ECE 463/663 - Test #3:   Name ______________________

Friday, April 26th   Closed Book, Closed Notes. Calculators Permitted

Euler LeGrange Equation: $F_y - \frac{d}{dt}(F_{\dot{y}}) = 0$

1) Determine the function, y(t), which minimizes the following funcitonal:

$$J = \int_0^{10} (9y^2 + \dot{y}^2) dt$$

subject to the constraint

y(0) = 1

y(10) = 0

2) Determine the function, y(t), which minimizes the following funcitonal:

$$J = \int_0^2 (9y^2 + u^2)dt$$

subject to the constraints

y(0) = 1   y(10) = 0 $\dot{y} - 2y = u$

2a) Solve the following Euler LeGrange equation

$$F = (9y^2 + u^2) + m(\dot{y} - 2y - u)$$

$$F_y - \frac{d}{dt}(F_{\dot{y}}) = 0$$

2b) Solve the following Euler LeGrange equation

$$F = (9y^2 + u^2) + m(\dot{y} - 2y - u)$$

$$F_u - \frac{d}{dt}(F_{\dot{u}}) = 0$$

2c) Solve the following Euler LeGrange equation

$$F = (9y^2 + u^2) + m(\dot{y} - 2y - u)$$

$$F_m - \frac{d}{dt}(F_{\dot{m}}) = 0$$

2d) Solve for y(t) given the endpoint constraints:

$y(0) = 1$

$y(10) = 0$

3)   A road is to be built between two points, A and B. The cost per mile to build the road is proportional to the square of the distance from they axis:

$$J = \int_a^b \left(x^2 \sqrt{1 + \dot{y}^2} \right) dx$$

$$F_y - \frac{d}{dx}(F_{\dot{y}}) = 0$$

Find the differential equation which describes the optimal path for the road. (You don't need to solve for y(x), just give the differential equation that y must satisfy.)

4) Express the following system in state-space form