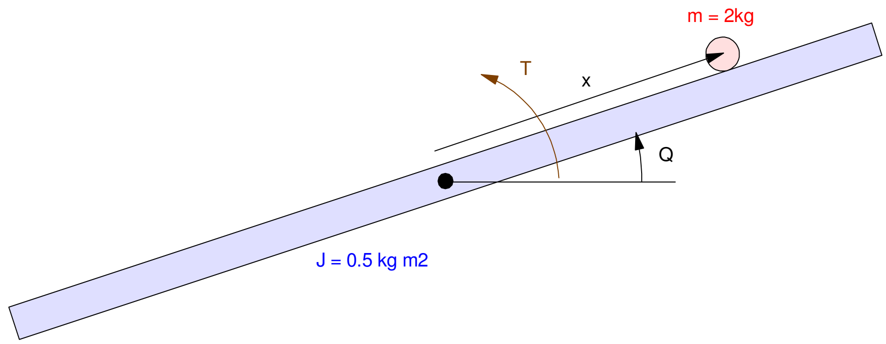
ECE 463/663 - Homework #6

Servo Compensators. Due Monday, March 2nd



The dynamics of a Ball and Beam System (homework set #4) with a disturbance are

$$s\begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -7 & 0 & 0 \\ -7.84 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.4 \end{bmatrix} T + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.4 \end{bmatrix} d$$

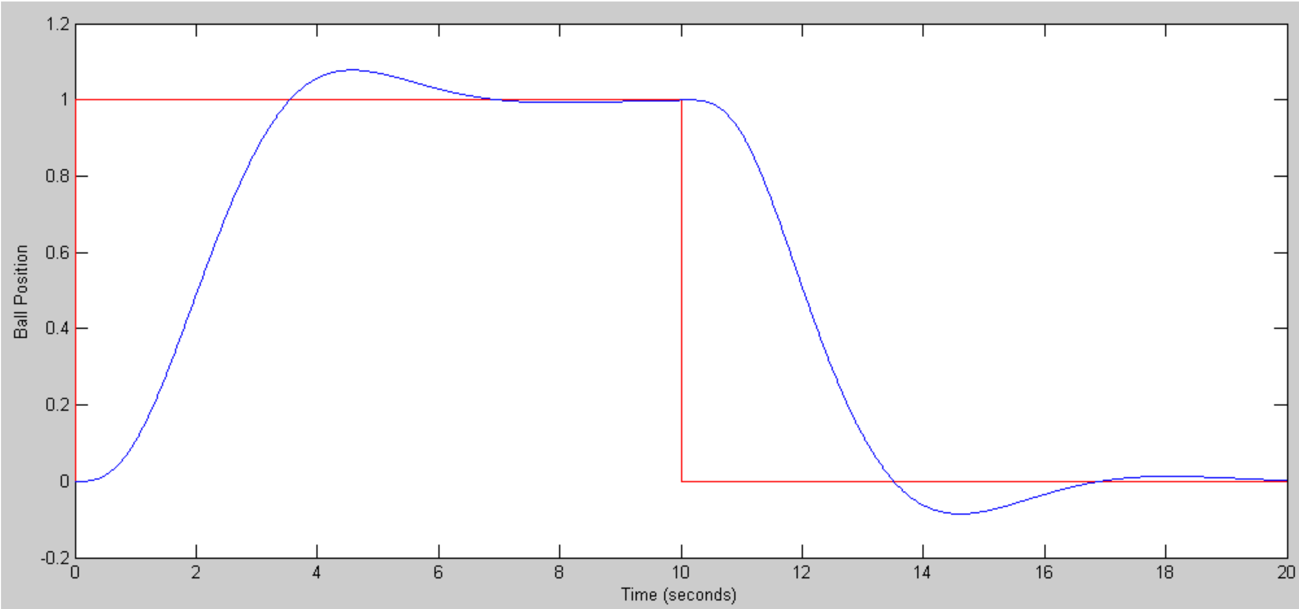
1) Use the feedaback control you you designed in homeowork #5 of the form

U = Kr * R - Kx * X

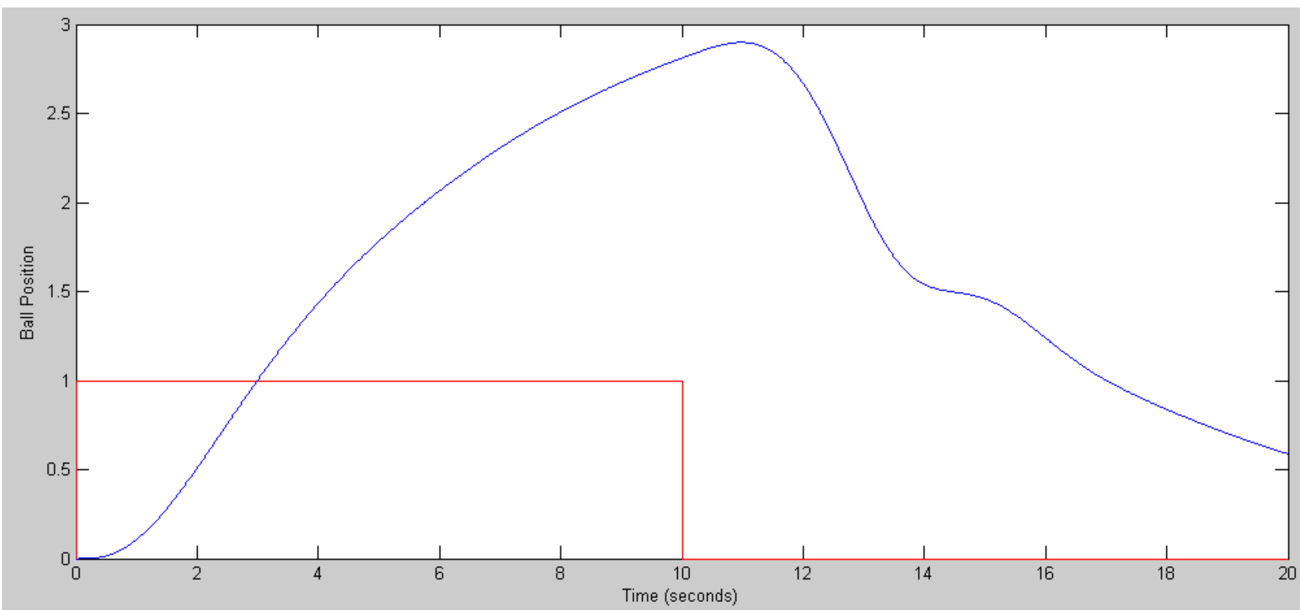
to determine the step response of the nonlinear system

- When the mass of the ball is 2.00kg (same as homework #5), and
- When the mass of the ball is 2.2kg

```
A = [0,0,1,0;0,0,0,1;0,-7,0,0;-7.84,0,0,0];
B = [0;0;0;0.4];
Kx = ppl(A, B, [-0.67 + j*0.91, -0.67 - j*0.91, -2, -3])

Kx = -22.3364    34.9425    -5.1518    15.8500

C = [1,0,0,0];
DC = -C*inv(A-B*Kx)*B
Kr = 1/DC

Kr =  -2.7364
```



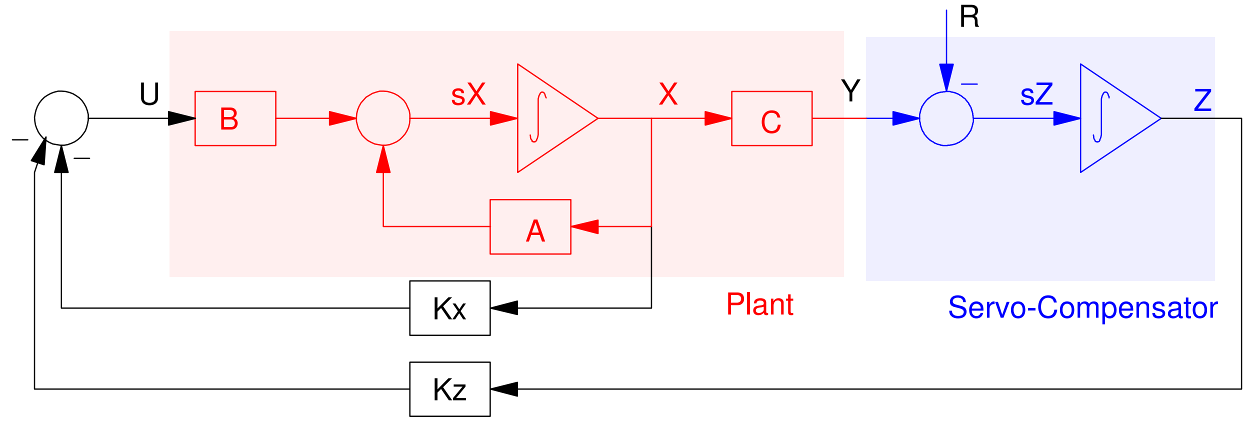
Step Response when m = 2.0 kg



Step Response when m = 2.2 kg

Constant Disturbance / Set Point

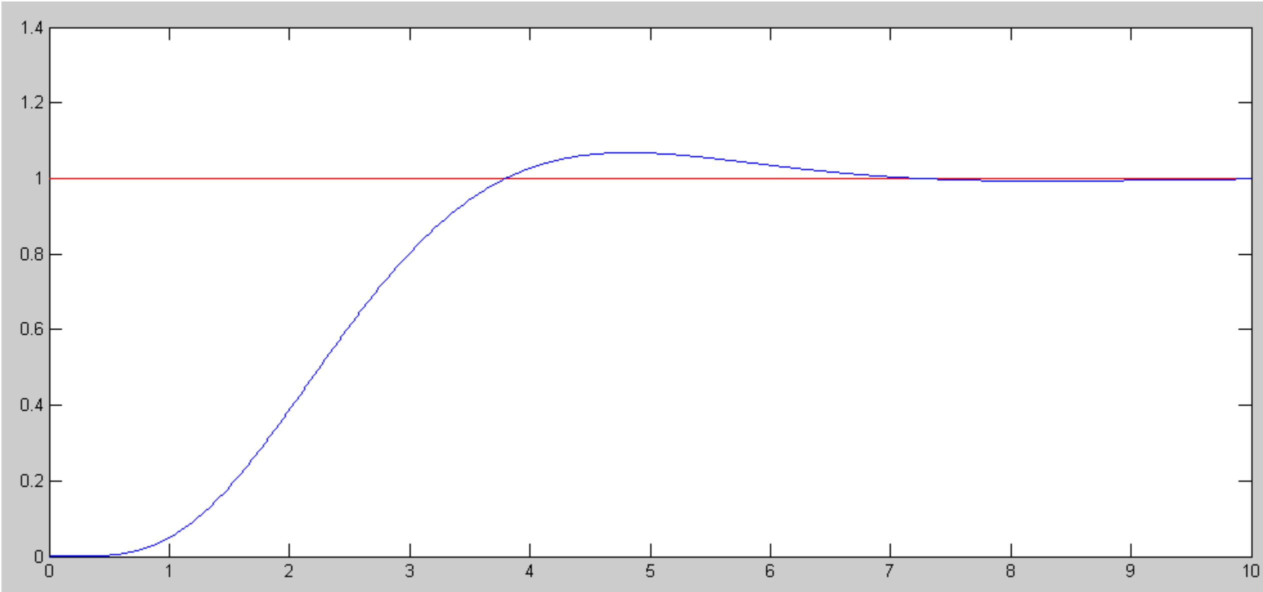
2) Design a servo-compensator so that it can track a contant set-point and reject a constant disturbance. Provide:



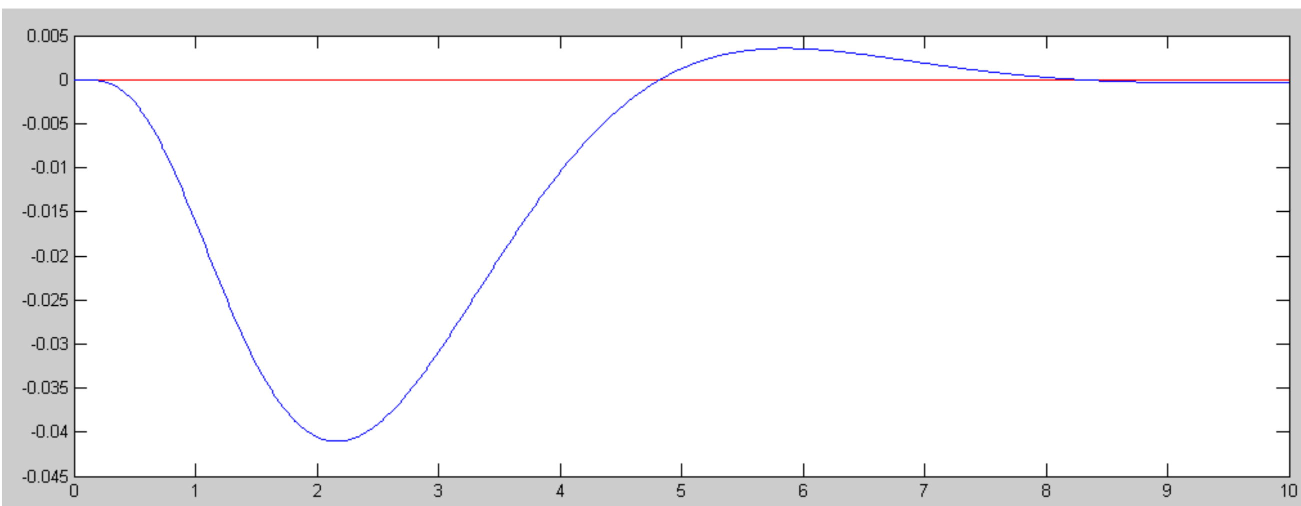
Block diagram of Plant & Servo Compensator

The feedback control law

```
A5 = [A, zeros(4,1) ; C, 0]

         0         0    1.0000         0         0
         0         0         0    1.0000         0
         0   -7.0000         0         0         0
   -7.8400         0         0         0         0
    1.0000         0         0         0         0

B5u = [B; 0];
B5r = [0*B ; -1];
C5 = [C, 0];
D5 = 0;

K5 = ppl(A5, B5u, [-0.67 + j*0.91, -0.67 - j*0.91, -2, -3. -4])

K5 =  -42.9436   98.3425  -25.1189   25.8500  -10.9457
                       Kx                           Kz

t = [0:0.01:10]';
R = 0*t+1;
y = step3(A5-B5u*K5, B5r, C5, D5, t, X0, R);

plot(t,R,'r',t,y,'b');
```

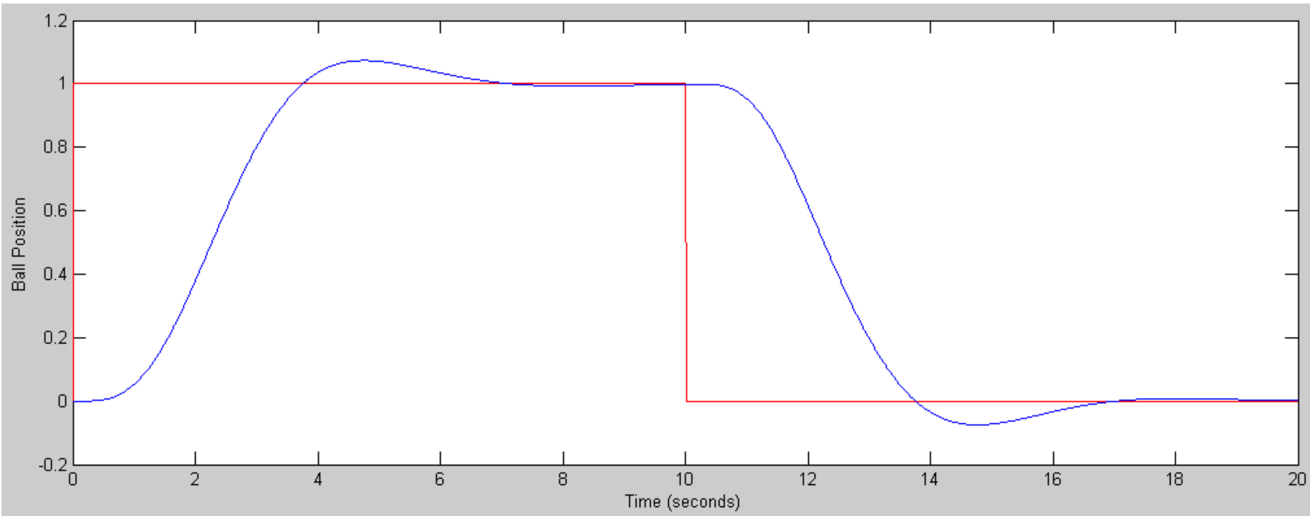



```
y = step3(A5-B5u*K5, B5u, C5, D5, t, X0, R);
plot(t,R*0,'r',t,y,'b');
```

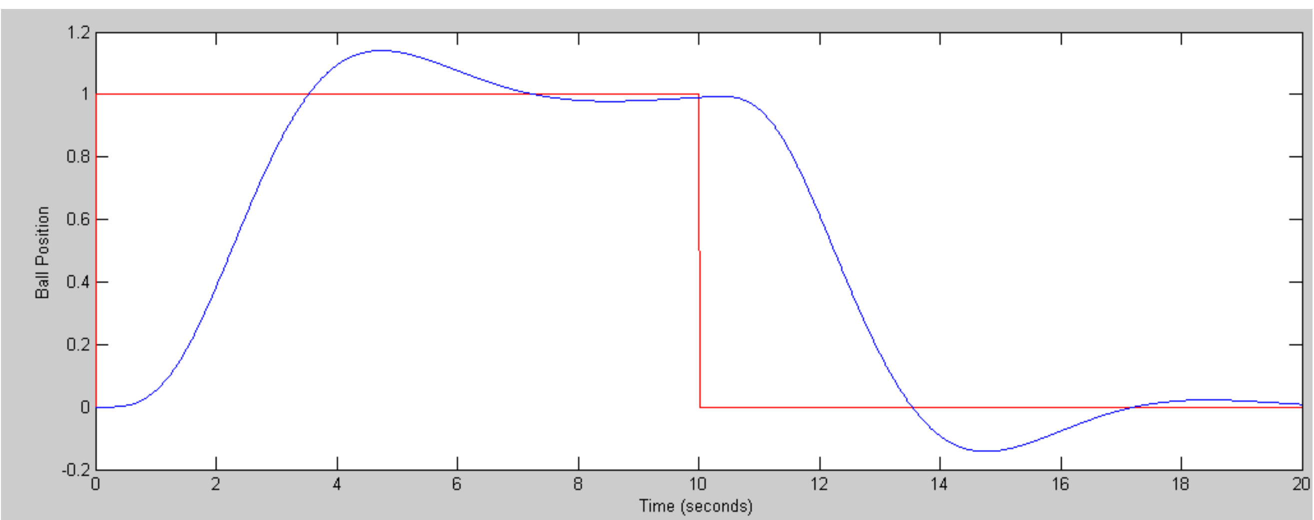


3) Add the servo-compensator to the nonlinear simulation and find the step response for

- m = 2.0kg, and
- m = 2.2kg



Response when m = 2.0 kg



Response when m = 2.2kg

Code:

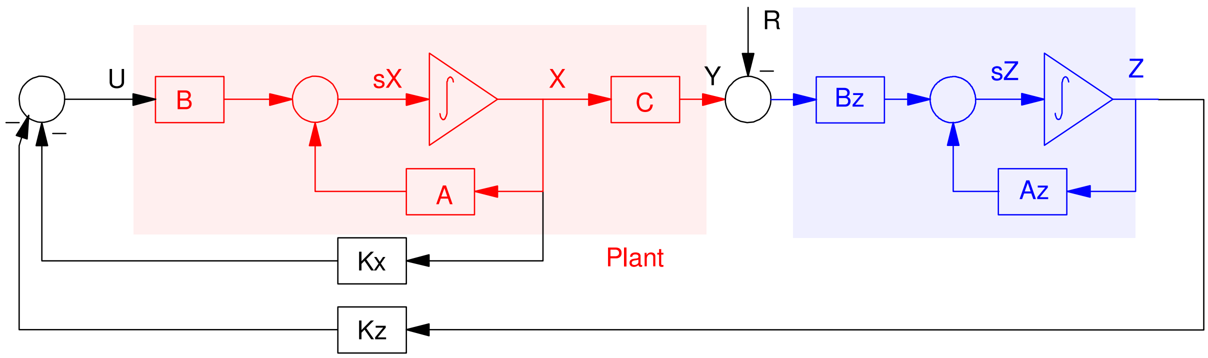
```
% Ball & Beam System
% Sp 20 Version
% m = 2kg
% J = 0.5 kg m^2

X = [0, 0, 0, 0]';
dt = 0.01;
t = 0;
Kx = [-42.9436   98.3425  -25.1189   25.8500];
Kz = -10.9457;
Z = 0;

y = [];

while(t < 20)
 Ref = 1 * (sin(0.1*pi*t) > 0);

 U = -Kz*Z - Kx*X;

 dX = BeamDynamics(X, U);
 dZ = X(1) - Ref;

 X = X + dX * dt;
 Z = Z + dZ * dt;

 y = [y ; Ref, X(1)];

 t = t + dt;
 BeamDisplay(X, Ref);
 end
```

Sinusoidal Disturbance / Setpoint

4) Design a servo-compensator so that it can track an 0.5 rad/sec sinusoidal set-point of the form

$$Ref = \sin(0.5t)$$

Provide: A block-diagram for the plant - servo compensator - full-state feedback system



The feedback control law, and

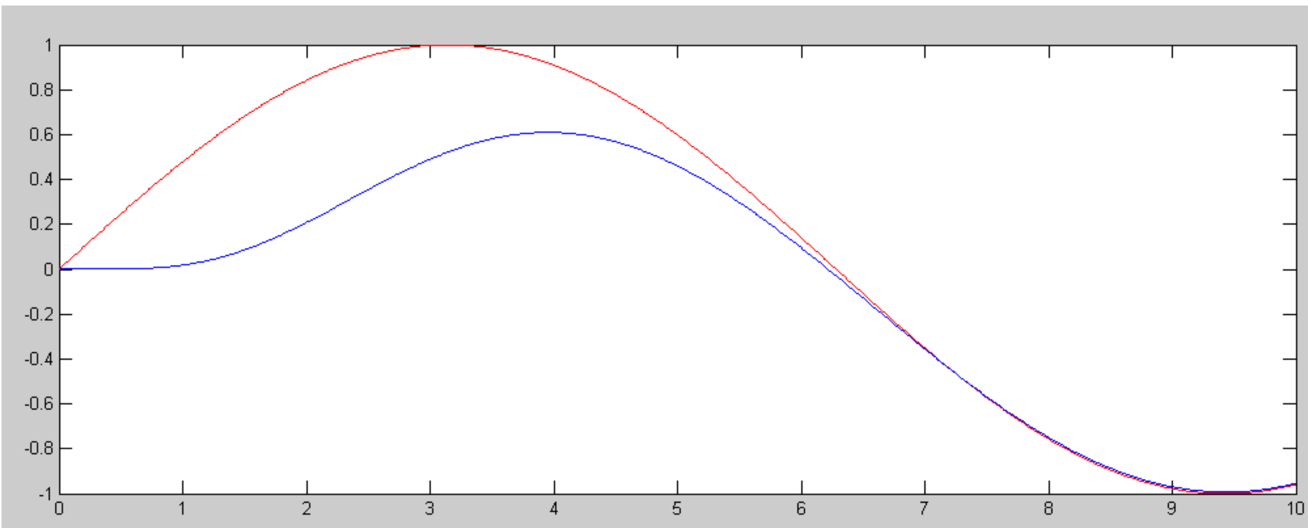
```
Az = [0,0.5 ; -0.5, 0]

         0    0.5000
   -0.5000         0

Bz = [1;1]

     1
     1

A6 = [A, zeros(4,2) ; Bz*C, Az]

         0         0    1.0000         0         0         0
         0         0         0    1.0000         0         0
         0   -7.0000         0         0         0         0
   -7.8400         0         0         0         0         0
    1.0000         0         0         0         0    0.5000
    1.0000         0         0         0   -0.5000         0

B6u = [B ; 0*Bz]

         0
         0
         0
    0.4000
         0
         0

B6r = [0*B; -Bz]

     0
     0
     0
     0
    -1
    -1

C6 = [C, zeros(1,2)];
D6 = 0;
```

```
X0 = zeros(6,1);
t = [0:0.01:10]';

K6 = ppl(A6, B6u, [-0.67 + j*0.5, -0.67 - j*0.5, -2, -3, -4, -5])
```

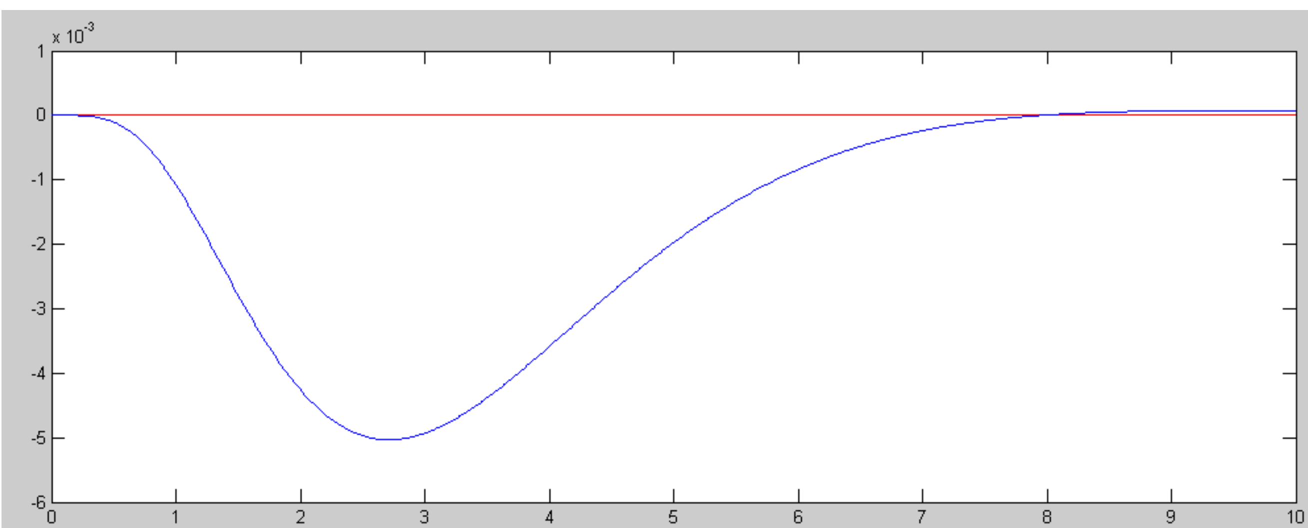
K6 = -145.8249 225.5222 -91.1034 38.3500 -34.9427 -38.1495

```
      -------------- Kx --------------------   ----- Kz -------
```

```
R = sin(0.5*t);
y = step3(A6-B6u*K6, B6r, C6, D6, t, X0, R);
plot(t,R,'r',t,y,'b');
```



Ref = sin(0.5*t), d = 0



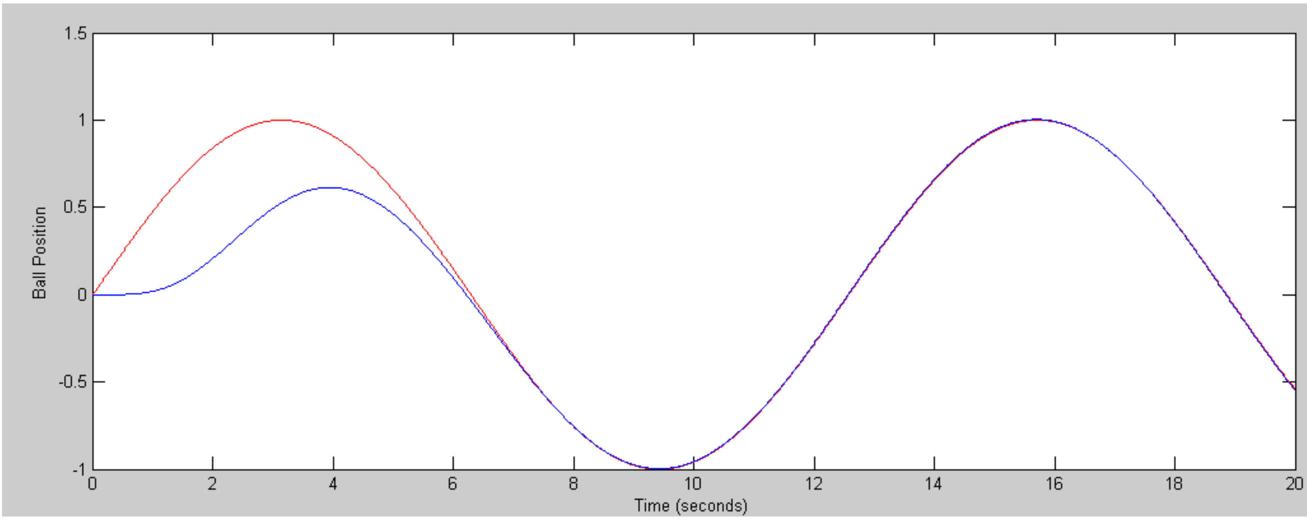
Response to R = 0, d = sin(0.5*t)

5) Add the servo-compensator to the nonlinear simulation and find the response for

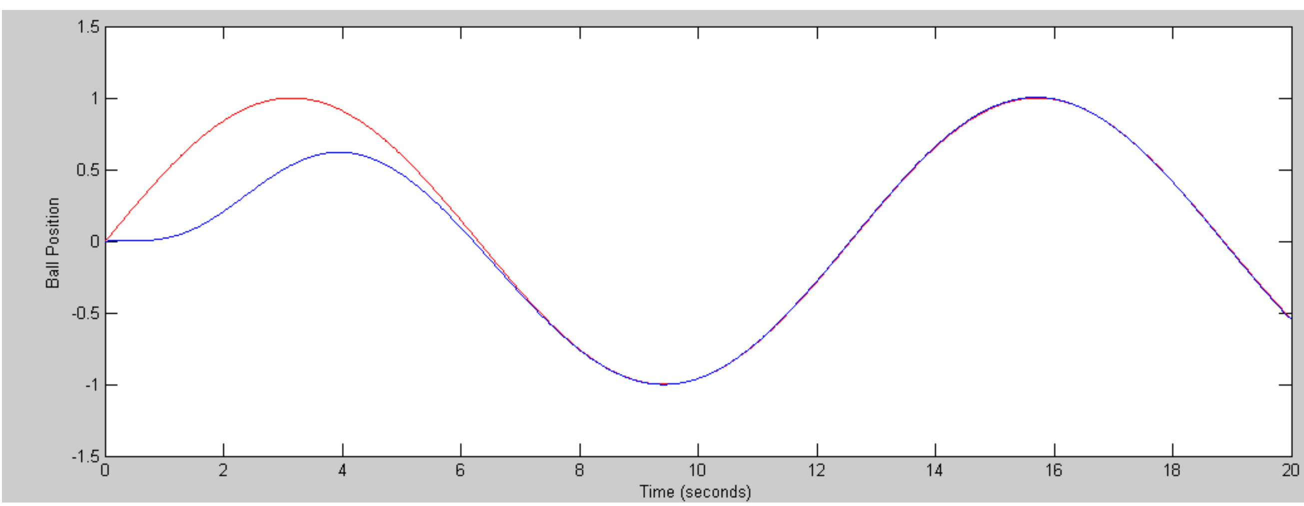
- Ref = sin(0.5t)

and

- m = 2.0kg, and
- m = 2.2kg



Nonlinear response: m = 2.0kg



Nonlinear Response: m = 2.2kg

Matlab Code:

```
% Ball & Beam System
% Sp 20 Version
% m = 2kg
% J = 0.5 kg m^2

X = [0, 0, 0, 0]';
dt = 0.01;
t = 0;
Kx = [ -145.8249  225.5222  -91.1034   38.350];
Kz = [-34.9427  -38.1495];
Z = zeros(2,1);
Az = [0, 0.5 ; -0.5, 0];
Bz = [1;1];


y = [];

while(t < 20)
 Ref = sin(0.5*t);

 U = -Kz*Z - Kx*X;

 dX = BeamDynamics(X, U);
 dZ = Az*Z + Bz*(X(1) - Ref);

 X = X + dX * dt;
 Z = Z + dZ * dt;

 y = [y ; Ref, X(1)];

 t = t + dt;
 BeamDisplay(X, Ref);
 end

t = [1:length(y)]' * dt;


plot(t,y(:,1),'r',t,y(:,2),'b');
xlabel('Time (seconds)');
ylabel('Ball Position');
```