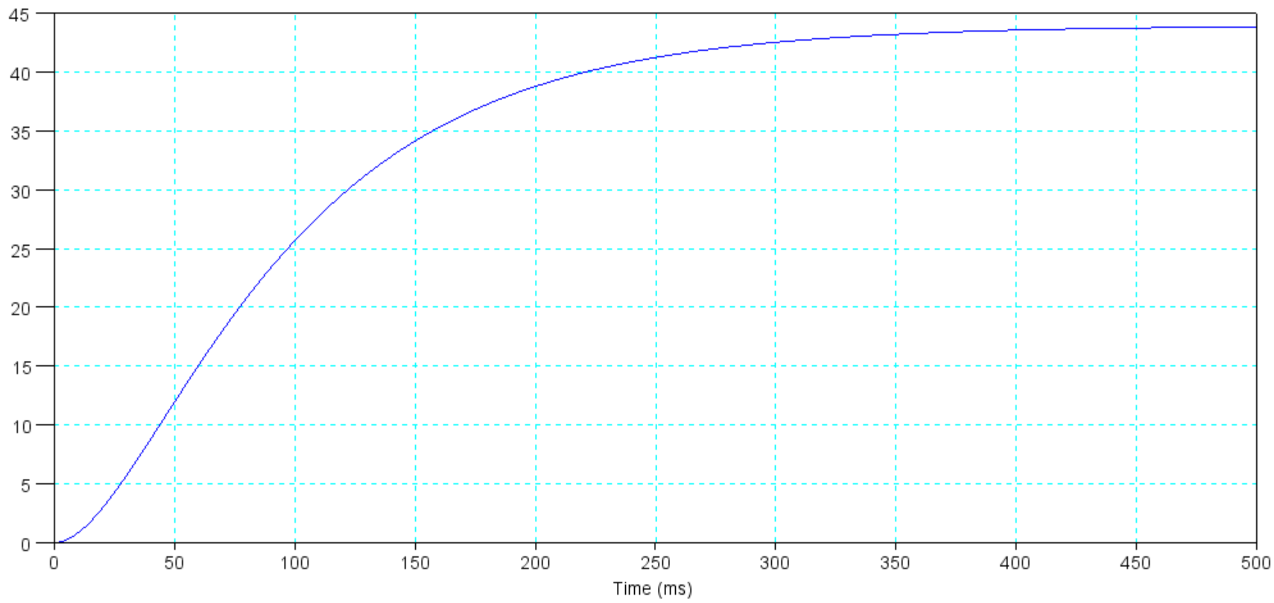
ECE 463/663 - Homework #1 Solution

LaPlace Transforms and Dominant Poles. Due Wednesday, Jan 22nd

1) Name That System! Give the transfer function for a system with the following step response.



This is a 1st-order system (no oscillations) - so the answer has to be in the form of

$$Y = \left(\frac{a}{s+b}\right)X$$

With two unknowns, we need to find two pieces of information from the above graph.

DC Gain: The DC gain of the system is about 44

$$\left(\frac{a}{s+b}\right)_{s=0} = \frac{a}{b} = 44$$

2% Settling Time: The system reaches steady-state (within 2% of its final value) in 350ms

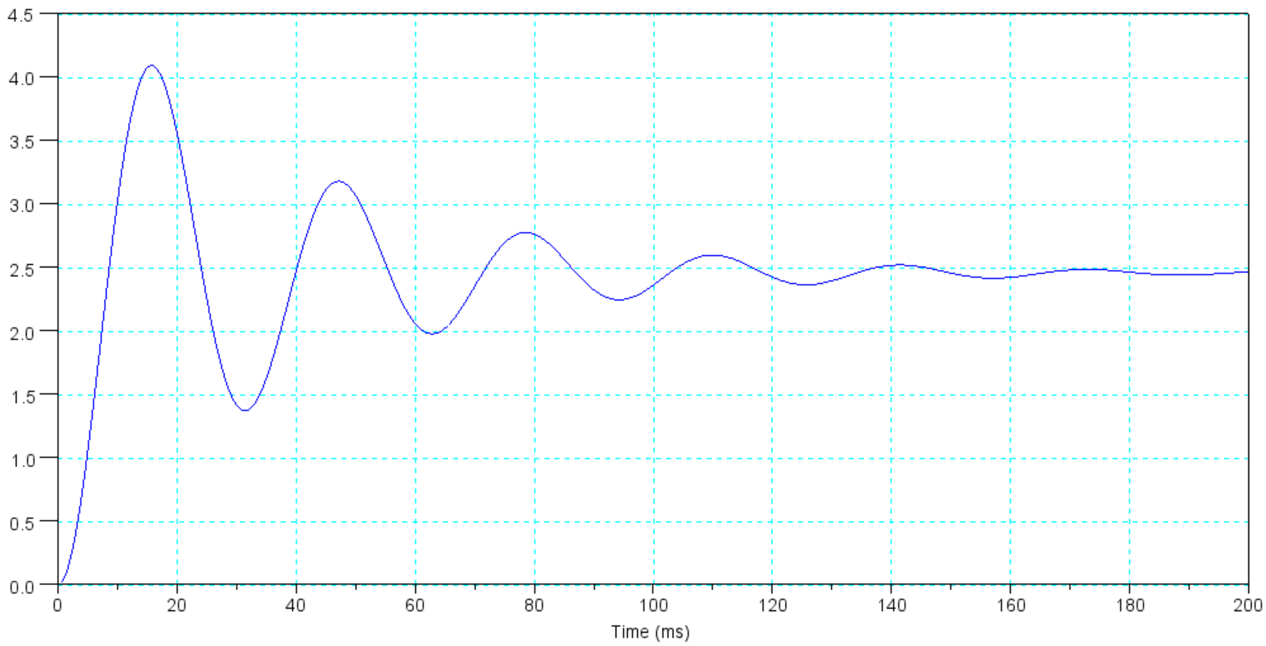
$$\left(e^{-bt}\right)_{t=0.35} = 0.02$$

$$b = \frac{4}{0.35} = 11.42$$

so

$$\boxed{G(s) \approx \left(\frac{502.8}{s+11.42}\right)}$$

2) Name That System! Give the transfer function for a system with the following step response.



This is a 2nd-order system (complex poles giving oscillations)

$$Y = \left(\frac{a}{(s+b+jc)(s+b-jc)}\right) X$$

DC Gain: 2.5

$$\left(\frac{a}{(s+b+jc)(s+b-jc)}\right)_{s=0} = \left(\frac{a}{b^2+c^2}\right) = 2.5$$

2% Settling Time: 160ms

$$\frac{4}{b} = 0.16$$

$$b = 25$$

Frequency of oscillation

$$c = \left(\frac{4\text{ cycles}}{125ms}\right) 2\pi = 201 \frac{rad}{\text{sec}}$$

so

$$\boxed{Y \approx \left(\frac{2.5\left(25^2+201^2\right)}{(s+25+j201)(s+25-j201)}\right) X = \left(\frac{102{,}627}{s^2+50s+41{,}026}\right) X}$$

Problem 3 - 6) Assume

$$Y = \left(\frac{300}{(s+2)(s+5)(s+12)}\right) X$$

3) What is the differential equation relating X and Y?

Multiply out and cross multiply

$$(s^3 + 19s^2 + 94s + 120)Y = 300X$$

'sY' means 'the derivative of Y'

$$\boxed{y''' + 19y'' + 94y' + 120y = 300x}$$

4) Determine y(t) assuming x(t) is a sinusoidal input:

$$x(t) = 2\cos(3t)$$

This is phasor analysis

$$s = j3$$

$$X = 2 + j0$$

$$Y = \left(\frac{300}{(s+2)(s+5)(s+12)}\right)_{s=j3} X$$

$$Y = (-0.2262 - j1.1312) \cdot (2 + j0)$$

$$Y = -0.4525 - j2.2624$$

meaning

$$\boxed{y(t) = -0.4525\cos(3t) + 2.2624\sin(3t)}$$

5) Determine y(t) assuming x(t) is a step input:

$$x(t) = u(t)$$

This is LaPlace transform analysis

$$X(s) = \frac{1}{s}$$

$$Y = \left(\frac{300}{(s+2)(s+5)(s+12)}\right)\left(\frac{1}{s}\right)$$

Use partial fractions

$$Y = \left(\frac{2.5}{s}\right) + \left(\frac{-5}{s+2}\right) + \left(\frac{2.8571}{s+5}\right) + \left(\frac{-0.3571}{s+12}\right)$$

Take the inverse LaPlace transform

$$\boxed{y(t) = 2.5 - 5e^{-2t} + 2.8571e^{-5t} - 0.3571e^{-12t}} \quad t > 0$$

6a) Determine a 1st-order approximation for this system

$$Y = \left(\frac{300}{(s+2)(s+5)(s+12)}\right) X \approx \left(\frac{a}{s+b}\right) X$$

Keep the dominant pole

b = -2

Keep the DC gain at 2.5

$$\left(\frac{300}{(s+2)(s+5)(s+12)}\right)_{s=0} = 2.5$$

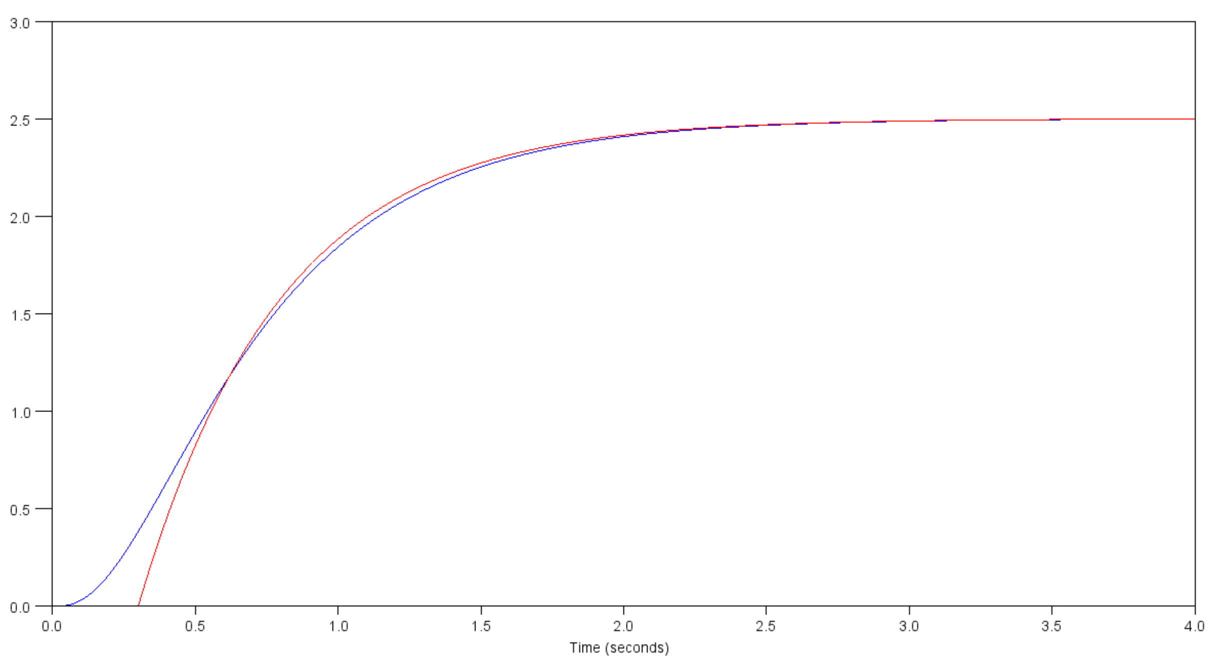
$$\left(\frac{a}{s+b}\right)_{s=0} = 2.5$$

This results in

$$\boxed{\left(\frac{300}{(s+2)(s+5)(s+12)}\right) \approx \left(\frac{5}{s+2}\right)}$$

6b) Compare the step response of your 1st-order model to the actual 3rd-order system in Matlab

```
t = [0:0.01:5]';
G1 = zpk([],[-2,-5,-12],300);
G2 = zpk([],-2,5);
y1 = step(G1,t);
y2 = step(G2,t);
plot(t,y1,'b',t+0.35,y2,'r');
xlabel('Time (seconds)');
```



3rd-Order System (blue) and 1st-Order Approximation (red) with a 0.35s delay