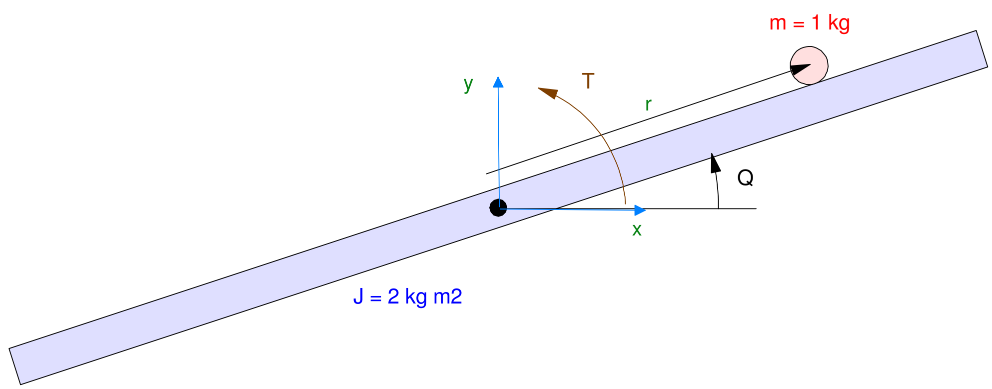
ECE 463/663: Test #2

Take-Home. Open-Book, Open-Notes, Open-Internet. Please work alone. Due March 25th.



Ball and Beam from homework #4

Use the dynamics for a ball and beam system from previous homework sets

- mass of ball = 0.9 - 1.1kg (1kg nominal)
- inertia of beam = 2 kg m2
- Closed-Loop System's 2% Settling Time = 6 seconds

A-Level (max score = 100%): Design a feedback control law to

- Track a constant and sinusoidal set-point, R

  $$R = a + b\sin(t)$$

- Reject a constant and sinusoidal disturbance

  $$\eta = c + d\sin(t + \phi)$$

- While only measuring the ball position (r) (meaning you need to use a full-order observer)

B-Level (max score = 90%): Design a feedback control law to

- **Track a constant set-point**

  $$R = a$$

- **Reject a constant disturbance**

  $$\eta = c$$

- **While only measuring the ball position (r) (meaning you need to use a full-order observer)**

C-Level (max score = 80%): Same as B-level but assume all states are measured (so there is no need for a full-order observer)

Problem 1) Give a block-diagram and state-space model for the

- Plant
- Servo Compensator,
- Full-order observer (A and B level), and
- Disturbance estimator (A and B level)

Problem 2) Using the linear model, design

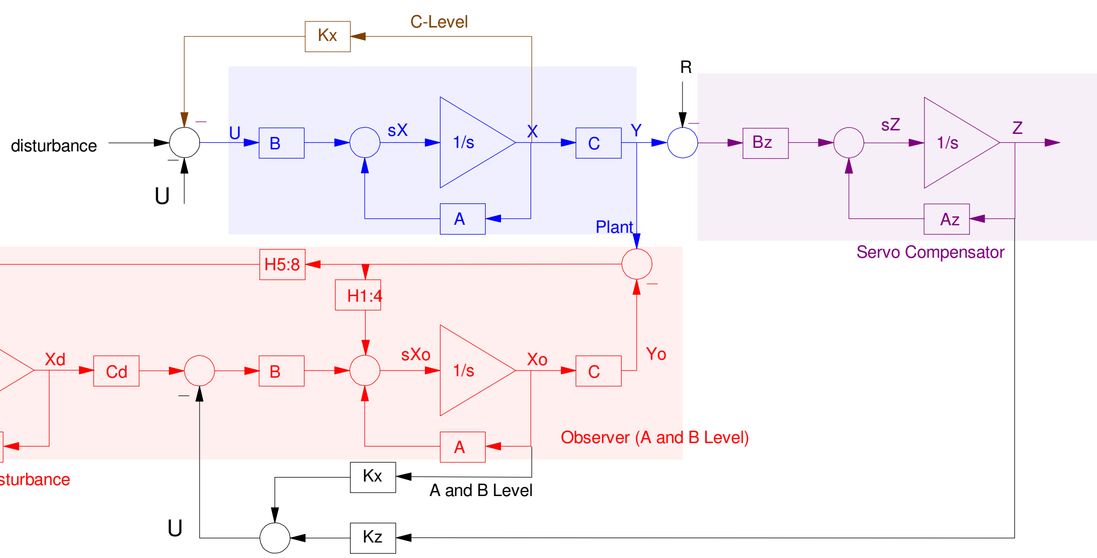
- A feedback control law to meet the requirements.
- A full-order observer to estimate the states and disturbances (A and B level)

Problem 3) Using the linear model, determine the response to

- A step input (R = 1)
- A step disturbance (n = 1)
- A sinuosidal setpoint (R = sin(t)   A level only)
- A sinusoidal disturbance (n = sin(t). A level only)

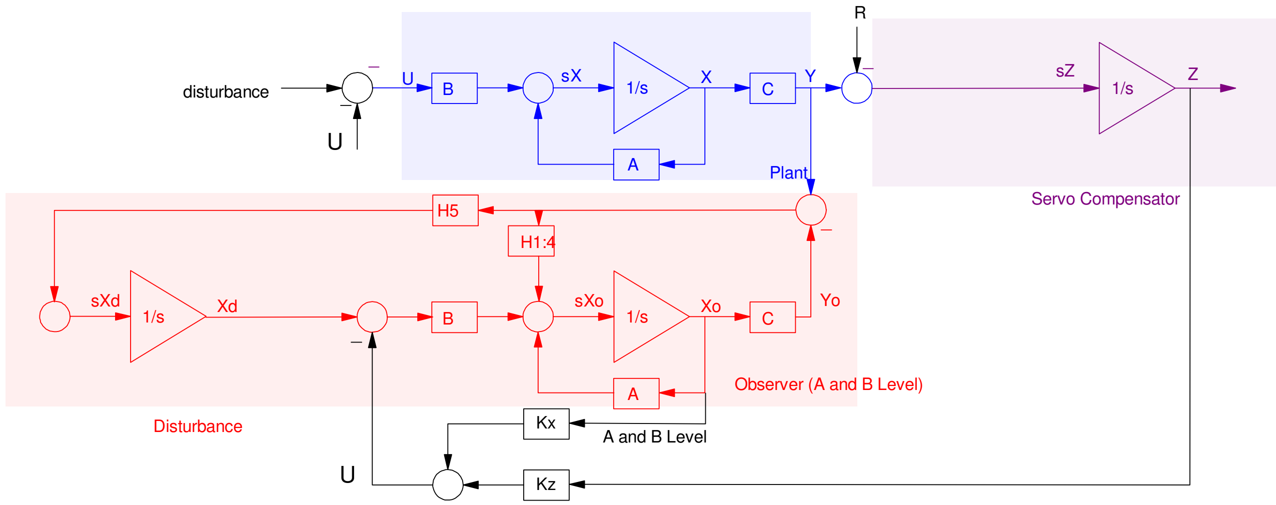
Problem 4) Modify the non-linear simularion and determine (and plot) the step response (R = 1) for

- m = 1.0kg (nominal)
- m = 0.9kg
- m = 11kg



B-Level

Block Diagram



System Matricies

$$\begin{bmatrix} sX \\ sZ \\ sX_o \\ sX_d \end{bmatrix} = \begin{bmatrix} A & -BK_z & -BK_x & 0 \\ C & 0 & 0 & 0 \\ H_{1:4}C & 0 & A - BK_x - H_{1:4}C & B \\ H_5C & 0 & -H_5C & 0 \end{bmatrix} \begin{bmatrix} X \\ Z \\ X_o \\ X_d \end{bmatrix} + \begin{bmatrix} B \\ 0 \\ 0 \\ 0 \end{bmatrix} d + \begin{bmatrix} 0 \\ -B_z \\ 0 \\ 0 \end{bmatrix} R$$

Step 1: Design the feedback control law.

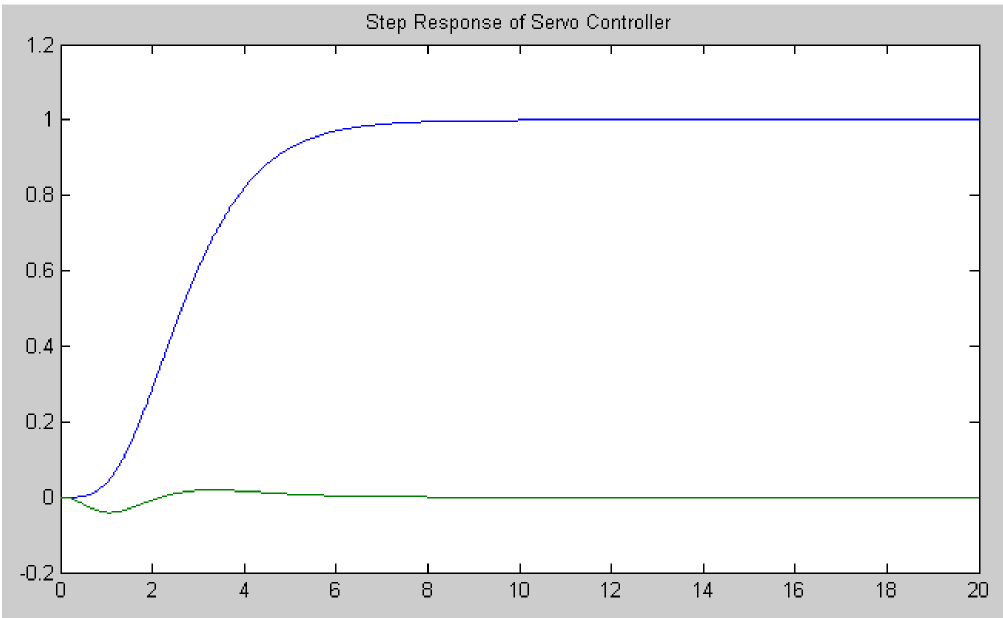
Use superposition to find the feedback gains.

Create the augmented system (plant & servo)

$$\begin{bmatrix} sX \\ sZ \end{bmatrix} = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix} \begin{bmatrix} X \\ Z \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} U$$

$$Y = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} X \\ Z \end{bmatrix}$$

```
A = [0,0,1,0 ; 0,0,0,1 ; 0,-7,0,0 ; -3.27,0,0,0]
B = [0;0;0;0.33]
C = [1,0,0,0]

A5 = [A, zeros(4,1) ; C, 0]
B5 = [B; 0]
K5 = ppl(A5, B5, [-1, -2, -2.1, -2.2, -2.3])
B5r = [zeros(4,1) ; -1];
C5 = [C,0];
D5 = [0];

C5xq = [1,0,0,0,0 ; 0,1,0,0,0];
D5xq = zeros(2,1);
t = [0:0.01:20]';
X0 = zeros(5,1);
R = 0*t + 1;

y = step3(A5-B5*K5, B5r, C5xq, D5xq, t, X0, R);
plot(t,y);
title('Step Response of Servo Controller');
```



Step 2: Design a full-order observer for the augmented plant (plant + disturbance)

$$\begin{bmatrix} sX \\ sX_d \end{bmatrix} = \begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix} \begin{bmatrix} X \\ X_d \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} U$$

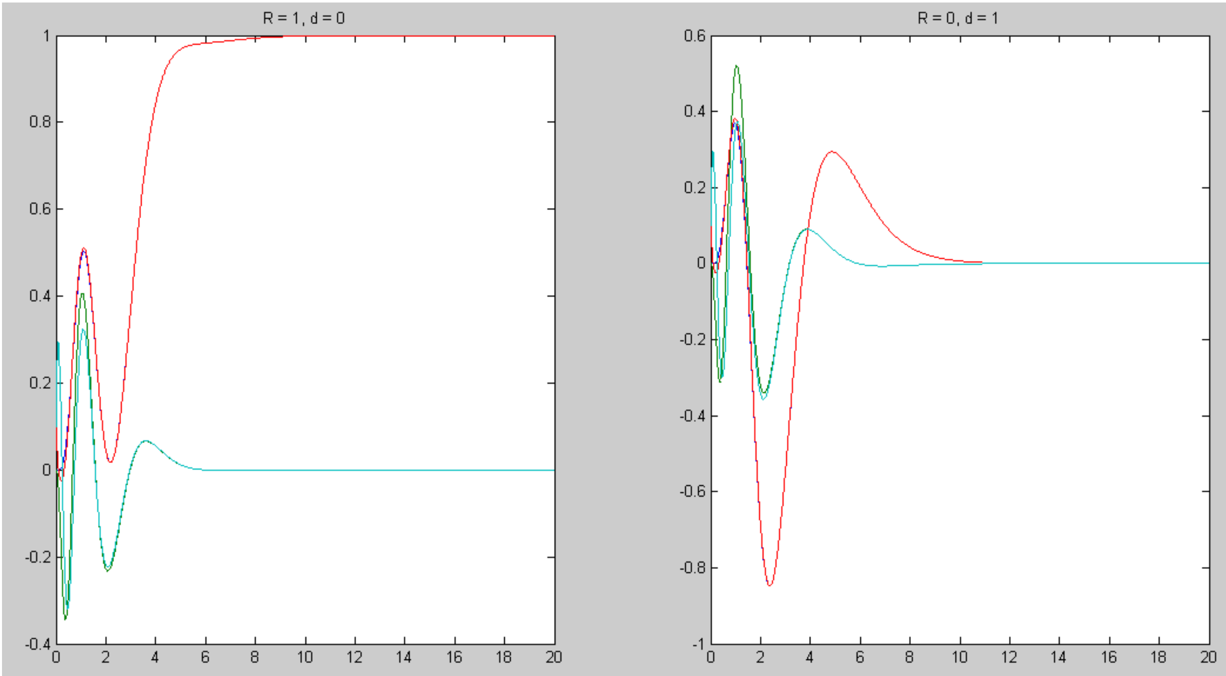
$$Y = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} X \\ X_d \end{bmatrix}$$

```
Ao = [A, B ; zeros(1,4), 0];
Bo = [B; 0];
Co = [C, 0];
Do = 0;

H = ppl(Ao', Co', [-3, -3.1, -3.2, -3.3, -3.4])'
```

Putting it all together

$$\begin{bmatrix} sX \\ sZ \\ sX_o \\ sX_d \end{bmatrix} = \begin{bmatrix} A & -BK_z & -BK_x & 0 \\ C & 0 & 0 & 0 \\ H_{1:4}C & -BK_z & A - BK_x - H_{1:4}C & B \\ H_5 C & 0 & -H_5 C & 0 \end{bmatrix} \begin{bmatrix} X \\ Z \\ X_o \\ X_d \end{bmatrix} + \begin{bmatrix} B \\ 0 \\ 0 \\ 0 \end{bmatrix} d + \begin{bmatrix} 0 \\ -B_z \\ 0 \\ 0 \end{bmatrix} R$$

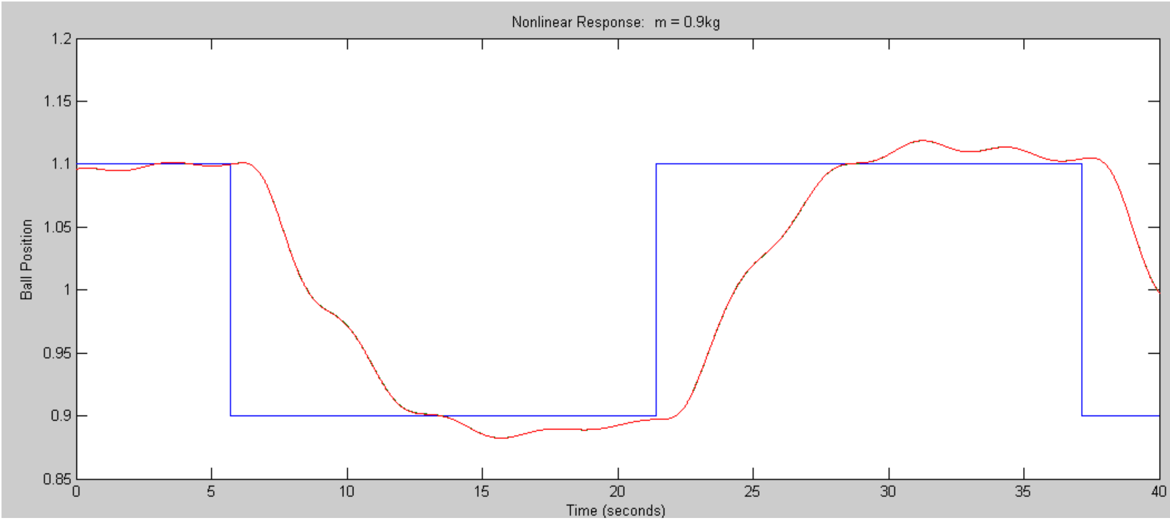
```
Kx = K5(1:4);
Kz = K5(5);

A10 = [ A, -B*Kz, -B*Kx, zeros(4,1) ;
        C,  0,  zeros(1,4), 0 ;
        H(1:4)*C, -B*Kz, A-B*Kx-H(1:4)*C, B ;
        H(5)*C, 0, -H(5)*C, 0];
B10 = [zeros(4,1) ; -1 ; zeros(4,1) ; 0];
C10 = [ 1,0,0,0,0,0,0,0,0,0 ;
        0,1,0,0,0,0,0,0,0,0 ;
        0,0,0,0,0,1,0,0,0,0 ;
        0,0,0,0,0,0,1,0,0,0 ];
D10 = zeros(4,1);

X0 = [zeros(5,1) ; 0.1*ones(5,1)];
R = 0*t + 1;

y = step3(A10, B10, C10, D10, t, X0, R);

subplot(121)
plot(t,y)
title('R = 1, d = 0');

B10 = [B ; zeros(6,1)];
y = step3(A10, B10, C10, D10, t, X0, R);

subplot(122)
plot(t,y)
title('R = 0, d = 1');
```

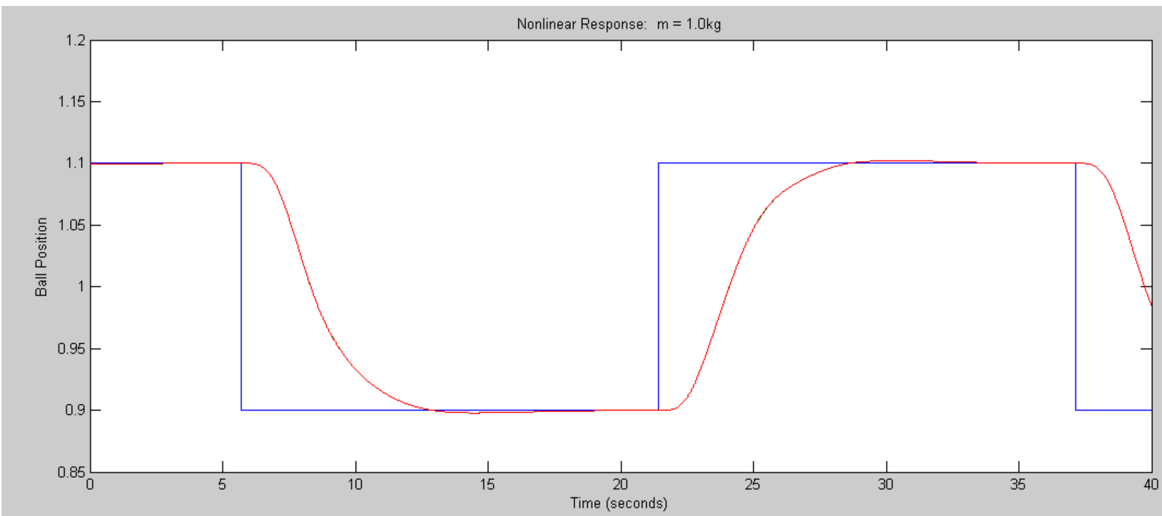


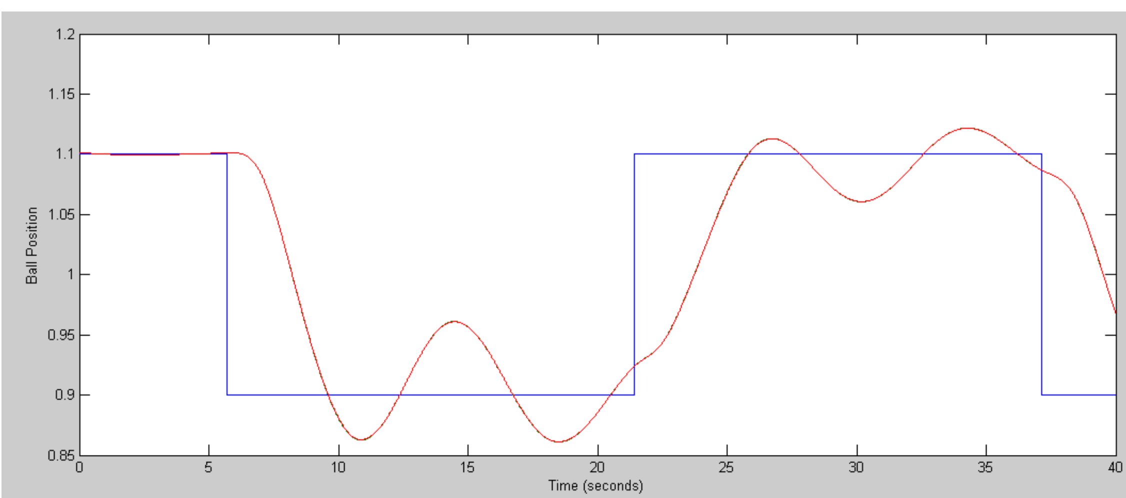
Response to R = 1 (left) and d = 1 (right)

Nonlinear Simulation

```
% Ball & Beam System
% Sp 19 Version
% Test #2
% m = 1kg
% J = 2 kg m^2

A = [0,0,1,0 ; 0,0,0,1 ; 0,-7,0,0 ; -3.27,0,0,0]
B = [0;0;0;0.33]
C = [1,0,0,0]

A5 = [A, zeros(4,1) ; C, 0]
B5 = [B; 0]
K5 = ppl(A5, B5, [-1, -2, -2.1, -2.2, -2.3])
Kx = K5(1:4);
Kz = K5(5);

X = zeros(4,1);
Z = 0;

dt = 0.01;
t = 0;

% Observer

Ad = 0;
Cd = 1;

Ao = [A, B*Cd ; zeros(1,4), Ad];
Bo = [B; 0];
Co = [C, 0];
Do = 0;
H = ppl(Ao', Co', [-3, -3.1, -3.2, -3.3, -3.4])'
Xo = zeros(5,1);

Ref = 0;
y = [];

while(t < 50)
 Ref =  1 + 0.1*sign(sin(0.2*t));

 if(t<10)
    U = - Kx*X - Kz*Z;
 else
    U = - [Kx, 0]*Xo - Kz*Z;
 end

 dX = BeamDynamics(X, U);
 dZ = Az*Z + Bz*(C*X - Ref);

 % 5th order observer
 dXo = Ao*Xo + Bo*U - H*(Co*Xo - C*X);

 X = X + dX * dt;
 Xo = Xo + dXo * dt;
 Z = Z + dZ * dt;


if(t>10)
   y = [y ; Ref, X(1), Xo(1)];
   end

 t = t + dt;
 %BeamDisplay(X, Xo, Ref);

 end

t = [1:length(y)]' * dt;


plot(t,y);
xlabel('Time (seconds)');
ylabel('Ball Position');
```


Nonlinear Response: m = 0.9kg
Ball Position
Time (seconds)
1.2
1.15
1.1
1.05
1
0.95
0.9
0.85
0
5
10
15
20
25
30
35
40


Nonlinear Response: m = 1.0kg
Ball Position
Time (seconds)
1.2
1.15
1.1
1.05
1
0.95
0.9
0.85
0
5
10
15
20
25
30
35
40


Ball Position
Time (seconds)
1.2
1.15
1.1
1.05
1
0.95
0.9
0.85
0
5
10
15
20
25
30
35
40