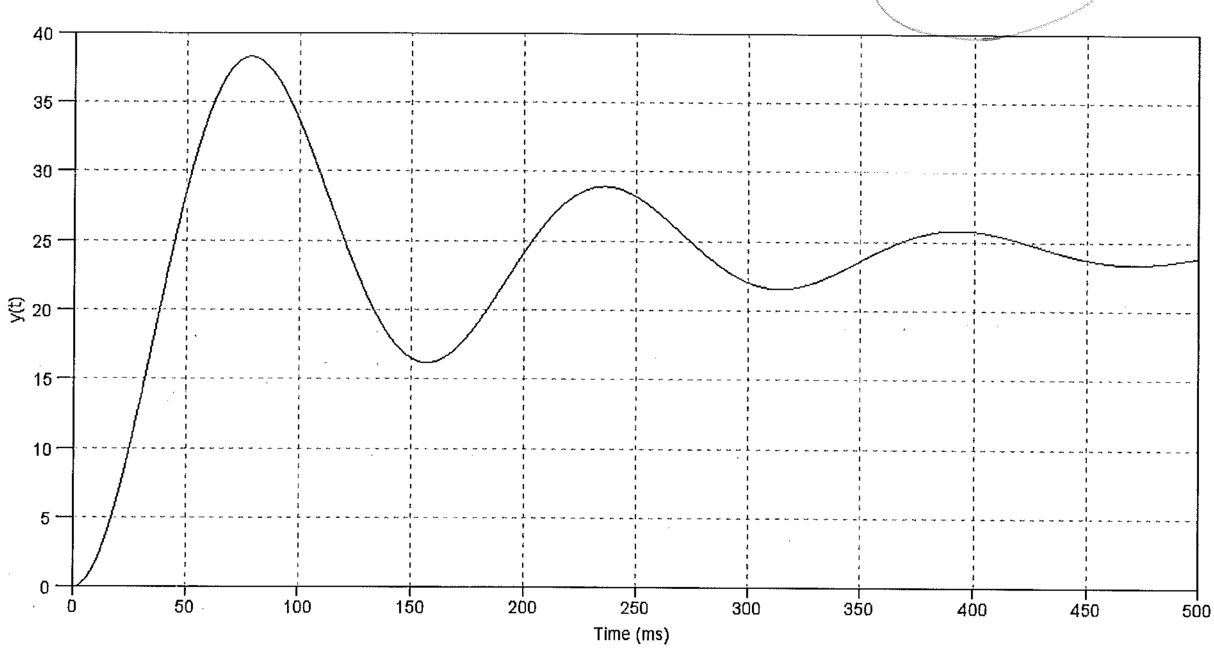
ECE 463/663: Test #1. Name ________________

Spring 2019

1) Find the transfer funciton for a system with the following step response

25 pt each



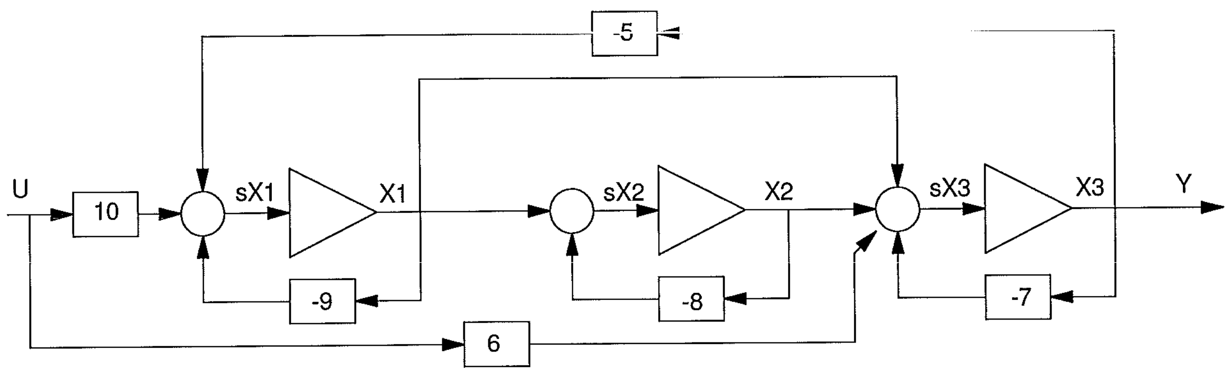
DC = 24

$\omega = \frac{3 \text{ cycles}}{470\text{ms}} \cdot 2\pi = 40.1$

$T_s = 500\text{ms}$ $\sigma = \frac{4}{500\text{ms}} = 8$

$$\frac{39\,936}{(s+8 \pm j40)}$$

2) Give the state-space model for the following system



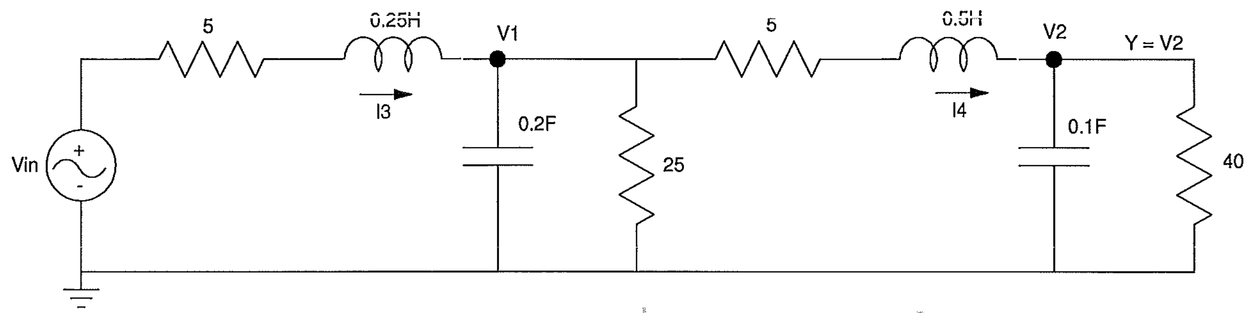
$$\begin{bmatrix} sX1 \\ sX2 \\ sX3 \end{bmatrix} = \begin{bmatrix} -9 & 0 & -5 \\ 1 & -8 & 0 \\ 1 & 1 & -7 \end{bmatrix} \begin{bmatrix} X1 \\ X2 \\ X3 \end{bmatrix} + \begin{bmatrix} 10 \\ 0 \\ 6 \end{bmatrix} U$$

$$Y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X1 \\ X2 \\ X3 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix}$$

2pt each

Problem 3) (work either this problem or the mass-spring problem)

3a) Write four coupled differential equations to describe the following circuit



$$0.2\dot{V}_1 = I_3 - I_4 - \frac{V_1}{25}$$

$$0.1\dot{V}_2 = I_4 - V_2/40$$

$$0.25\dot{I}_3 = V_{in} - 5I_3 - V_1$$

$$0.5\dot{I}_4 = V_1 - 5I_4 - V_2$$

$$\dot{V}_1 = 5I_3 - 5I_4 - .2V_1$$

$$\dot{V}_2 = 10I_4 - .25V_2$$

$$\dot{I}_3 = 4V_{in} - 20I_3 - 4V_1$$

$$\dot{I}_4 = 2V_1 - 10I_4 - 2V_2$$

3b) Express these dynamics in state-space form

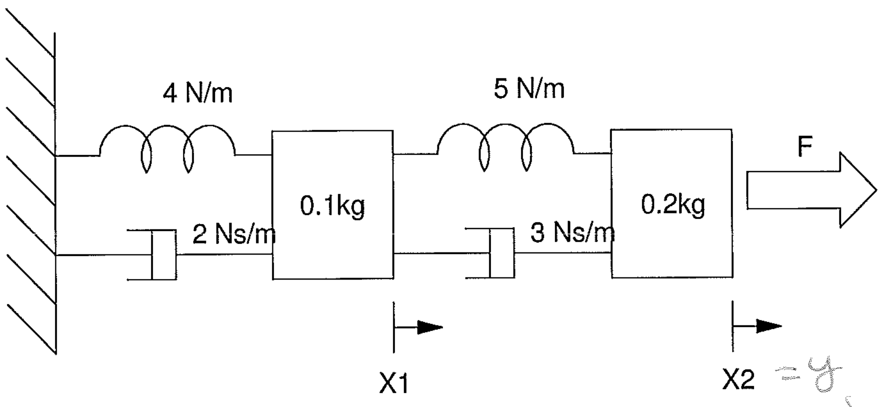
$$\begin{bmatrix} sV1 \\ sV2 \\ sI3 \\ sI4 \end{bmatrix} = \begin{array}{cccc} V_1 & V_2 & I_3 & I_4 \\ \hline \end{array}$$

$$\begin{bmatrix} -.2 & 0 & 5 & -5 \\ 0 & -.25 & 0 & 10 \\ -4 & 0 & -20 & 0 \\ 2 & -2 & 0 & -10 \end{bmatrix} \begin{bmatrix} V1 \\ V2 \\ I3 \\ I4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 4 \\ 0 \end{bmatrix} Vin$$

$$Y = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} V1 \\ V2 \\ I3 \\ I4 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} Vin$$

Problem 3) (work either this problem or the circuit problem)

3a) Write two coupled 2nd-order differential equations to describe the following mass spring system



$$0.1\ddot{x}_1 = 4(0-x_1)+5(x_2-x_1)+2s(-x_1)+3s(x_2-x_1)$$

$$0.2\ddot{x}_2 = 5(x_1-x_2)+3s(x_1-x_2)+F$$

$$\ddot{x}_1 = -90x_1+50x_2-50\dot{x}_1+30\dot{x}_2$$

$$\ddot{x}_2 = 25x_1-25x_2+15\dot{x}_1-15\dot{x}_2+5F$$

3b) Express these dynamics in state-space form

$$s\begin{bmatrix} X1 \\ X2 \\ sX1 \\ sX2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -90 & 50 & -50 & 30 \\ 25 & -25 & 15 & -15 \end{bmatrix}\begin{bmatrix} X1 \\ X2 \\ sX1 \\ sX2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 5 \end{bmatrix} F$$

$$Y = \begin{bmatrix} \; & \; & \; & \; \end{bmatrix}\begin{bmatrix} X1 \\ X2 \\ sX1 \\ sX2 \end{bmatrix} + \begin{bmatrix} \; \end{bmatrix} F$$

4) Assume the LaGrangian is:

$$L = \frac{3}{2}\dot{x}^2 + \dot{x}\dot{\theta}\cos\theta + \frac{1}{2}\dot{\theta}^2 + 9.8\cos\theta$$

Determine

$$T = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \left(\frac{\partial L}{\partial \theta}\right)$$

$$T = \overbrace{\frac{d}{dt}\left(\dot{x}\cos\theta + \dot{\theta}\right)}^{8pt} - \overbrace{\left(-\dot{x}\dot{\theta}\sin\theta - 9.8\sin\theta\right)}^{8pt}$$

$$T = \overbrace{\ddot{x}\cos\theta - \dot{x}\dot{\theta}\sin\theta + \ddot{\theta}}^{9pt} + \dot{x}\dot{\theta}\sin\theta + 9.8\sin\theta$$

$$\boxed{T = \ddot{x}\cos\theta + \ddot{\theta} + 9.8\sin\theta}$$