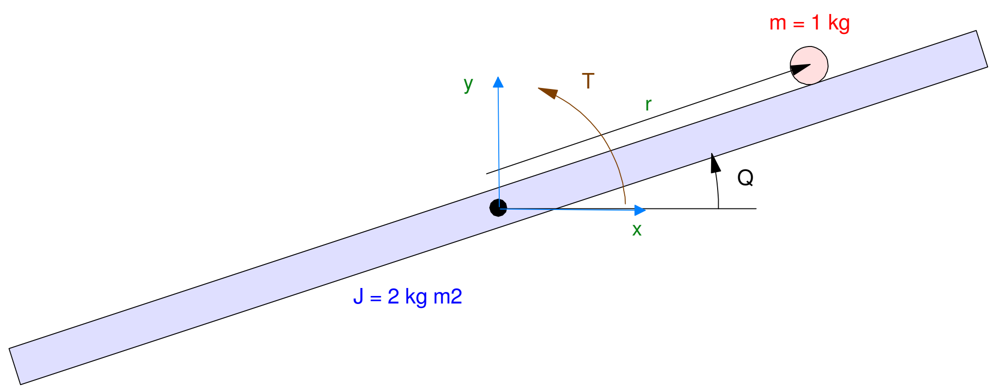
ECE 463/663: Test #2 - A Level

Take-Home. Open-Book, Open-Notes, Open-Internet. Please work alone. Due March 25th.



Ball and Beam from homework #4

Use the dynamics for a ball and beam system from previous homework sets

- mass of ball = 0.9 - 1.1kg (1kg nominal)
- inertia of beam = 2 kg m2
- Closed-Loop System's 2% Settling Time = 6 seconds

A-Level (max score = 100%): Design a feedback control law to

- **Track a constant and sinusoidal set-point, R**

  $$R = a + b\sin(t)$$

- **Reject a constant and sinusoidal disturbance**

  $$\eta = c + d\sin(t + \phi)$$

- **While only measuring the ball position (r) (meaning you need to use a full-order observer)**

B-Level (max score = 90%): Design a feedback control law to

- Track a constant set-point

  $$R = a$$

- Reject a constant disturbance

  $$\eta = c$$

- While only measuring the ball position (r) (meaning you need to use a full-order observer)

C-Level (max score = 80%): Same as B-level but assume all states are measured (so there is no need for a full-order observer)

Problem 1) Give a block-diagram and state-space model for the

- Plant
- Servo Compensator,
- Full-order observer (A and B level), and
- Disturbance estimator (A and B level)

Problem 2) Using the linear model, design

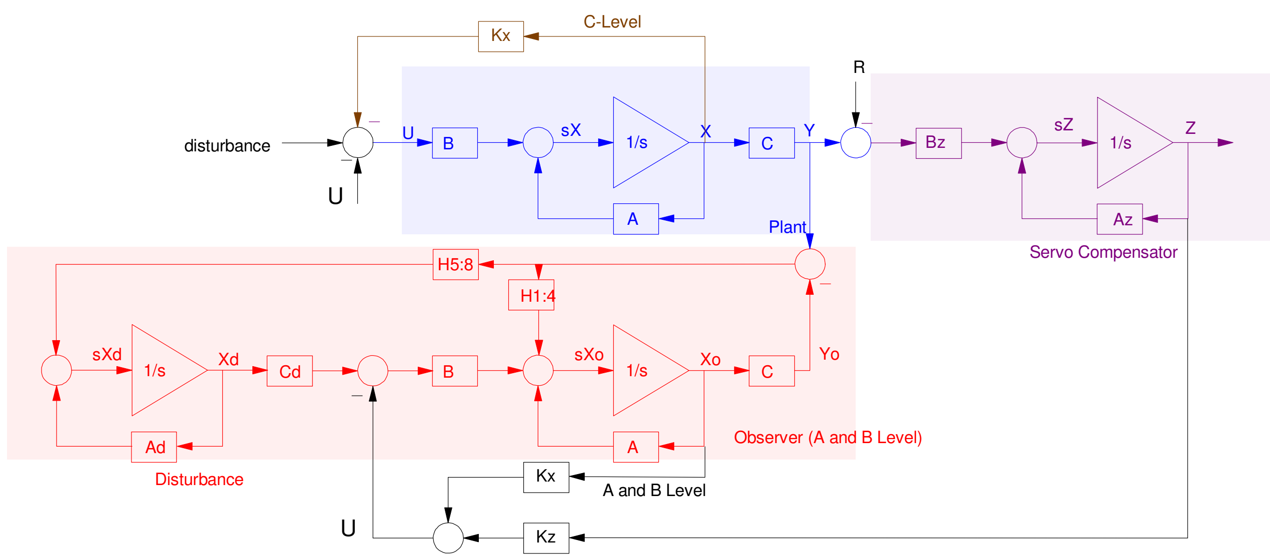
- A feedback control law to meet the requirements.
- A full-order observer to estimate the states and disturbances (A and B level)

Problem 3) Using the linear model, determine the response to

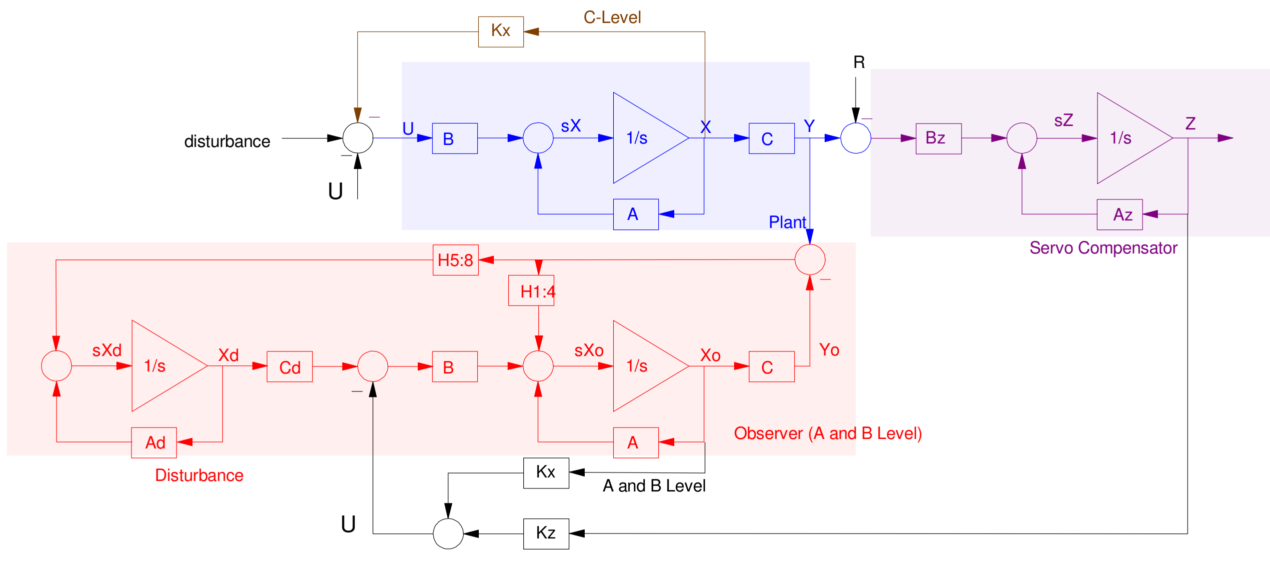
- A step input (R = 1)
- A step disturbance (n = 1)
- A sinuosidal setpoint (R = sin(t)   A level only)
- A sinusoidal disturbance (n = sin(t). A level only)

Problem 4) Modify the non-linear simularion and determine (and plot) the step response (R = 1) for

- m = 1.0kg (nominal)
- m = 0.9kg
- m = 11kg



Block Diagram



System Matricies

$$\begin{bmatrix} sX \\ sZ \\ sX_o \\ sX_d \end{bmatrix} = \begin{bmatrix} A & -BK_z & -BK_x & 0 \\ B_zC & A_z & 0 & 0 \\ H_{1:4}C & -BK_z & A-BK_x-H_{1:4}C & BC_d \\ H_{5:7}C & 0 & -H_{5:7}C & A_d \end{bmatrix} \begin{bmatrix} X \\ Z \\ X_o \\ X_d \end{bmatrix} + \begin{bmatrix} B \\ 0 \\ 0 \\ 0 \end{bmatrix} d + \begin{bmatrix} 0 \\ -B_z \\ 0 \\ 0 \end{bmatrix} R$$

Step 1: Design the feedback control law.

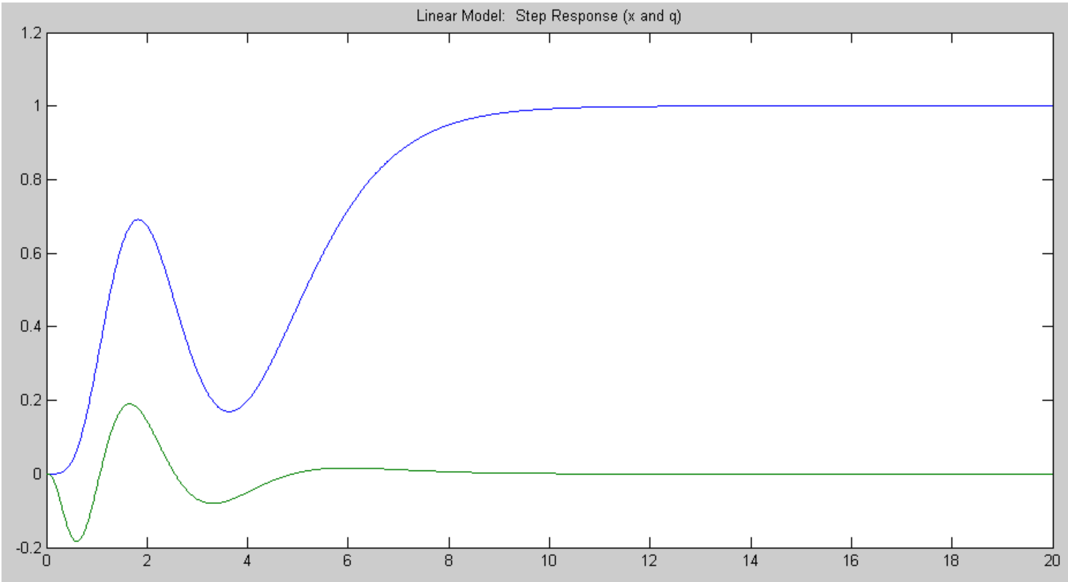
Use superposition to find the feedback gains.

Create the augmented system (plant & servo)

$$\begin{bmatrix} sX \\ sZ \end{bmatrix} = \begin{bmatrix} A & 0 \\ B_z C & A_z \end{bmatrix} \begin{bmatrix} X \\ Z \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} U$$

$$Y = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} X \\ Z \end{bmatrix}$$

```
A = [0,0,1,0 ; 0,0,0,1 ; 0,-7,0,0 ; -3.27,0,0,0]
B = [0;0;0;0.33]
Az = [0,0,0 ; 0,0,1 ; 0,-1,0]
Bz = [1;1;1]
C = [1,0,0,0]

A7 = [A, zeros(4,3) ; Bz*C, Az]
B7 = [B; zeros(3,1)]
K7 = ppl(A7, B7, [-1, -2, -2.1, -2.2, -2.3, -2.4, -2.5])
B7r = [zeros(4,1) ; -Bz];
C7 = [C,0,0,0];
D7 = [0];

C7xq = [1,0,0,0,0,0,0 ; 0,1,0,0,0,0,0];
D7xq = zeros(2,1);
t = [0:0.01:20]';
X0 = zeros(7,1);
R = 0*t + 1;

y = step3(A7-B7*K7, B7r, C7xq, D7xq, t, X0, R);
plot(t,y);
```



Step 2: Design a full-order observer for the augmented plant (plant + disturbance)

$$\begin{bmatrix} sX \\ sX_d \end{bmatrix} = \begin{bmatrix} A & BC_d \\ 0 & A_d \end{bmatrix} \begin{bmatrix} X \\ X_d \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} U$$

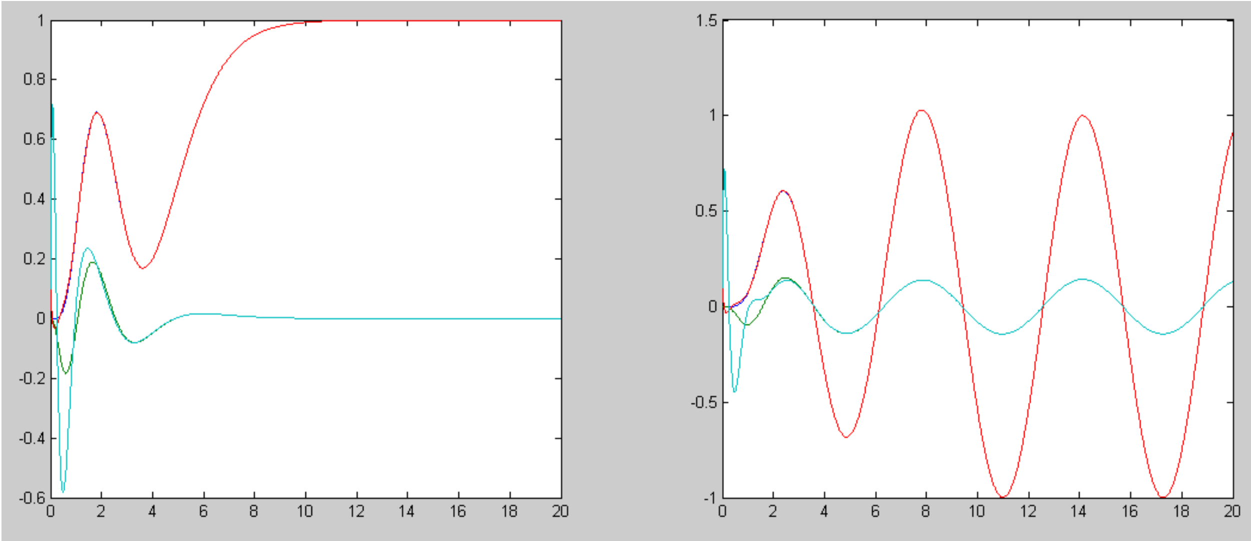
$$Y = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} X \\ X_d \end{bmatrix}$$

```
Ad = Az;
Cd = [1,1,1];

Ao = [A, B*Cd ; zeros(3,4), Ad];
Bo = [B; zeros(3,1)];
Co = [C, zeros(1,3)];
Do = 0;

H = ppl(Ao', Co', [-3, -3.1, -3.2, -3.3, -3.4, -3.5, -3.6])'
```

Putting it together

$$\begin{bmatrix} sX \\ sZ \\ sX_o \\ sX_d \end{bmatrix} = \begin{bmatrix} A & -BK_z & -BK_x & 0 \\ B_z C & A_z & 0 & 0 \\ H_{1:4}C & -BK_z & A - BK_x - H_{1:4}C & BC_d \\ H_{5:7}C & 0 & -H_{5:7}C & A_d \end{bmatrix} \begin{bmatrix} X \\ Z \\ X_o \\ X_d \end{bmatrix} + \begin{bmatrix} B \\ 0 \\ 0 \\ 0 \end{bmatrix} d + \begin{bmatrix} 0 \\ -B_z \\ 0 \\ 0 \end{bmatrix} R$$

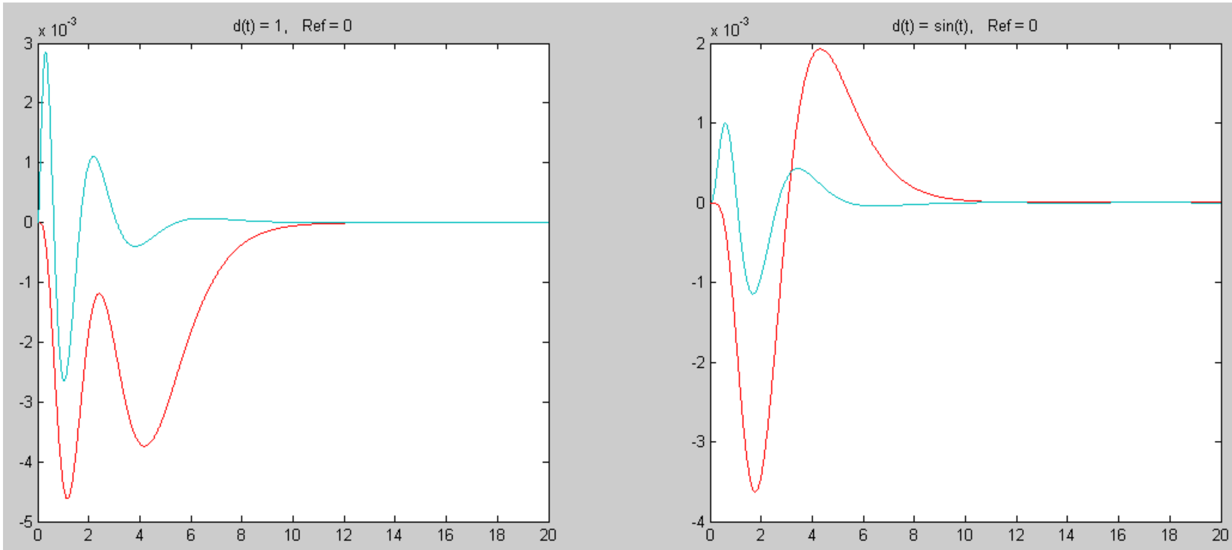
```
Kx = K7(1:4)
Kz = K7(5:7);

a1 = [ A, -B*Kz, -B*Kx, zeros(4,3)]
a2 = [ Bz*C,  Az,  zeros(3,4), zeros(3,3)]
a3 = [H(1:4)*C, -B*Kz, A-B*Kx-H(1:4)*C, B*Cd ]
a4 = [H(5:7)*C, zeros(3,3), -H(5:7)*C, Ad]
A14 = [a1;a2;a3;a4];
B14 = [zeros(4,1) ; -Bz ; zeros(4,1) ; zeros(3,1)];
C14 = [ 1,0,0,0,0,0,0, 0,0,0,0,0,0,0 ;
        0,1,0,0,0,0,0, 0,0,0,0,0,0,0 ;
        0,0,0,0,0,0,0, 1,0,0,0,0,0,0 ;
        0,0,0,0,0,0,0, 0,1,0,0,0,0,0 ];
D14 = zeros(4,1);

X0 = [zeros(7,1) ; 0.1*ones(7,1)];
R = 0*t + 1;

y = step3(A14, B14, C14, D14, t, X0, R);

subplot(121);
plot(t,y)
title('R = 1,   d = 0');

R = sin(t);

y = step3(A14, B14, C14, D14, t, X0, R);
subplot(122);
plot(t,y)
title('R = sin(t),  d = 0');
```



Response to R = 1 (left) and R = sin(t) (right)

Response to a disturbabance:

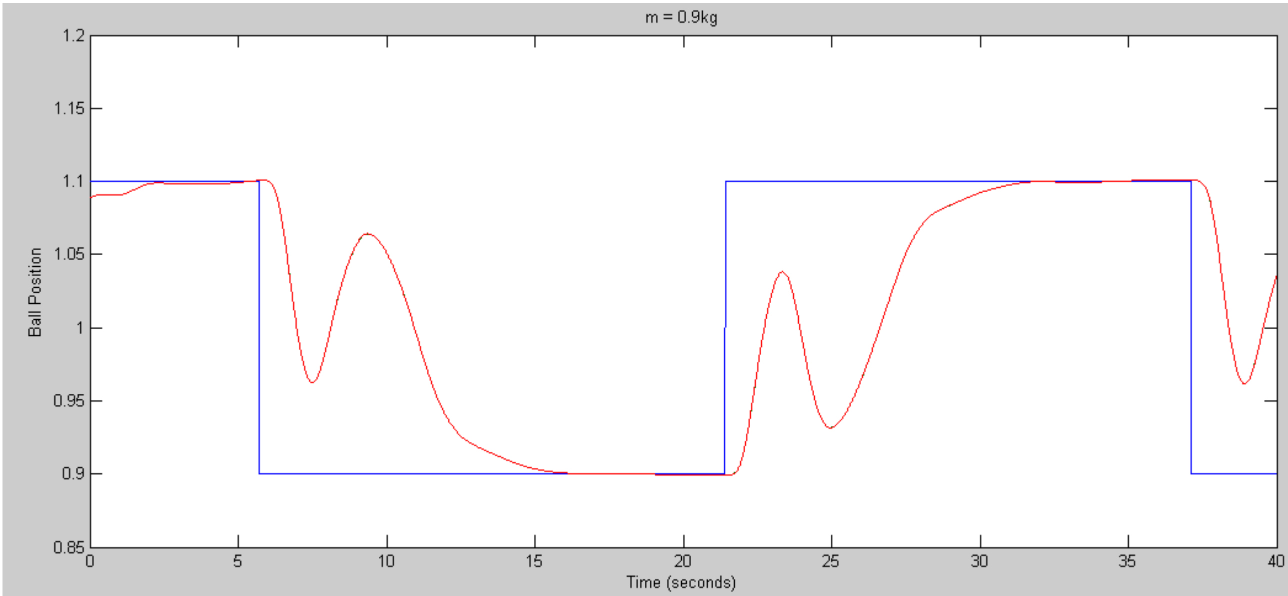
```
B14 = [B ; zeros(3,1) ; B ; zeros(3,1)];
X0 = [zeros(14,1)];
R = 0*t + 1;

y = step3(A14, B14, C14, D14, t, X0, R);

subplot(121);
plot(t,y)
title('d(t) = 1,   Ref = 0');

R = sin(t);

y = step3(A14, B14, C14, D14, t, X0, R);
subplot(122);
plot(t,y)
title('d(t) = sin(t),   Ref = 0');
```

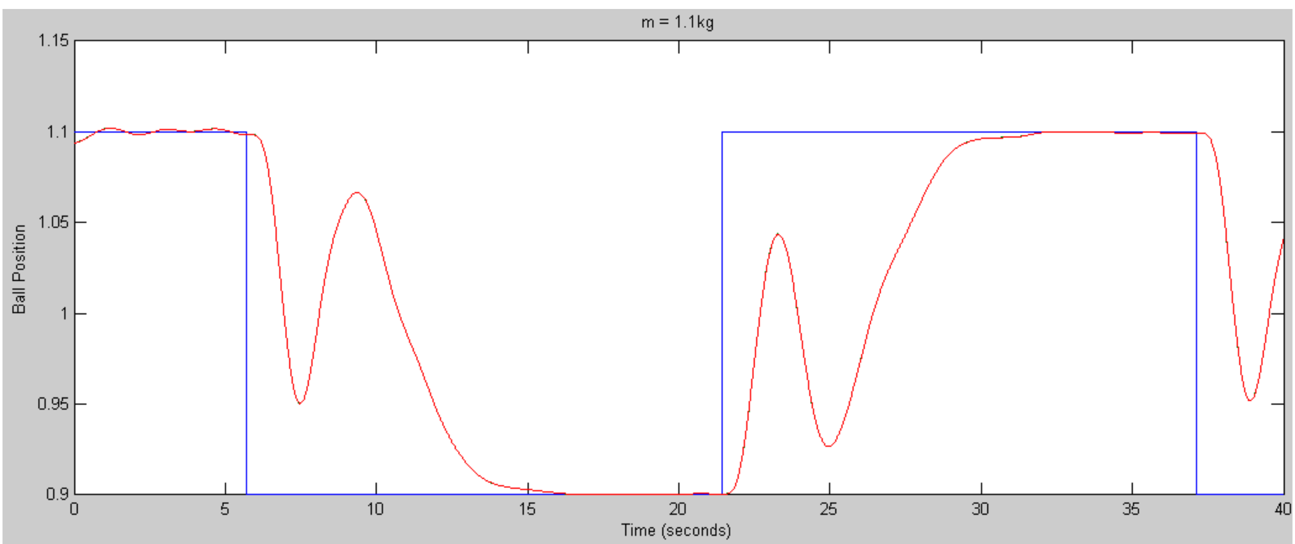

x 10-3
d(t) = 1,   Ref = 0
3
2
1
0
-1
-2
-3
-4
-5
0 2 4 6 8 10 12 14 16 18 20
x 10-3
d(t) = sin(t),   Ref = 0
2
1
0
-1
-2
-3
-4
0 2 4 6 8 10 12 14 16 18 20

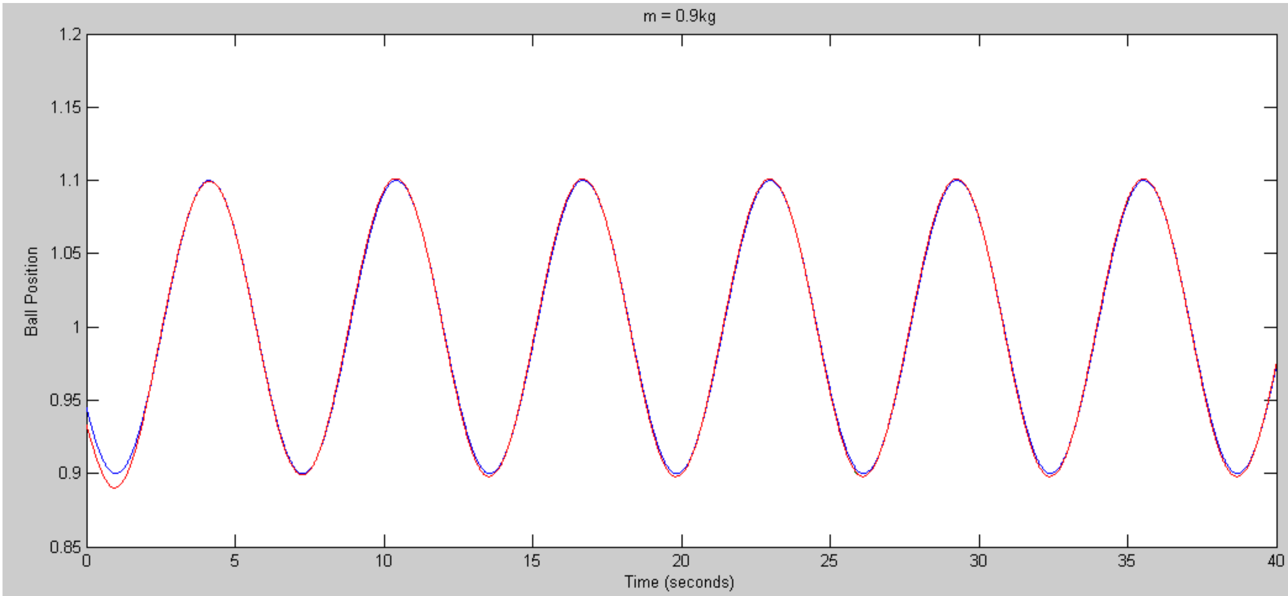
Nonlinear Simulation

```
% Ball & Beam System
% Sp 19 Version
% Test #2
% m = 1kg
% J = 2 kg m^2

A = [0,0,1,0 ; 0,0,0,1 ; 0,-7,0,0 ; -3.27,0,0,0]
B = [0;0;0;0.33]

% Servo Comp
Az = [0,0,0 ; 0,0,1 ; 0,-1,0]
Bz = [1;1;1]
C = [1,0,0,0]

A7 = [A, zeros(4,3) ; Bz*C, Az]
B7 = [B; zeros(3,1)]
K7 = ppl(A7, B7, [-1, -2, -2.1, -2.2, -2.3, -2.4, -2.5])
Kx = K7(1:4);
Kz = K7(5:7);

X = zeros(4,1);
Z = zeros(3,1);

dt = 0.01;
t = 0;

% Observer

Ad = Az;
Cd = [1,1,1];

Ao = [A, B*Cd ; zeros(3,4), Ad];
Bo = [B; zeros(3,1)];
Co = [C, zeros(1,3)];
Do = 0;
H = ppl(Ao', Co', [-3, -3.1, -3.2, -3.3, -3.4, -3.5, -3.6])'
Xo = zeros(7,1);

Ref = 0;
y = [];

while(t < 50)
 Ref =  1 + 0.1*sin(t);

 if(t<10)
    U = - Kx*X(1:4) - Kz*Z;
 else
    U = - Kx*Xo(1:4) - Kz*Z;
 end

 dX = BeamDynamics(X, U);
 dZ = Az*Z + Bz*(C*X - Ref);

 % 7th order observer
 dXo = Ao*Xo + Bo*U - H*(Co*Xo - C*X);

 X = X + dX * dt;
 Xo = Xo + dXo * dt;
 Z = Z + dZ * dt;
```

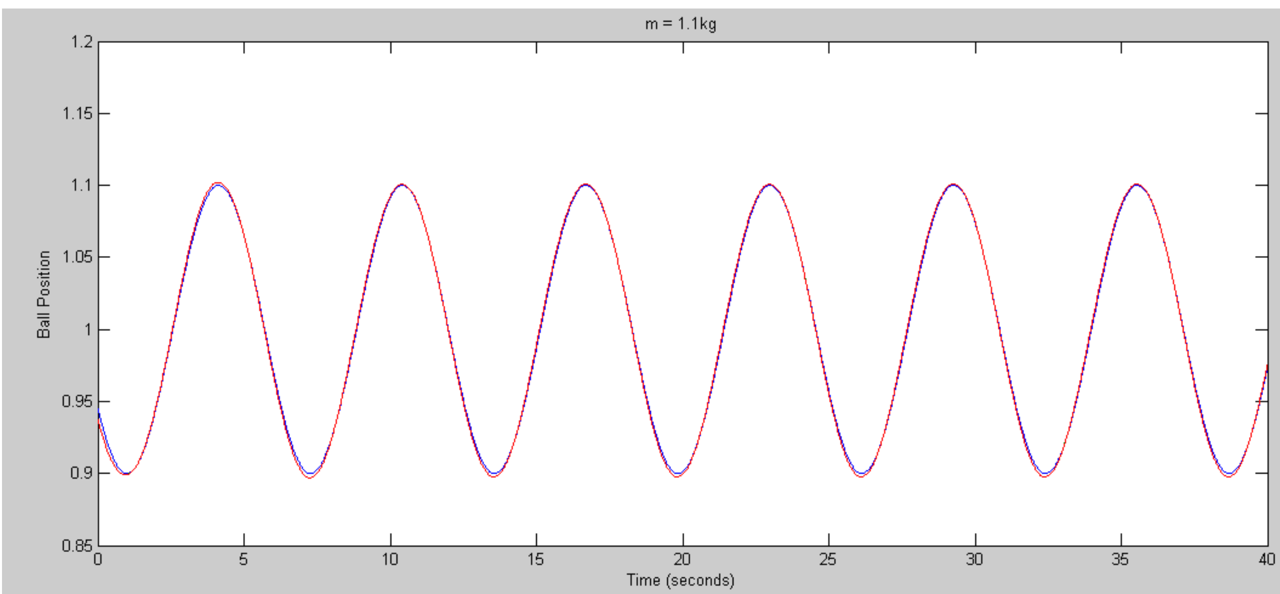
```
if(t>10)
   y = [y ; Ref, X(1), Xo(1)];
   end

 t = t + dt;
 BeamDisplay(X, Xo, Ref);

 end

t = [1:length(y)]' * dt;


plot(t,y);
xlabel('Time (seconds)');
ylabel('Ball Position');
```


m = 0.9kg
Ball Position
Time (seconds)
1.2
1.15
1.1
1.05
1
0.95
0.9
0.85
0
5
10
15
20
25
30
35
40


m = 1.1kg
Ball Position
Time (seconds)
1.15
1.1
1.05
1
0.95
0.9
0
5
10
15
20
25
30
35
40



m = 0.9kg
Ball Position
Time (seconds)
1.2
1.15
1.1
1.05
1
0.95
0.9
0.85
0
5
10
15
20
25
30
35
40



m = 1.1kg
Ball Position
Time (seconds)
1.2
1.15
1.1
1.05
1
0.95
0.9
0.85
0
5
10
15
20
25
30
35
40