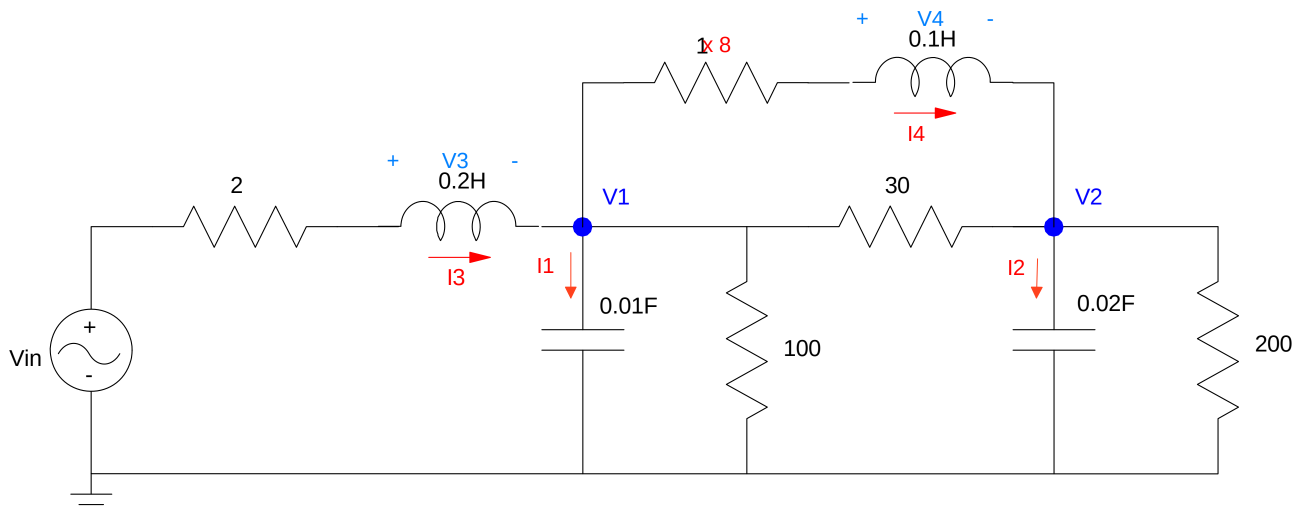
ECE 463/663 - Homework #2 Solution

State-Space, Eigenvalues, Eigenvectors. Due Monday, Jan 27th

Please make the subject "ECE 463/663 HW#2" if submitting homework electronically to Jacob_Glower@yahoo.com (or on blackboard)

1) For the following RLC circuit



Specify the dynamics for the system (write N coupled differential equations)

$$I_1 = 0.01\dot{V}_1 = I_3 - I_4 - \left(\frac{V_1 - V_2}{30}\right) - \left(\frac{V_1}{100}\right)$$

$$I_2 = 0.02\dot{V}_2 = I_4 + \left(\frac{V_1 - V_2}{30}\right) - \left(\frac{V_2}{200}\right)$$

$$V_3 = 0.2\dot{I}_3 = V_{in} - 2I_3 - V_1$$

$$V_4 = 0.1\dot{I}_4 = V_1 - 8I_4 - V_2$$

Express these dynamics in state-space form.

Group terms

$$\dot{V}_1 = 100I_3 - 100I_4 - 4.333V_1 + 3.333V_2$$

$$\dot{V}_2 = 50I_4 + 1.667V_1 - 1.917V_2$$

$$\dot{I}_3 = 5V_{in} - 10I_3 - 5V_1$$

$$\dot{I}_4 = 10V_1 - 80I_4 - 10V_2$$

place in matrix (state-space) form

$$\begin{bmatrix} \dot{V}_1 \\ \dot{V}_2 \\ \dot{I}_3 \\ \dot{I}_4 \end{bmatrix} = \begin{bmatrix} -4.333 & 3.333 & 100 & -100 \\ 1.667 & -1.917 & 0 & 50 \\ -5 & 0 & -10 & 0 \\ 10 & -10 & 0 & -80 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ I_3 \\ I_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 5 \\ 0 \end{bmatrix} V_{in}$$

$$Y = V_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ I_3 \\ I_4 \end{bmatrix}$$

Determine the transfer function from Vin to V2

```
A = [-4.333,3.333,100,-100 ; 1.667,-1.917,0,50 ; -5,0,-10,0 ; 10,-10,0,-80]

  - 4.333    3.333    100.   - 100.
    1.667  - 1.917    0.       50.
  - 5.       0.     - 10.      0.
    10.    - 10.      0.     - 80.

B = [0;0;5;0];
C = [0,1,0,0];
D = 0;

G = ss(A,B,C,D);

zpk(G)
```

$$G(s) = \left(\frac{316{,}680}{(s+8.45)(s+17.84 \pm j21.09)(s+52.11)} \right)$$

2) For the transfer function from Vin to V2

Determine a 1st or 2nd-order approximation for this trasfer function

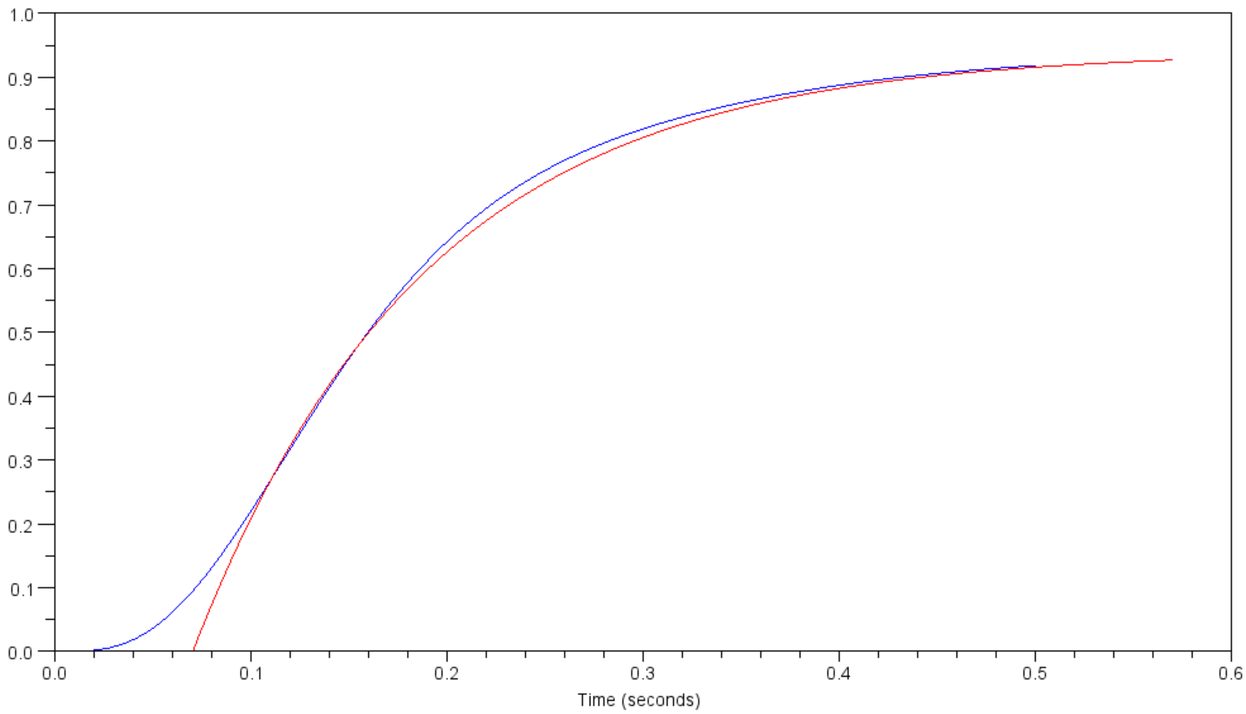
Plot the step response of the actual 4th-order system and its approximation

$$G(s) = \left(\frac{316{,}680}{(s+8.45)(s+17.84 \pm j21.09)(s+52.11)} \right) \approx \left(\frac{a}{s+b} \right)$$

Keep the dominant pole (-8.45)

Match the DC gain (0.9414)

$$G(s) = \left(\frac{316{,}680}{(s+8.45)(s+17.84 \pm j21.09)(s+52.11)} \right) \approx \left(\frac{7.95}{s+8.45} \right)$$



Step Response of 4th-Order System (blue) vs. 1st-Order Approximation with a 0.7 second delay (red)

3) For this circuit...

What initial condition will the energy in the system decay as slowly as possible?

What initial condition will the energy in the system decay as fast as possible?

```
[a,b] = eig(A)

eigenvalues


  - 52.108299    0                           0                           0
    0          - 17.842309 + 21.094206i      0                           0
    0            0                         - 17.842309 - 21.094206i      0
    0            0                           0                         - 8.4570824

eigenvectors

    0.7636156    0.8801024                   0.8801024                 - 0.0506288
  - 0.4637214  - 0.2555863 - 0.2997336i    - 0.2555863 + 0.2997336i    - 0.9766095
    0.0906728    0.0681390 + 0.1832799i      0.0681390 - 0.1832799i      0.1640685
    0.4400366    0.1785161 - 0.0123608i      0.1785161 + 0.0123608i      0.1294301

       fast                                                                slow
```

Fast Response:

```
V1 =  0.7636156
V2 = -0.4637214
I3 =  0.0906728
I4 =  0.4400366
```

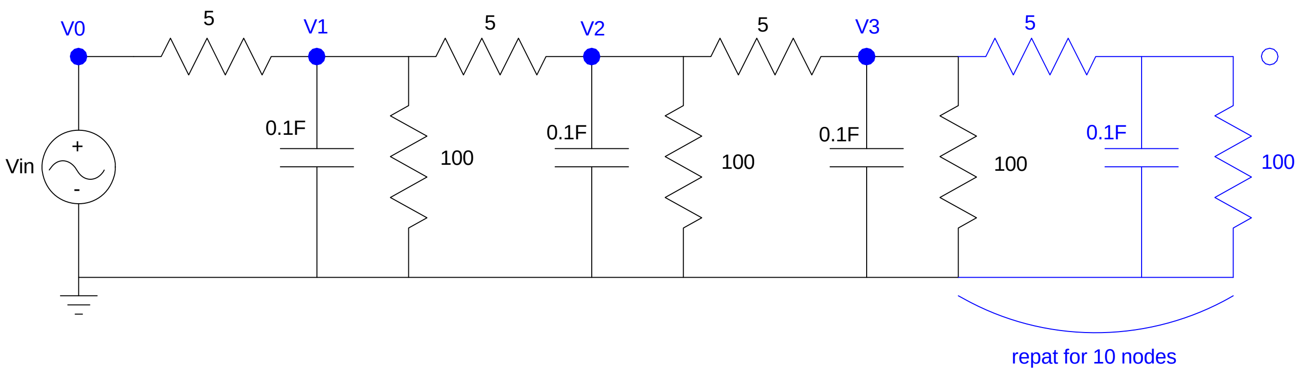
Slow Response

```
V1 = - 0.0506288
V2 = - 0.9766095
I3 =   0.1640685
I4 =   0.1294301
```

Problem 4-7: 10-Stage RC Filter.

note: You can turn in the Matlab code along with screen shots of the plots if you like.

4) For the following 10-stage RC circuit



Specify the dynamics for the system (write N coupled differential equations)

- note: Nodes 1..9 have the same form. Just write the node equation for node 1 and node 10.

$$0.1\dot{V}_1 = \left(\frac{V_0 - V_1}{5}\right) + \left(\frac{V_2 - V_1}{5}\right) - \left(\frac{V_1}{100}\right)$$

$$0.1\dot{V}_{10} = \left(\frac{V_9 - V_{10}}{5}\right) + \left(\frac{V_{10}}{100}\right)$$

Grouping terms

$$\dot{V}_1 = 2V_0 - 4.1V_1 + 2V_2$$

$$\dot{V}_2 = 2V_1 - 4.1V_2 + 2V_3$$

$$\dot{V}_3 = 2V_2 - 4.1V_3 + 2V_4$$

$$\vdots$$

$$\dot{V}_9 = 2V_8 - 4.1V_9 + 2V_{10}$$

$$\dot{V}_{10} = 2V_9 - 2.1V_{10}$$

Express these dynamics in state-space form

$$sX = AX + BU$$

$$Y = CX + DU$$

```
A = zeros(10,10);

for i=1:9
  A(i,i) = -4.1;
  A(i,i+1) = 2;
  A(i+1,i) = 2;
  end

A(10,10) = -2.1

  - 4.1    2.      0.      0.      0.      0.      0.      0.      0.      0.
    2.   - 4.1     2.      0.      0.      0.      0.      0.      0.      0.
    0.     2.    - 4.1     2.      0.      0.      0.      0.      0.      0.
    0.     0.      2.    - 4.1     2.      0.      0.      0.      0.      0.
    0.     0.      0.      2.    - 4.1     2.      0.      0.      0.      0.
    0.     0.      0.      0.      2.    - 4.1     2.      0.      0.      0.
    0.     0.      0.      0.      0.      2.    - 4.1     2.      0.      0.
    0.     0.      0.      0.      0.      0.      2.    - 4.1     2.      0.
    0.     0.      0.      0.      0.      0.      0.      2.    - 4.1     2.
    0.     0.      0.      0.      0.      0.      0.      0.      2.    - 2.1

B = [10;0;0;0;0;0;0;0;0;0]

    10.
    0.
    0.
    0.
    0.
    0.
    0.
    0.
    0.
    0.

C = [0,0,0,0,0,0,0,0,0,1]

    0.    0.    0.    0.    0.    0.    0.    0.    0.    1.

D = 0;
```

Determine the transfer function from Vin to V10

```
G = ss(A,B,C,D);
zpk(G)
```

$$Y = \left(\frac{5347}{(s+0.496)(s+1.167)(s+2.100)(s+3.21)(s+4.40)(s+5.56)(s+6.59)(s+7.40)(s+7.92)} \right) V_{in}$$

5) For the transfer function for problem #4

$$Y = \left(\frac{5347}{(s+0.145)(s+0.496)(s+1.167)(s+2.100)(s+3.21)(s+4.40)(s+5.56)(s+6.59)(s+7.40)(s+7.92)} \right) V_{in}$$

Determine a 2nd-order approximation for this trasfer function

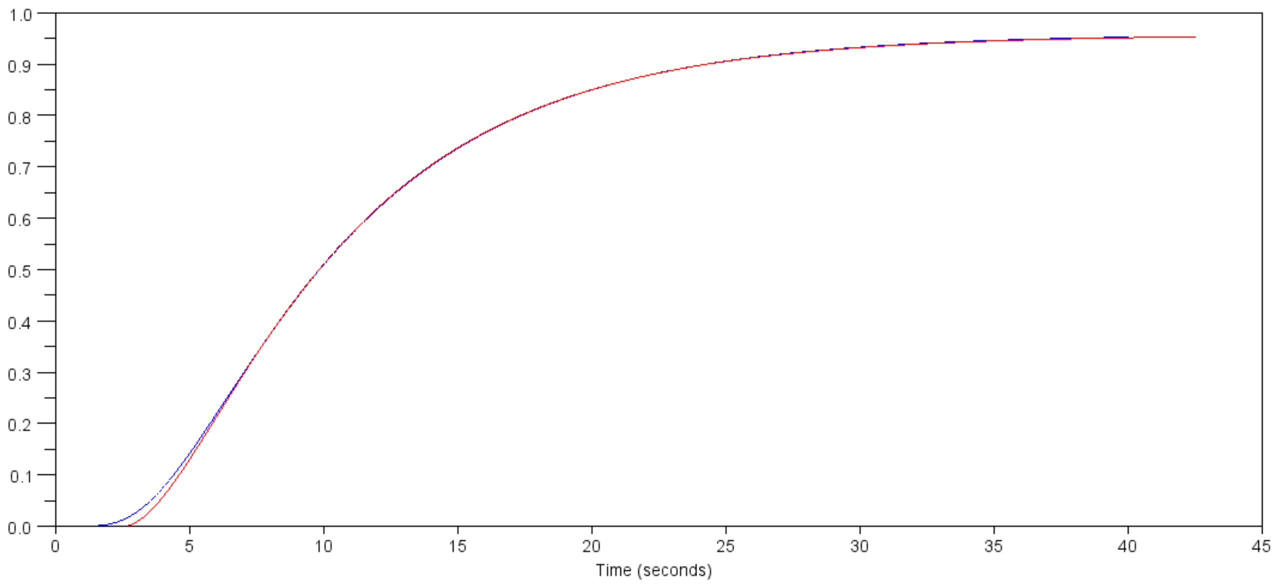
Keep the dominant poles (-0.145, -0.496)

Keep the DC gain (0.9575)

$$Y \approx \left(\frac{0.0689}{(s+0.145)(s+0.496)} \right) V_{in}$$

Plot the step response of the actual 10th-order system and its 2nd-order approximation

```
G2 = zpk([],[-0.145,-0.496],0.0689);
t = [0:0.01:40]';
y = step(G,t);
y2 = step(G2, t);
plot(t,y,'b', t+2.5,y2,'r');
```



Step Response of 10th-Order System (blue) and 2nd-Order Approximation (red - with a 2.5 second delay added)

6) For the circuit for problem #4

- What initial condition will decay as slowly as possible?
- What initial condition will decay as fast as possible?

```
-->[a,b] = spec(A)
 b  =

  - 7.9222912    0.           0.           0.           0.           0.           0.     0.           0.           0.
    0.         - 7.4049551    0.           0.           0.           0.           0.     0.           0.           0.
    0.           0.         - 6.5939592    0.           0.           0.           0.     0.           0.           0.
    0.           0.           0.         - 5.5613641    0.           0.           0.     0.           0.           0.
    0.           0.           0.           0.         - 4.3989204    0.           0.     0.           0.           0.
    0.           0.           0.           0.           0.         - 3.2099163    0.     0.           0.           0.
    0.           0.           0.           0.           0.           0.         - 2.1    0.           0.           0.
    0.           0.           0.           0.           0.           0.           0.   - 1.1677925    0.           0.
    0.           0.           0.           0.           0.           0.           0.     0.         - 0.4961245    0.
    0.           0.           0.           0.           0.           0.           0.     0.           0.         - 0.1446767
 a  =

  - 0.1286417  - 0.2458530    0.3412192    0.4062666    0.4352154    0.4254934    0.3779645    0.2968517  - 0.1893624    0.0650474
    0.2458530    0.4062666  - 0.4254934  - 0.2968517  - 0.0650474    0.1893624    0.3779645    0.4352154  - 0.3412192    0.1286417
  - 0.3412192  - 0.4254934    0.1893624  - 0.1893624  - 0.4254934  - 0.3412192    1.208D-16    0.3412192  - 0.4254934    0.1893624
    0.4062666    0.2968517    0.1893624    0.4352154    0.1286417  - 0.3412192  - 0.3779645    0.0650474  - 0.4254934    0.2458530
  - 0.4352154  - 0.0650474  - 0.4254934  - 0.1286417    0.4062666    0.1893624  - 0.3779645  - 0.2458530  - 0.3412192    0.2968517
    0.4254934  - 0.1893624    0.3412192  - 0.3412192  - 0.1893624    0.4254934    3.518D-16  - 0.4254934  - 0.1893624    0.3412192
  - 0.3779645    0.3779645    7.828D-16    0.3779645  - 0.3779645  - 7.917D-17    0.3779645  - 0.3779645    3.573D-16    0.3779645
    0.2968517  - 0.4352154  - 0.3412192    0.0650474    0.2458530  - 0.4254934    0.3779645  - 0.1286417    0.1893624    0.4062666
  - 0.1893624    0.3412192    0.4254934  - 0.4254934    0.3412192  - 0.1893624    2.584D-16    0.1893624    0.3412192    0.4254934
    0.0650474  - 0.1286417  - 0.1893624    0.2458530  - 0.2968517    0.3412192  - 0.3779645    0.4062666    0.4254934    0.4352154
     fast                                                                                                                  slow
```

7) Modify the program *heat.m* to match the dynamics you calculated for this problem.

- Give the program listing
- Give the response for Vin = 0 and the initial conditions being
  - The slowest eigenvector
  - The fastest eigenvector
  - A random set of voltages