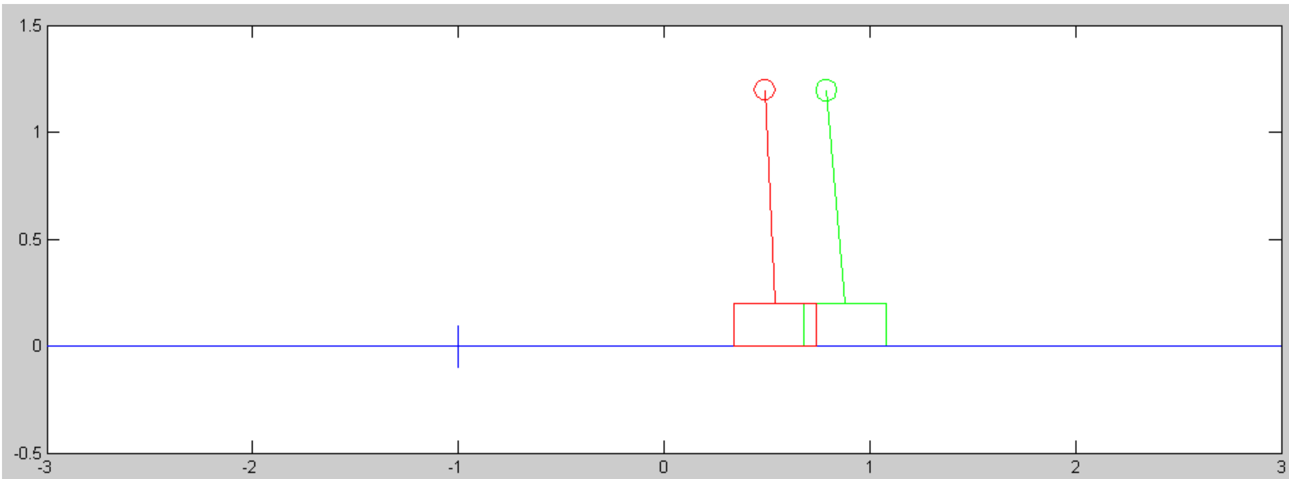
ECE 463: Homework #7

Linear Observers. Due Monday March 23rd



Cart and Pendulum from homework #4 with a state estimator (green)

Use the dynamics for the cart and pendulum from homework set #4

1) Design a full-state feedback control law of the form

$$U = F = K_r R - K_x X$$

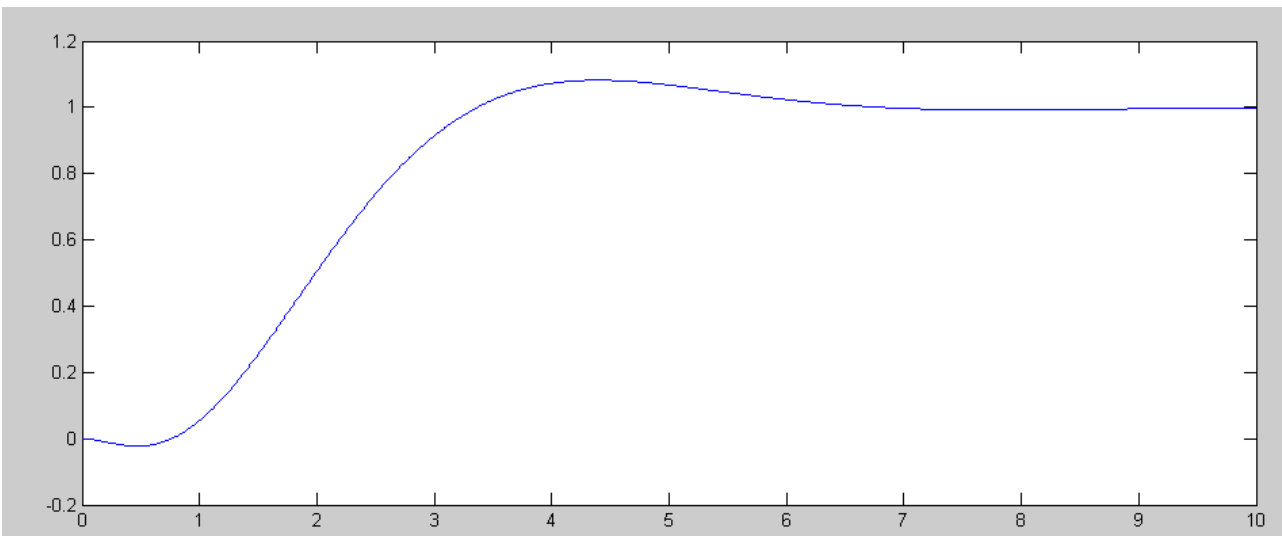
so that the closed-loop system has

- A 2% settling time of 6 seconds, and
- 10% overshoot for a step input.

Plot the step response of the linarized system in Matlab.

From homework #5:

```
A = [0,0,1,0;0,0,0,1;0,-6.533,0,0;0,16.333,0,0]
B = [0;0;0.333;-0.333]
C = [1,0,0,0]
D = 0;

Kx = ppl(A, B, [-0.67 + j*0.91, -0.67 - j*0.91, -2, -3])

Kx =     -2.3479   -93.3689   -4.4202   -23.4593

DC = -C*inv(A-B*Kx)*B
Kr = 1/DC

Kr =    -2.3479

G = ss(A-B*Kx, B*Kr, C, 0);
t = [0:0.01:10]';
y = step(G, t);
plot(t,y);
```



2) Assume you can only measure the cart position as well as beam angle.

2a) Design a full-order observer to estimate all four states so that the observer is 2-5 times faster than the plant. You may use either cart position or beam angle (or both) as measurements.

```
H = ppl(A', C', [-2, -3, -4, -5])'

   14.0000
  -58.5737
   87.3330
 -236.7075
```

2b) Give the state-space model of the closed loop system using the actual states:

Control

$$U = F = K_r R - K_x X$$

Plant

$$sX = AX + BU$$

Observer

$$sX_o = AX_o + BU + H(CX - CX_o)$$

Putting all three together

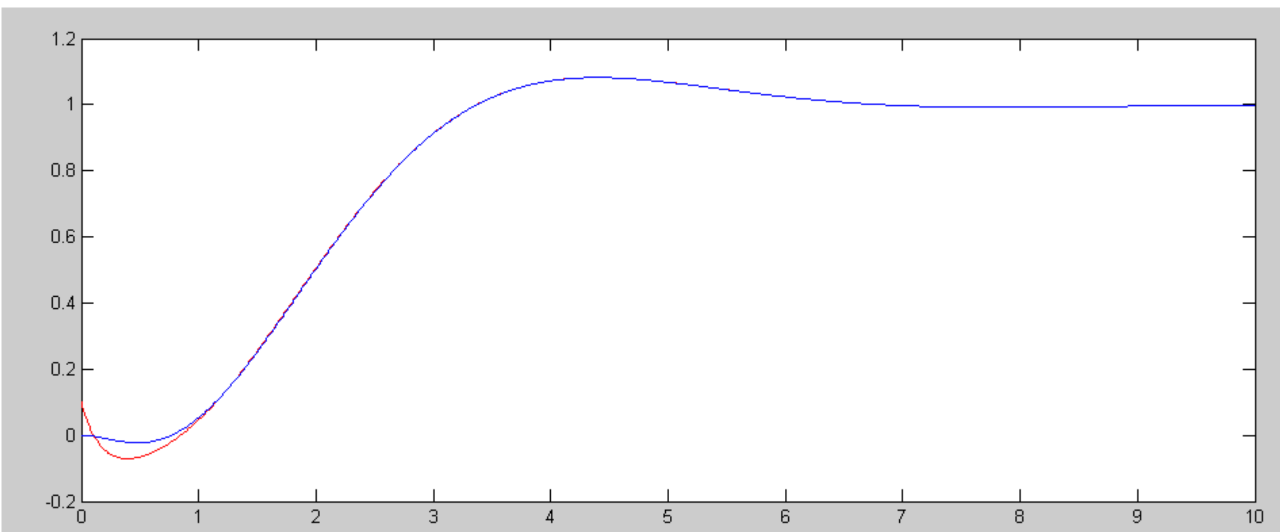
$$\begin{bmatrix} sX \\ sX_o \end{bmatrix} = \begin{bmatrix} A - BK_x & 0 \\ HC - BK_x & A - HC \end{bmatrix} \begin{bmatrix} X \\ X_o \end{bmatrix} + \begin{bmatrix} BK_r \\ BK_r \end{bmatrix} R$$

and plot the step response with initial conditions of

X(0) = [0, 0, 0, 0]' $X_{observer}(0)$ = [0.1, 0.1, 0.1, 0.1]'

```
A8 = [A-B*Kx, zeros(4,4) ; H*C - B*Kx,  A-H*C]

         0         0    1.0000         0         0         0         0         0
         0         0         0    1.0000         0         0         0         0
    0.7818   24.5588    1.4719    7.8119         0         0         0         0
   -0.7818  -14.7588   -1.4719   -7.8119         0         0         0         0
   14.0000         0         0         0  -14.0000         0    1.0000         0
  -58.5737         0         0         0   58.5737         0         0    1.0000
   88.1148   31.0918    1.4719    7.8119  -87.3330   -6.5330         0         0
 -237.4893  -31.0918   -1.4719   -7.8119  236.7075   16.3330         0         0

eig(A8)

  -5.0000
  -4.0000
  -0.6700 + 0.9100i
  -0.6700 - 0.9100i
  -3.0000
  -3.0000
  -2.0000
  -2.0000
```

```
B8 = [B*Kr ; B*Kr]

         0
         0
    0.7818
   -0.7818
         0
         0
    0.7818
   -0.7818

C8 = [C, 0*C ; 0*C, C]

     1     0     0     0     0     0     0     0
     0     0     0     0     1     0     0     0

D8 = [0;0];
X0 = [0;0;0;0 ; 0.1;0.1;0.1;0.1]

         0
         0
         0
         0
    0.1000
    0.1000
    0.1000
    0.1000

R = 0*t+1;
y = step3(A8, B8, C8, D8, t, X0, R);
plot(t,y)
```



2c) Give the state-space model of the closed loop system using the state estimates:

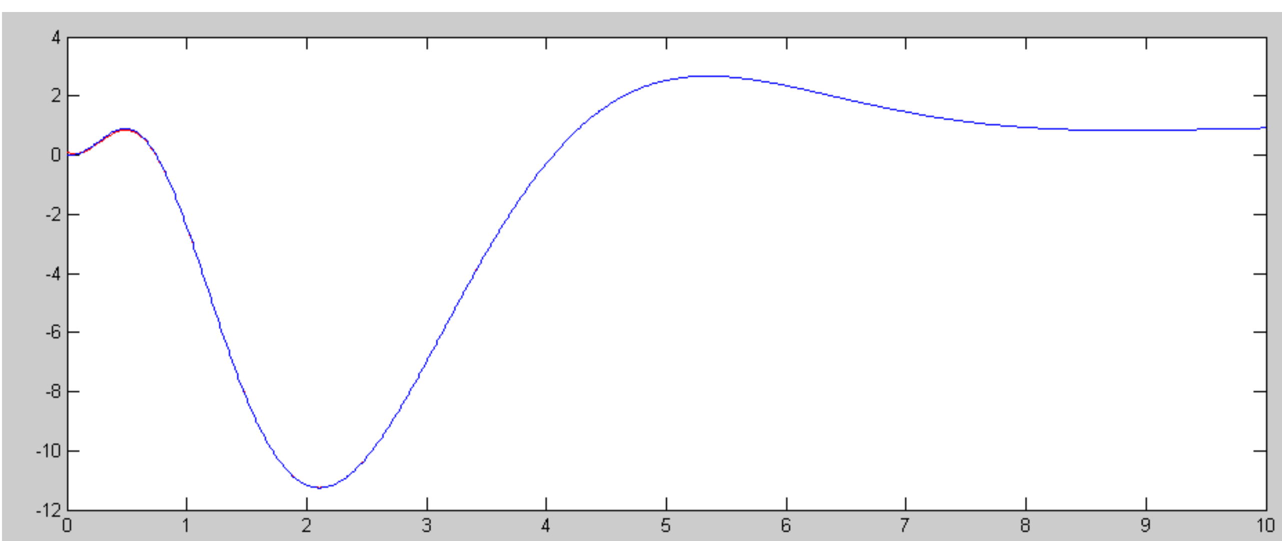
$$U = K_r R - K_x X_{observer}$$

and plot the step response with initial conditions of

X(0) = [0, 0, 0, 0]' $X_{observer}(0)$ = [0.1, 0.1, 0.1, 0.1]'

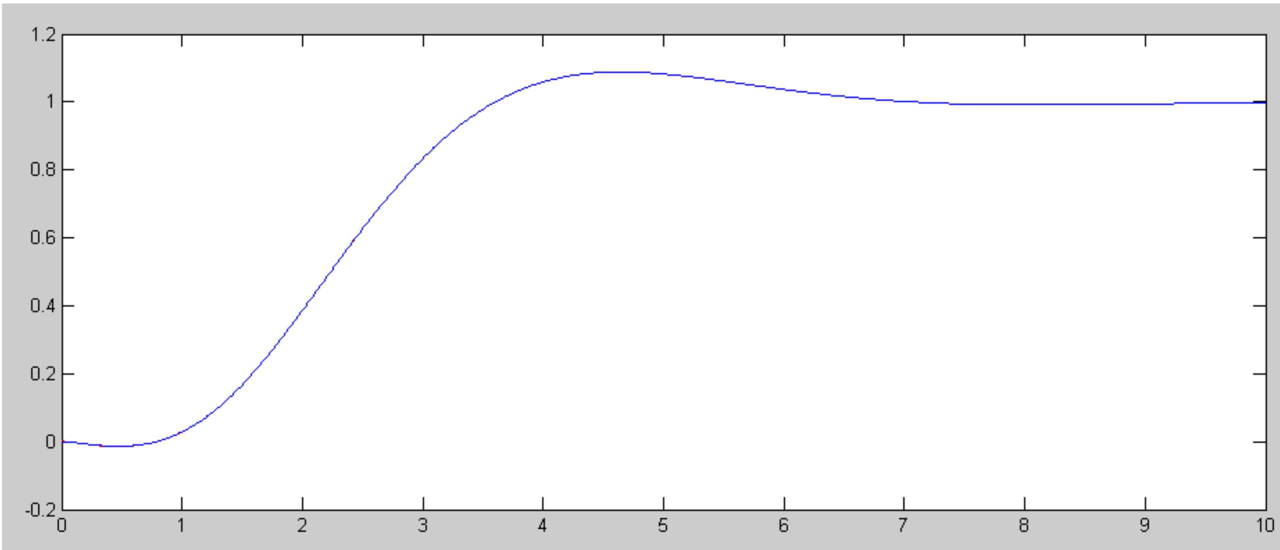
$$\begin{bmatrix} sX \\ sX_o \end{bmatrix} = \begin{bmatrix} A & -BK_x \\ HC & A-HC-BK_x \end{bmatrix}\begin{bmatrix} X \\ X_o \end{bmatrix} + \begin{bmatrix} BK_r \\ BK_r \end{bmatrix} R$$

```
A8 = [A, -B*Kx ; H*C,  A-H*C - B*Kx]
         0         0    1.0000         0         0         0         0         0
         0         0         0    1.0000         0         0         0         0
         0   -6.5330         0         0    0.7818   31.0918    1.4719    7.8119
         0   16.3330         0         0   -0.7818  -31.0918   -1.4719   -7.8119
   14.0000         0         0         0  -14.0000         0    1.0000         0
  -58.5737         0         0         0   58.5737         0         0    1.0000
   87.3330         0         0         0  -86.5512   24.5588    1.4719    7.8119
 -236.7075         0         0         0  235.9256  -14.7588   -1.4719   -7.8119

eig(A8)

  -5.0000
  -4.0000
  -3.0000
  -3.0000
  -2.0000
  -2.0000
  -0.6700 + 0.9100i
  -0.6700 - 0.9100i

y = step3(A8, B8, C8, D8, t, X0, R);
plot(t,y)
```



If you make the initial conditions closer

```
y = step3(A8, B8, C8, D8, t, X0 * 0.01, R);
plot(t,y(:,2),'r',t,y(:,1),'b');
```



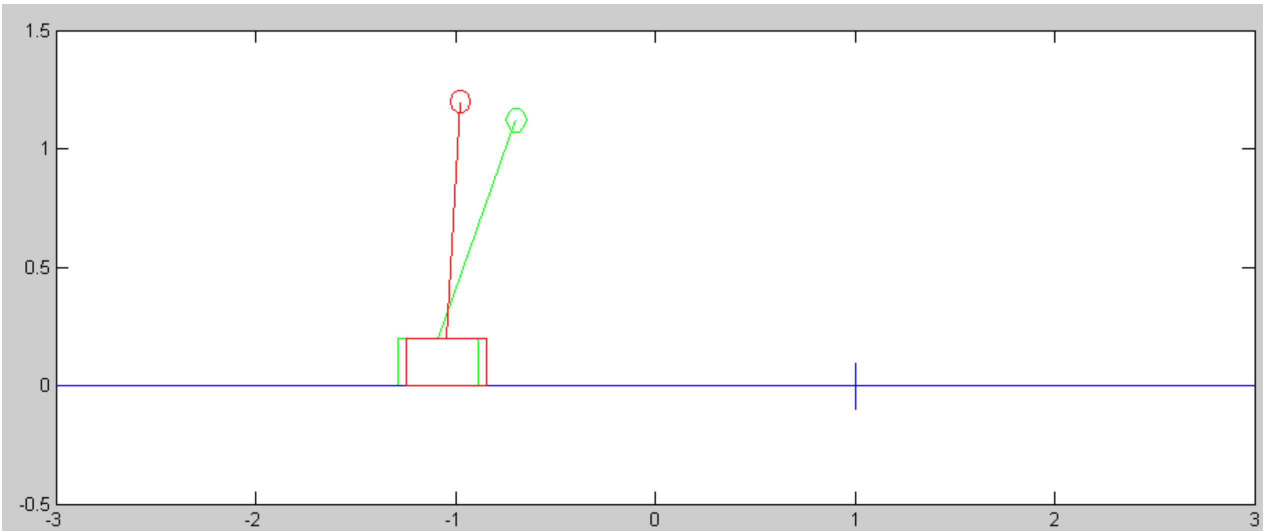
3) Modify the cart and pendulum system to include

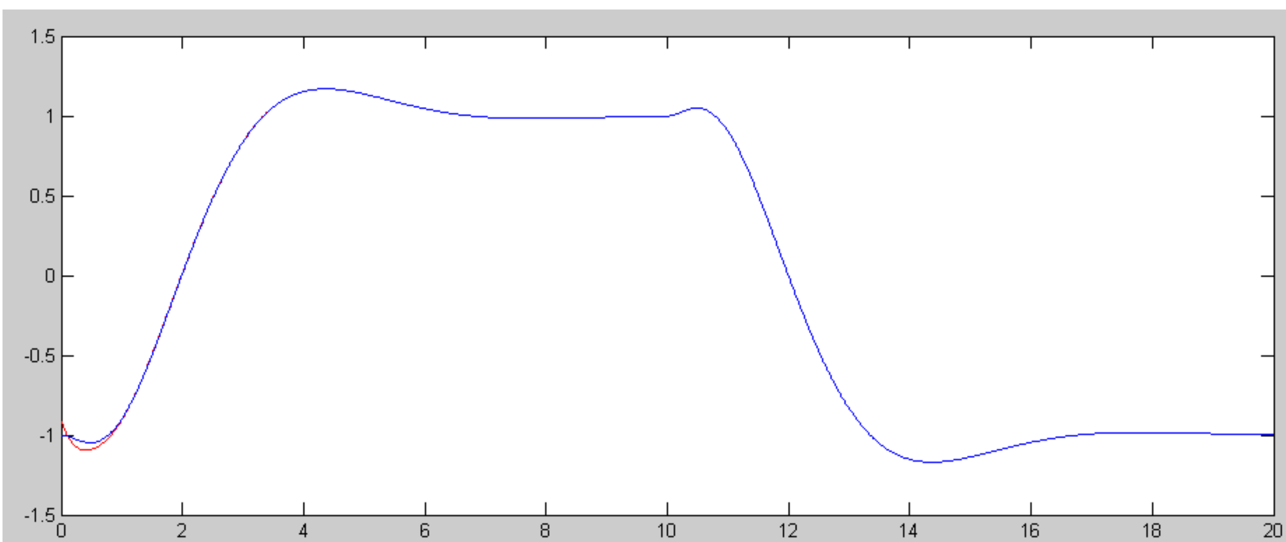
- your control law, and
- A full-order observer

using only cart position and/or beam angle

Find the step response when U is defined as

$$U = K_r R - K_x X$$

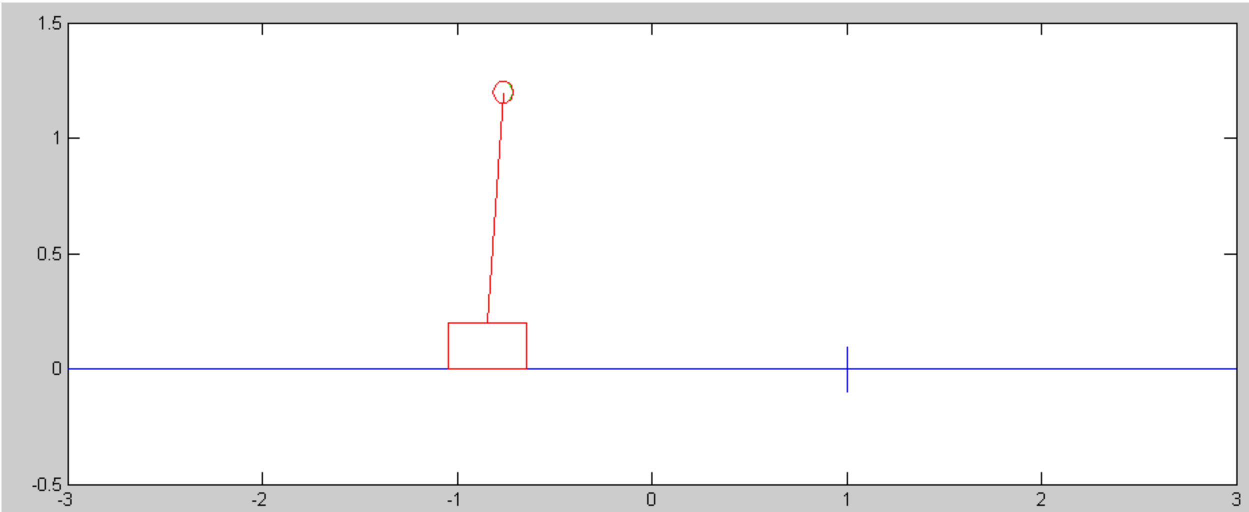


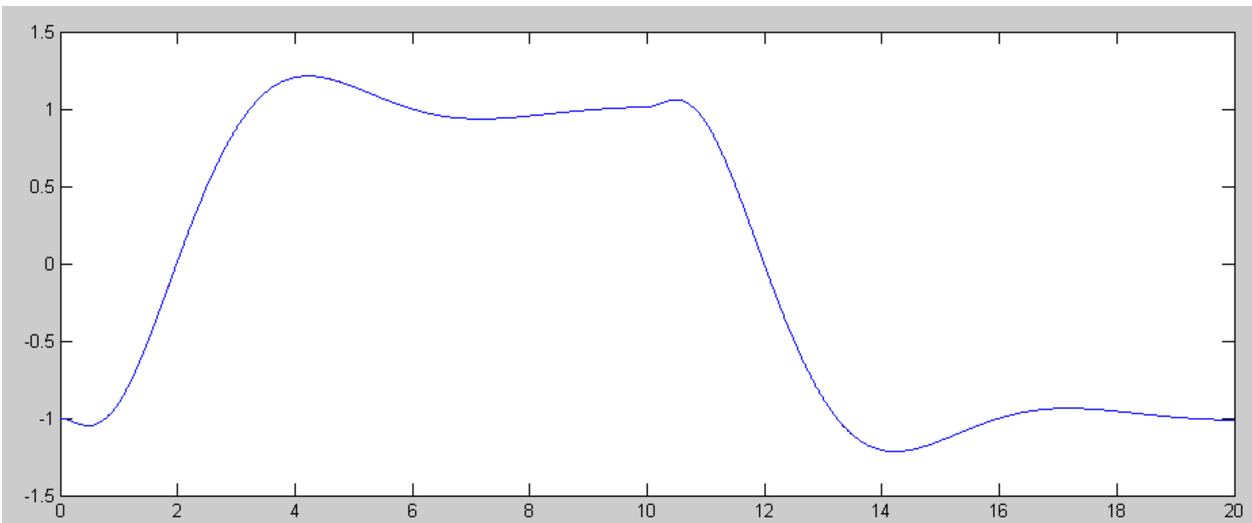


and

$$U = K_r R - K_x X_{observer}$$

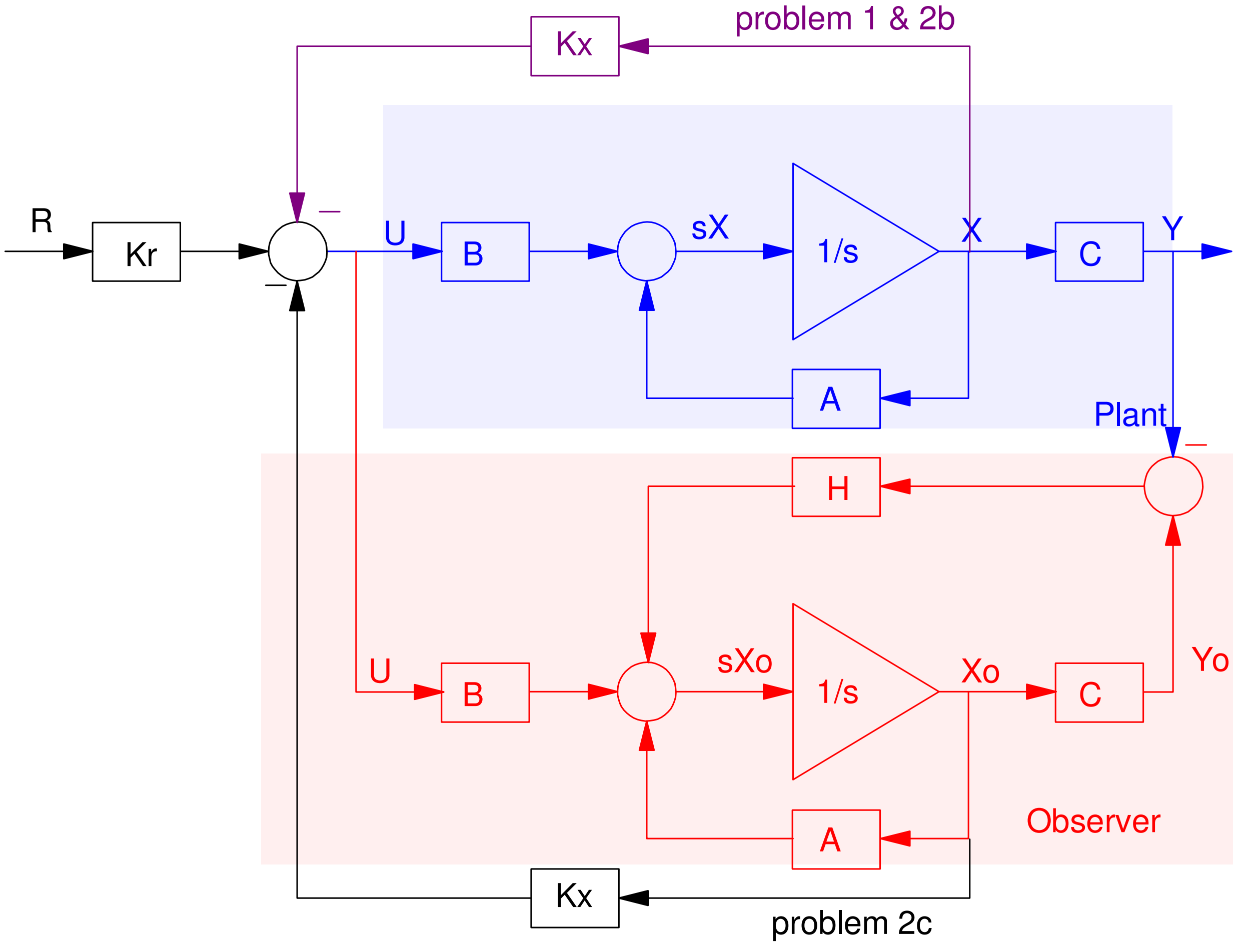
Doesn't work in this case: the initial conditions are too far off. If you set Xo = X at t=0, it does work:





Matlab Code:

```
% Cart and Pendulum (sp20 version)
X = [-1; 0 ; 0 ; 0];
dX = zeros(4,1);
Ref = 1;
dt = 0.01;
U = 0;
t = 0;
Kx = [ -2.3479  -93.3689   -4.4202  -23.4593];
Kr = -2.3479;

% Observer
 A = [0,0,1,0;0,0,0,1;0,-6.533,0,0;0,16.333,0,0];
 B = [0;0;0.333;-0.333];
 C = [1,0,0,0];
 D = 0;
 H = [  14.0000  -58.5737   87.3330 -236.7075]';
 Xo = [-0.9 ; 0.1; 0.1; 0.1];

y = [];
while(t < 20)
 Ref = sign(sin(0.1*pi*t));
 U = Kr*Ref - Kx*X;

 dX = CartDynamics(X, U);
 dXo = A*Xo + B*U + H*(C*X - C*Xo);

 X = X + dX * dt;
 Xo = Xo + dXo * dt;

  t = t + dt;
 CartDisplay3(X, Xo, Ref);
 y = [y; X(1), Xo(1)];
end

clf
t = [1:length(y)]' * 0.01;
plot(t, y);
```



problem 1 & 2b
Kx
R
Kr
U
B
sX
1/s
X
C
Y
A
Plant
H
U
B
sXo
1/s
Xo
C
Yo
A
Observer
Kx
problem 2c