ECE 463/663 - Homework #5

Pole Placement. Due Monday, March 2nd

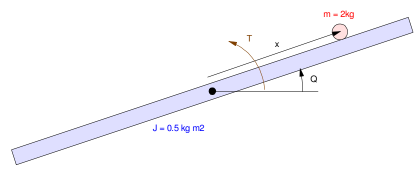
1) Write a Matlab m-file which is passed

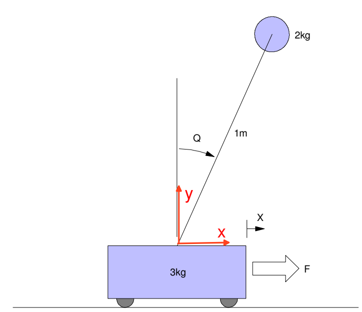
- The system dynamics (A, B),
- The desired pole locations (P)

and then returns the feedback gains, Kx, so that roots(A - B Kx) = P

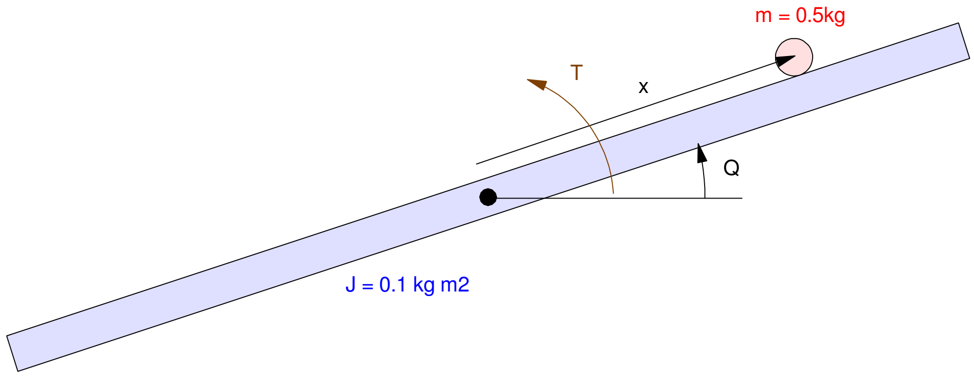
```
function [Kx] = ppl(A, B, P)
```

```
function [ Kx ] = ppl( A, B, P0)

N = length(A);

T1 = [];
for i=1:N
   T1 = [T1, (A^(i-1))*B];
end


P = poly(eig(A));
T2 = [];
for i=1:N
    T2 = [T2; zeros(1,i-1), P(1:N-i+1)];
end

T3 = zeros(N,N);
for i=1:N
    T3(i, N+1-i) = 1;
end

T = T1*T2*T3;

Pd = poly(P0);

dP = Pd - P;

Flip = [N+1:-1:2]';
Kz = dP(Flip);
Kx = Kz*inv(T);

end
```





Problem 2) (20pt) The dynamics of a Ball and Beam System (homework set #4) are



$$s\begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -7 & 0 & 0 \\ -7.84 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.4 \end{bmatrix}T$$

Design a feedback control law of the form

U = Kr * R - Kx * X

so that the closed-loop system has

- A 2% settling time of 6 seconds, and
- 10% overshoot for a step input

Step 1: Translate the requirements to pole locations:

- 10% overshoot means the damping ratio is 0.5910 (angle of poles are 53.77 degrees)
- 6 second settling time means the real part of the dominant pole is at -0.67

Place the dominant poles at

s = -0.67 + j0.91

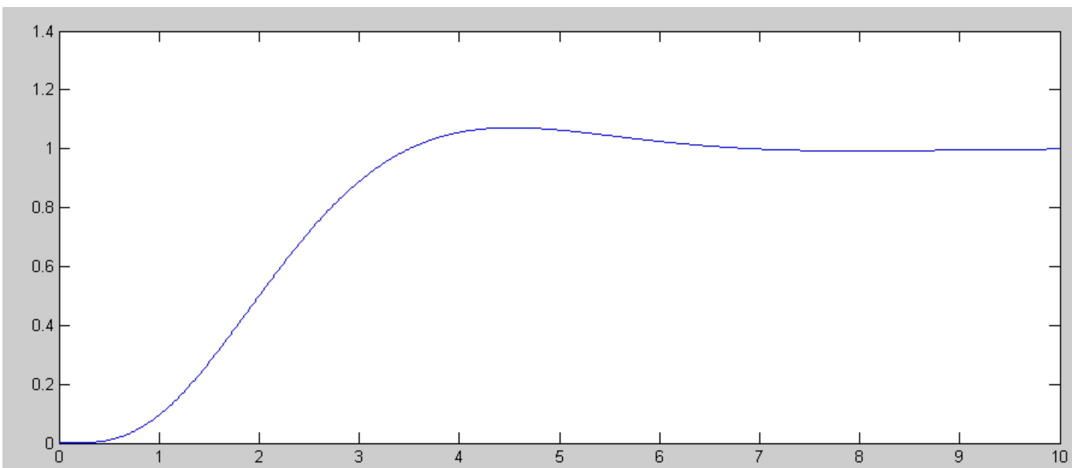
Step 2: Find the feedback gains

```
A = [0,0,1,0;0,0,0,1;0,-7,0,0;-7.84,0,0,0]

         0         0    1.0000         0
         0         0         0    1.0000
         0   -7.0000         0         0
   -7.8400         0         0         0

B = [0;0;0;0.4];
C = [1,0,0,0];

Kx = ppl(A, B, [-0.67 + j*0.91, -0.67 - j*0.91, -2, -3])
```

```
Kx =   -22.3364    34.9425    -5.1518    15.8500

DC = -C*inv(A-B*Kx)*B

DC =    -0.3654

Kr = 1/DC

Kr =    -2.7364
```

Check the step response of the linear system in Matlab

```
Gcl = ss(A-B*Kx, B*Kr, C, 0);
zpk(Gcl)

              7.662
---------------------------------
(s+3) (s+2) (s^2 + 1.34s + 1.277)

t = [0:0.01:10]';
y = step(Gcl, t);
plot(t,y);
```

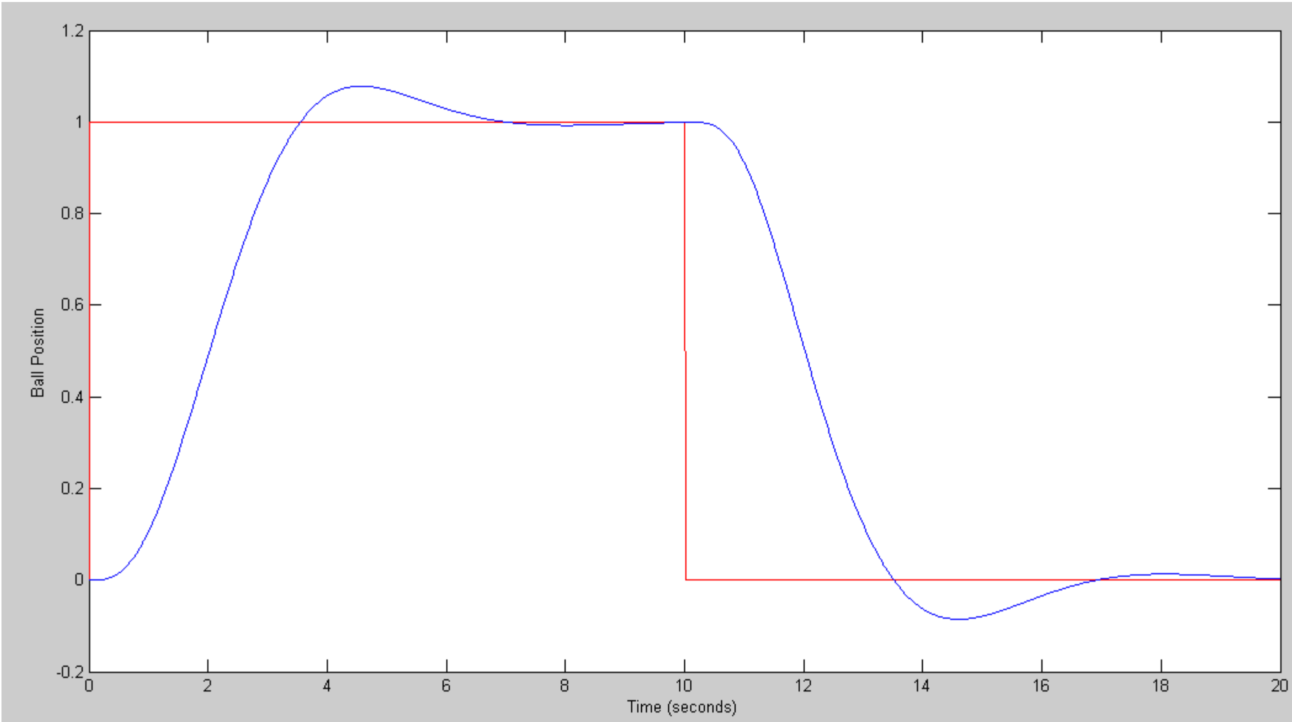


Check the step response of the nonlinear system

Modify the code for beam.m to be

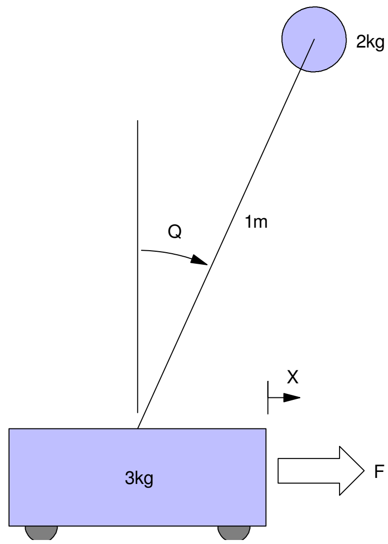
```
% Ball & Beam System
% Sp 20 Version
% m = 2kg
% J = 0.5 kg m^2

X = [0, 0, 0, 0]';
dt = 0.01;
t = 0;
Kx = [  -39.2255   38.4402  -14.6815   13.8637];
Kr = -19.8535;

y = [];
```

```
while(t < 20)
```



1.2
1
0.8
0.6
0.4
0.2
0
-0.2
Ball Position
0
2
4
6
8
10
12
14
16
18
20
Time (seconds)

Problem 3) (20pt) The dynamics of a cart and pendulum (homework set #4) are



$$s\begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -6.533 & 0 & 0 \\ 0 & 16.333 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0.333 \\ -0.333 \end{bmatrix}F$$

Design a feedback control law so that the closed-loop system has

- A 2% settling time of 6 seconds, and
- 10% overshoot for a step input

Follow the same steps for problem #3

```
A = [0,0,1,0;0,0,0,1;0,-6.533,0,0;0,16.333,0,0]

         0         0    1.0000         0
         0         0         0    1.0000
         0   -6.5330         0         0
         0   16.3330         0         0

B = [0;0;0.333;-0.333]

         0
         0
    0.3330
   -0.3330

C = [1,0,0,0];
Kx = ppl(A, B, [-0.67 + j*0.91, -0.67 - j*0.91, -2, -3])
```

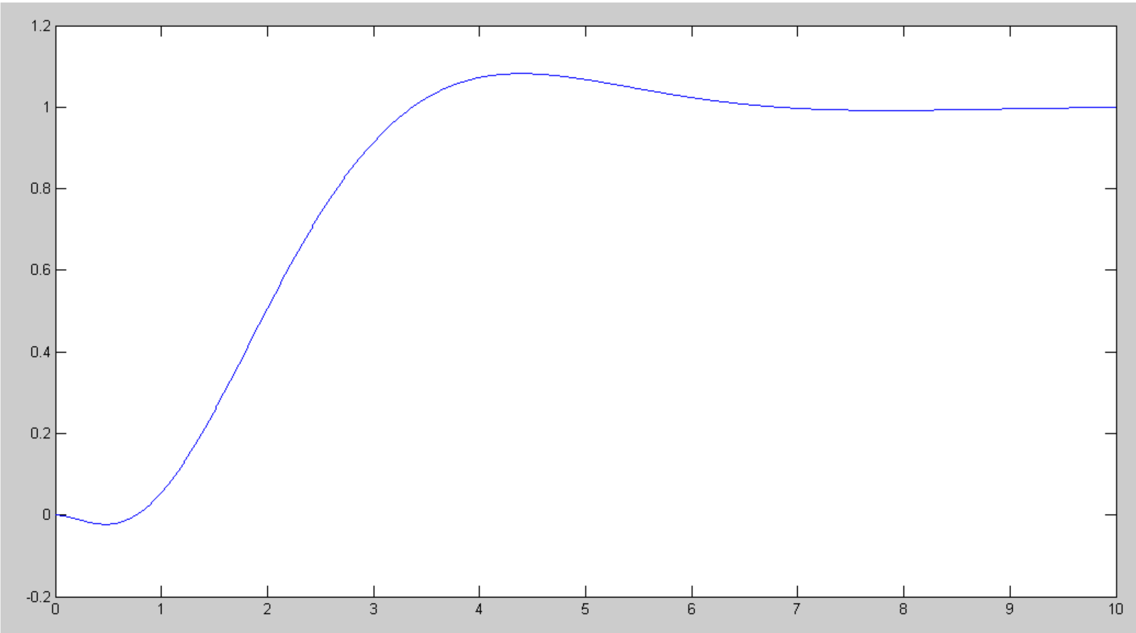
Kx =   -2.3479 -93.3689   -4.4202 -23.4593

```
DC = -C*inv(A-B*Kx)*B
Kr = 1/DC
```

Kr =   -2.3479

Check the step response of the linear system in Matlab

```
Gcl = ss(A-B*Kx, B*Kr, C, 0);
zpk(Gcl)

    -0.78184 (s-3.13) (s+3.13)
---------------------------------
(s+3) (s+2) (s^2 + 1.34s + 1.277)

t = [0:0.01:10]';
y = step(Gcl, t);
plot(t,y);
```

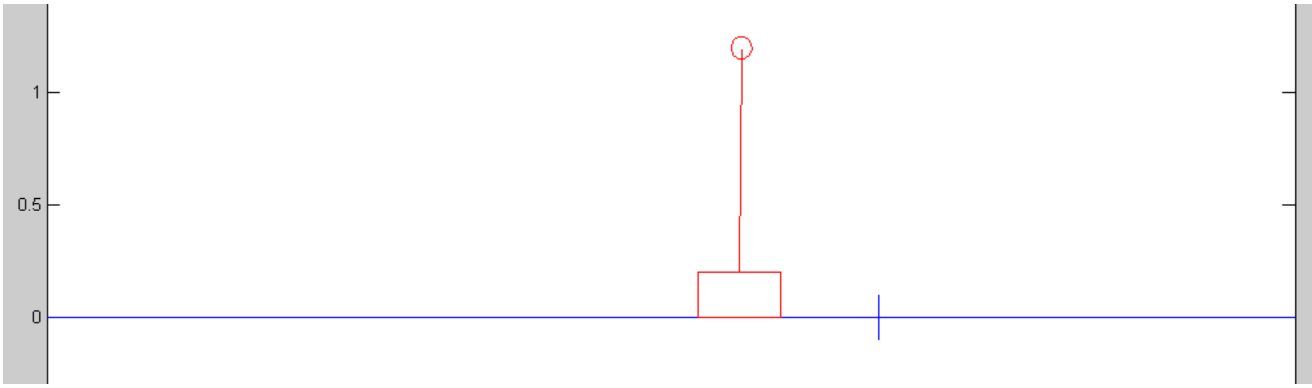


Check the step response of the nonlinear system

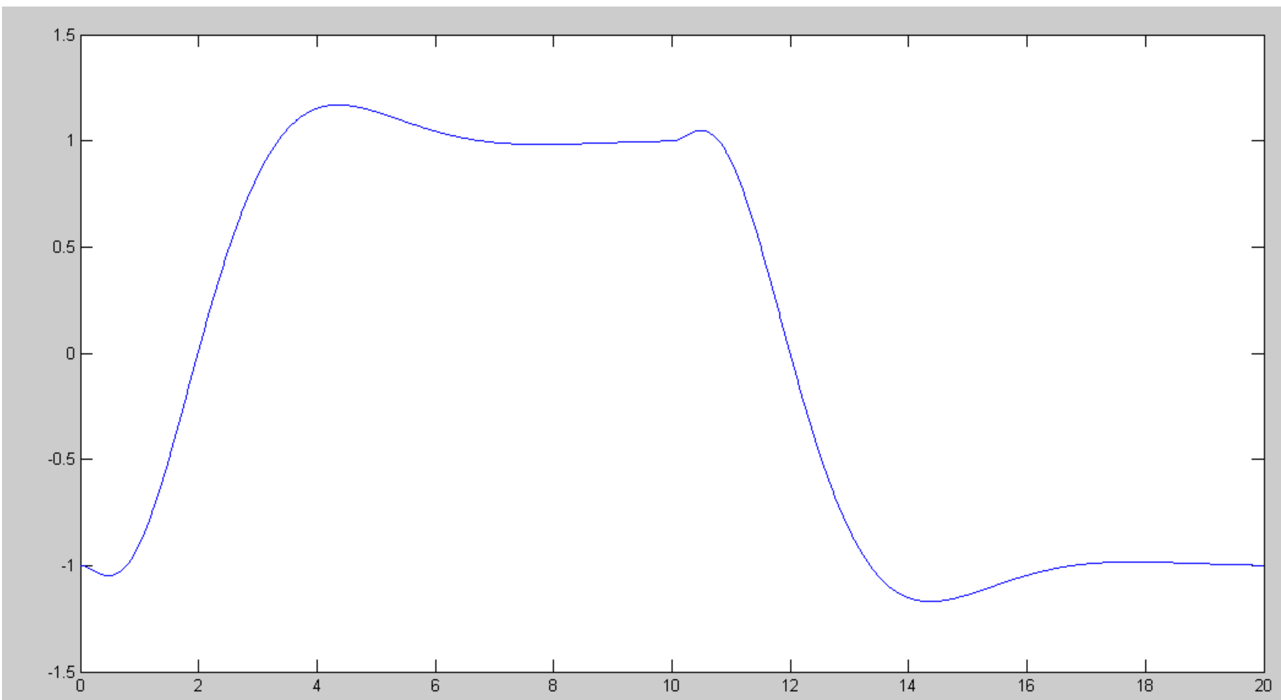
Modify the Cart.m routine

```
% Cart and Pendulum (sp20 version)
X = [-1, 0, 0, 0]';
dX = zeros(4,1);
Ref = 1;
dt = 0.01;
U = 0;
t = 0;
Kx = [-2.3479  -93.3689   -4.4202  -23.4593];
Kr = -2.3479;
Z = 0;
y = [];
while(t < 20)
```

Run the simulation:

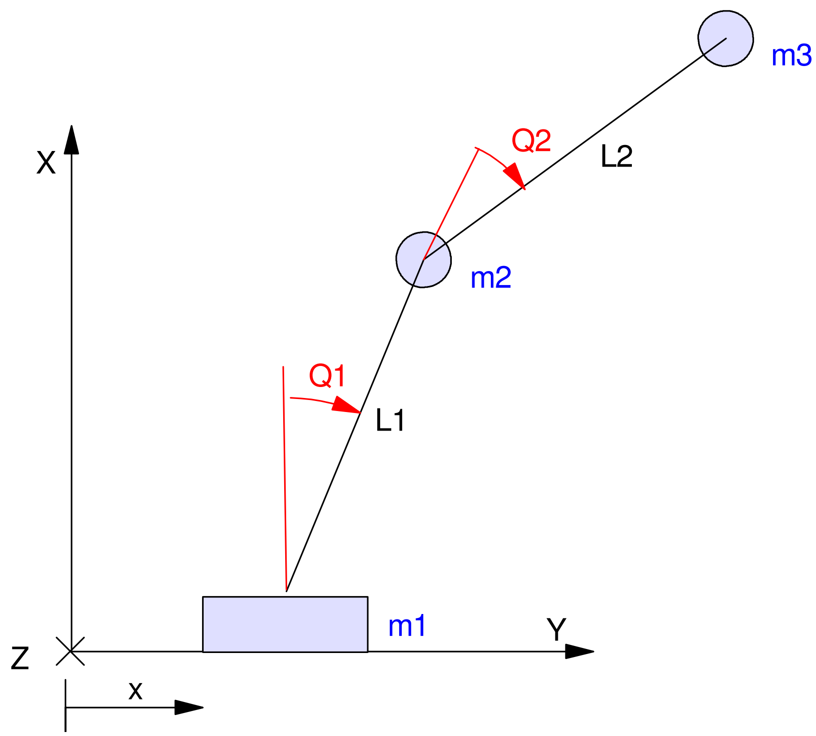


1
0.5
0



1.5
1
0.5
0
-0.5
-1
-1.5
0
2
4
6
8
10
12
14
16
18
20

Problem #4 (20pt): The dynamics of a double pendulum are



$$s\begin{bmatrix} x \\ \theta_1 \\ \theta_2 \\ \dot{x} \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & -2g & 0 & 0 & 0 & 0 \\ 0 & 3g & -g & 0 & 0 & 0 \\ 0 & -3g & 3g & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \theta_1 \\ \theta_2 \\ \dot{x} \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ -1 \\ 1 \end{bmatrix} F$$

Design a feedback control law of the form

U = Kr * R - Kx * X

so that the closed-loop system has

- A 2% settling time of 6 seconds, and
- 10% overshoot for a step input

```
g = 9.8;
A = [0,0,0,1,0,0;0,0,0,0,1,0;0,0,0,0,0,1;0,-2*g,0,0,0,0;0,3*g,-g,0,0,0;0,-3*g,3*g,0,0,0]

         0          0          0     1.0000          0          0
         0          0          0          0     1.0000          0
         0          0          0          0          0     1.0000
         0   -19.6000          0          0          0          0
         0    29.4000    -9.8000          0          0          0
         0   -29.4000    29.4000          0          0          0

B = [0;0;0;1;-1;1];
C = [1,0,0,0,0,0];
Kx = ppl(A, B, [-0.67 + j*0.91, -0.67 - j*0.91, -2, -3, -4, -5])
```

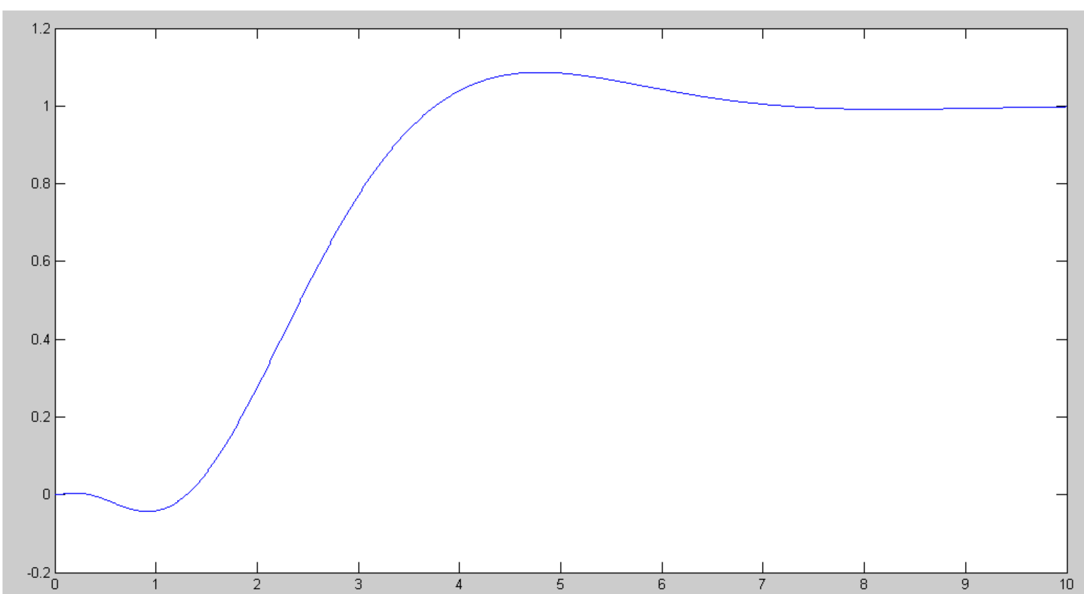
Kx =   0.7978   -6.5275 142.5117   1.8610   17.3453   30.8244

```
DC = -C*inv(A-B*Kx)*B
Kr = 1/DC
```

Kr = 0.7978

Determine the step response of the linear system in Matlab

```
Gcl = ss(A-B*Kx, B*Kr, C, 0);
zpk(Gcl)

0.79779 (s-5.784) (s-2.396) (s+2.396) (s+5.784)
-----------------------------------------------
 (s+5) (s+4) (s+3) (s+2) (s^2 + 1.34s + 1.277)

t = [0:0.01:10]';
y = step(Gcl, t);
plot(t,y);
```



Determine the step response of the nonlinear system

Modify Cart2.m as follows:

```
X = [-1, 0, 0, 0, 0, 0]';
Ref = 1;
dt = 0.01;
U = 0;
t = 0;
Kx = [0.7978   -6.5275  142.5117    1.8610   17.3453   30.8244];
Kr = 0.7978;
while(t < 20)
```

Run the step response: