ECE 463/663 - Homework #9

LQG Control with Servo Compensators. Due Wednesday, April 15th

LQG Control

Cart and Pendulum (HW #5): Use LQG methods to design a full-state feedback control law of the form

$$U = -K_z Z - K_x X$$

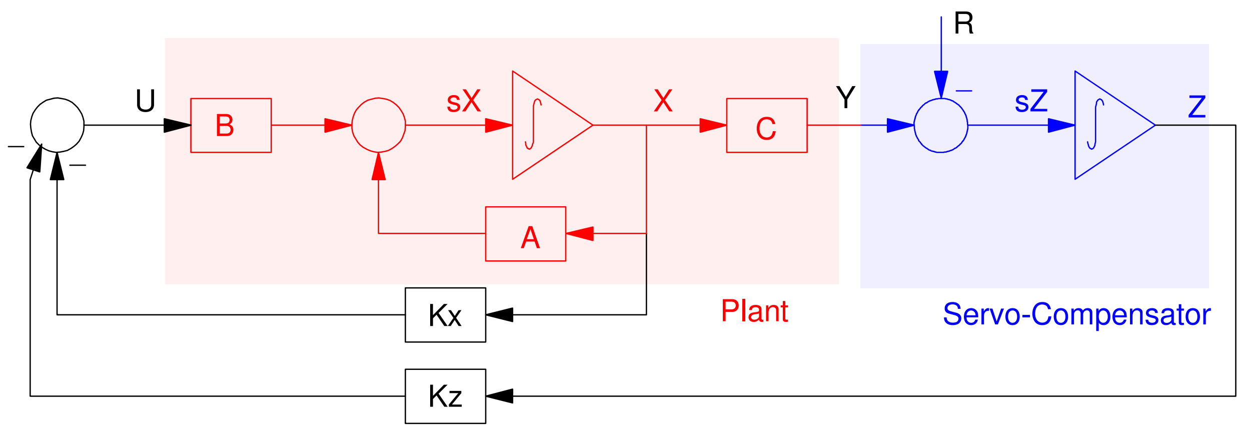
$$\dot{Z} = (x - R)$$

for the cart and pendulum system from homework #5 using LQG control so that

- You track constant setpoints,
- You reject constant disturbances,
- The 2% settling time is 4 seconds, and
- There is no overshoot for a step input.

1) Give the control law (Kx and Kz) and explain how you chose Q and R

Create the augmented system (plant plus servo compensator - same as homework #6)



The plant plus servo compensator

$$s\begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \\ z \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & -6.533 & 0 & 0 & 0 \\ 0 & 16.333 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0.333 \\ -0.333 \\ 0 \end{bmatrix}F + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{bmatrix}R$$

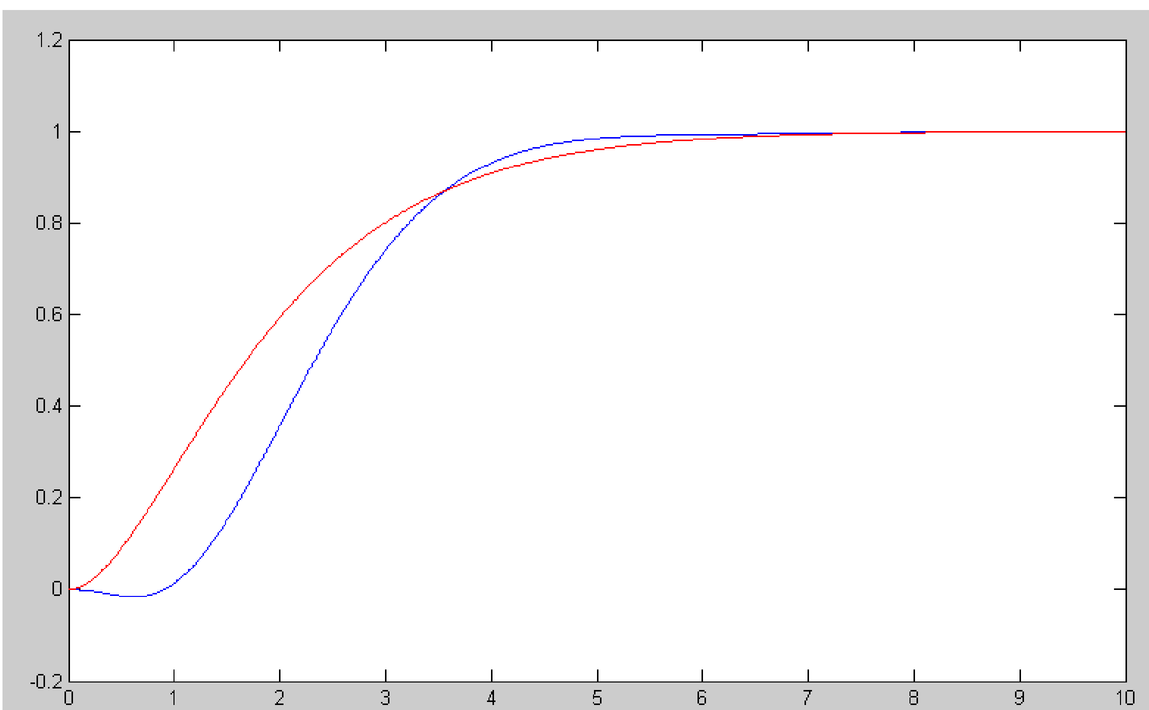
Now use LQR methods to force the system to behave like

$$G_d = \left(\frac{1}{s^2+2s+1}\right)$$

```
A5 = [A, zeros(4,1) ; C, 0]

         0         0    1.0000         0         0
         0         0         0    1.0000         0
         0   -7.0000         0         0         0
   -7.8400         0         0         0         0
    1.0000         0         0         0         0

B5u = [B; 0];
B5r = [0*B ; -1];
C5 = [C, 0];
D5 = 0;

t = [0:0.01:10]';
Gd = tf(1,[1,2,1]);
Yd = step(Gd,t);
```

After some trial and error, a reasonable Q and R were:

```
K5 = lqr(A5, B5u, diag([100,0,0,0,100]),1);
y = step3(A5-B5u*K5, B5r, C5, D5, t, X0, Ref);
plot(t,y,'b',t,Yd,'r');
```



so the resulting feedback gains are:

```
K5 =  -24.8289 -204.2766  -25.8237  -58.6530  -10.0000
```

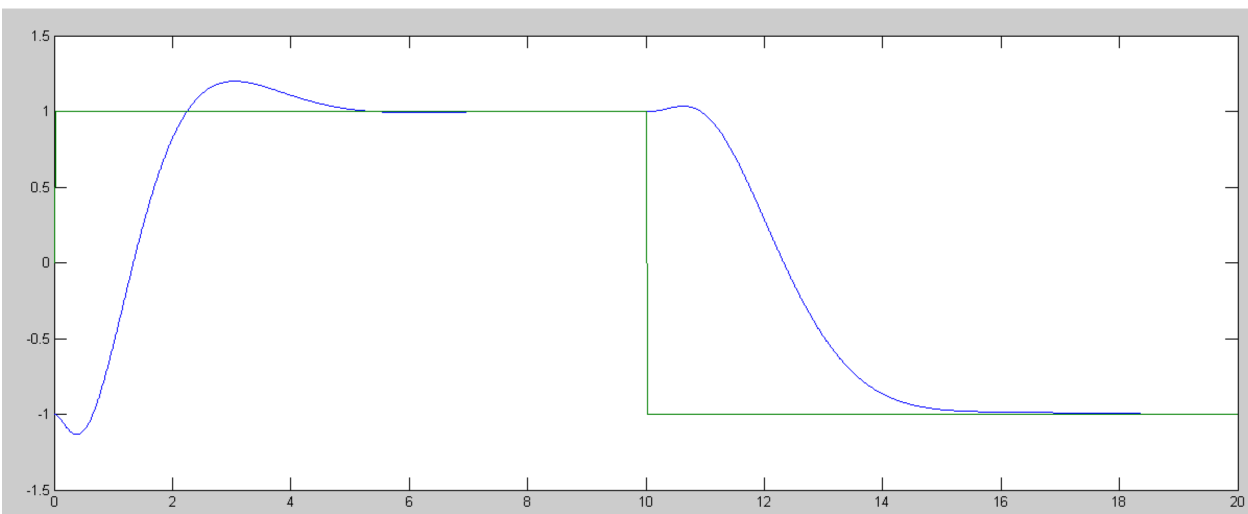
```
>> eig(A5-B5u*K5)

ans =

  -4.0250 + 0.1594i
  -4.0250 - 0.1594i
  -0.9166
  -0.9827 + 1.1083i
  -0.9827 - 1.1083i
```

2) Plot the step response of the linear system

   see problem #1

   done while itterating to find Q and R

3) Check your design with the nonlinear simulation of the cart and pendulum system.



```
% Cart and Pendulum (sp20 version)
X = [-1; 0 ; 0 ; 0];
dX = zeros(4,1);
Ref = 1;
dt = 0.01;
U = 0;
t = 0;
Kx = [ -24.8289 -204.2766  -25.8237  -58.6530];
Kz = -10;
Z = 0;

y = [];
while(t < 20)
 Ref = sign(sin(0.1*pi*t));
 U = -Kz*Z - Kx*X;

 dX = CartDynamics(X, U);
 dZ = X(1) - Ref;

 X = X + dX * dt;
 Z = Z + dZ * dt;

  t = t + dt;
 CartDisplay(X, Ref);
 y = [y; X(1), Ref];
end

clf
t = [1:length(y)]' * 0.01;
plot(t, y);
```

Ball and Beam (HW #6): Use LQG methods to design a full-state feedback control law of the form

$$U = -K_z Z - K_x X$$

$$\dot{Z} = (x - R)$$

for the ball and beam system from homework #5 using LQG control so that

- You track constant setpoints,
- You reject constant disturbances,
- The 2% settling time is 4 seconds, and
- There is no overshoot for a step input.

4) Give the control law (Kx and Kx) and explain how you chose Q and R

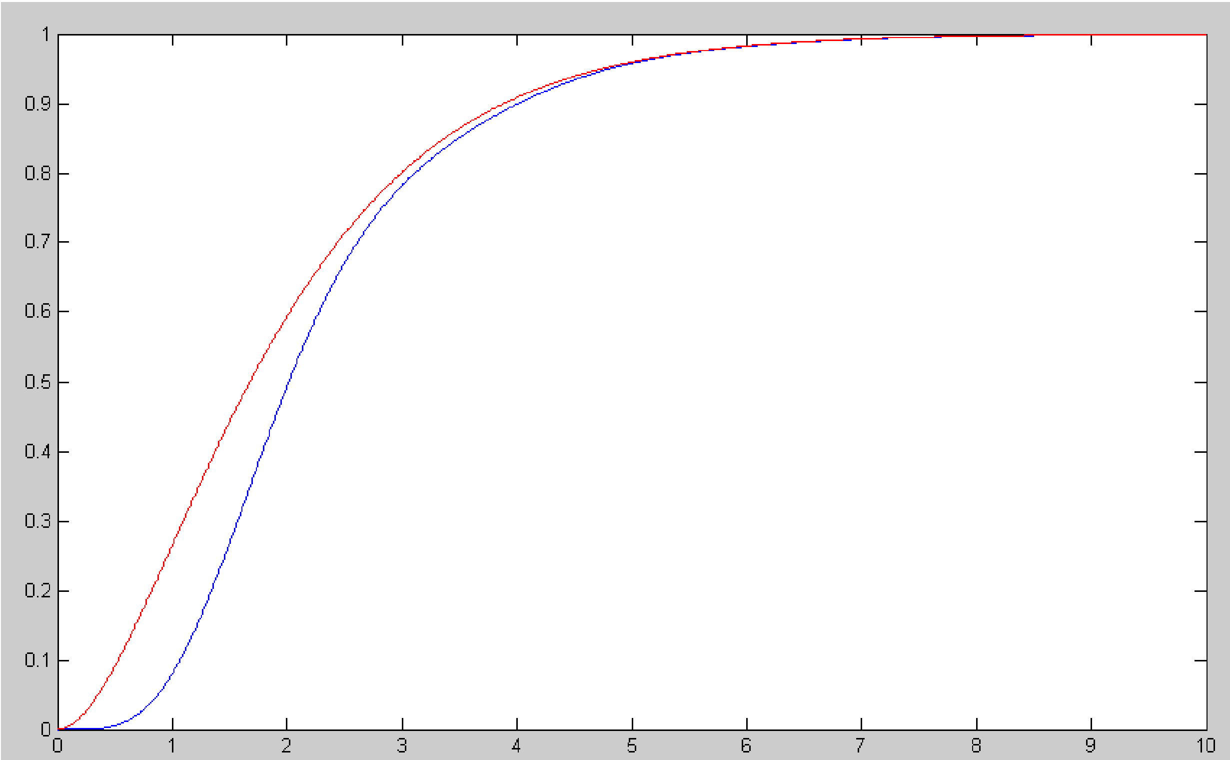
The augmented system in this case is

$$s\begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \\ \cdots \\ z \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 & \vdots & 0 \\ 0 & 0 & 0 & 1 & \vdots & 0 \\ 0 & -7 & 0 & 0 & \vdots & 0 \\ -7.84 & 0 & 0 & 0 & \vdots & 0 \\ \cdots & \cdots & \cdots & \cdots & & \cdots \\ 1 & 0 & 0 & 0 & \vdots & 0 \end{bmatrix}\begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \\ \cdots \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.4 \\ \\ 0 \end{bmatrix} U + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \\ -1 \end{bmatrix} R$$

Repeating the previous trial-and-error for guessing Q to find a 'good' response

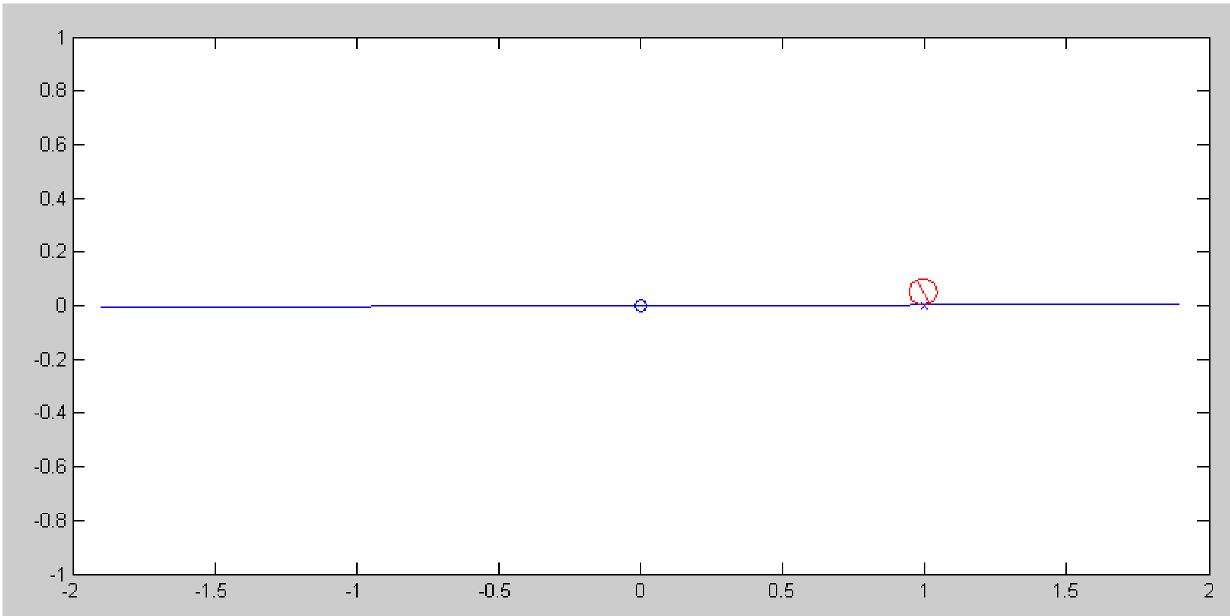
```
K5 = lqr(A5, B5, diag([100,0,100,100,300]),1);
y = step3(A5-B5*K5, B5r, C5, D5, t, X0, Ref);
plot(t,y,'b',t,Yd,'r');
eig(A5-B5*K5)

K5 =  -59.6985   93.4079  -32.4393   23.8126  -17.3205

eig(A5-B5*K5)

  -4.7115
  -1.1838 + 2.4747i
  -1.1838 - 2.4747i
  -1.5806
  -0.8654
```

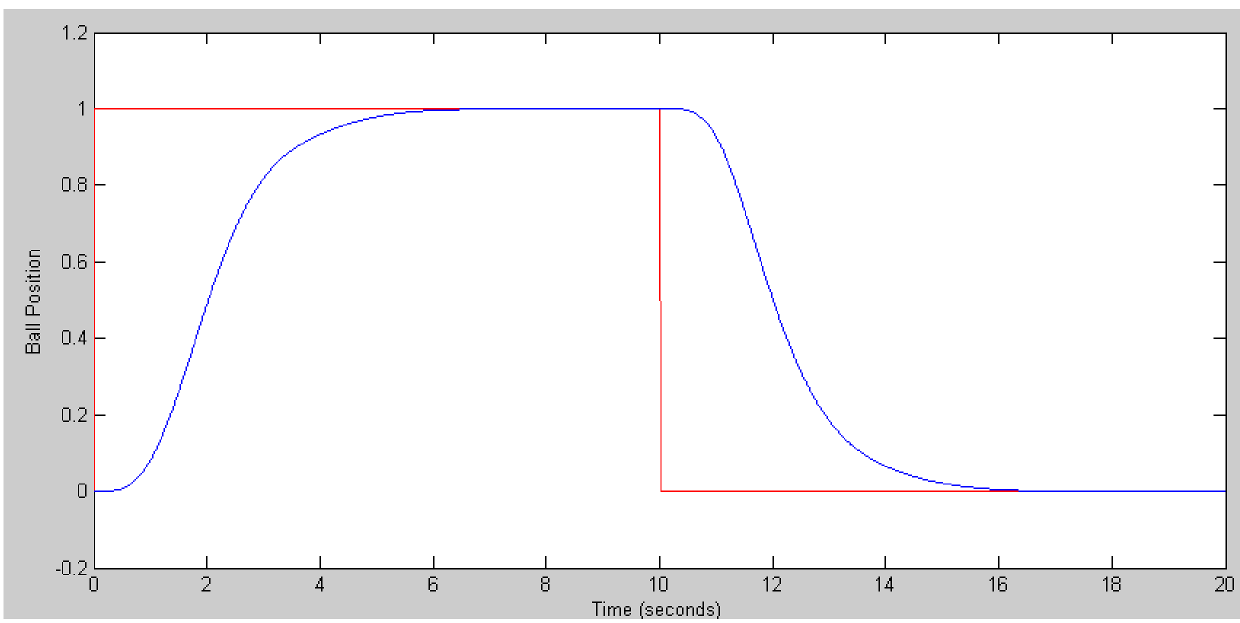
5) Plot the step response of the linear system



1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
1
2
3
4
5
6
7
8
9
10

6) Check your design with the nonlinear simulation of the cart and pendulum system.





Code:

```
% Ball & Beam System
% Sp 20 Version
% m = 2kg
% J = 0.5 kg m^2

X = [0, 0, 0, 0]';
dt = 0.01;
t = 0;
Kx = [-59.6985   93.4079  -32.4393   23.8126 ];
Kz = [ -17.3205];
Z = 0;

y = [];

while(t < 20)
 Ref = 1 * (sin(2*pi*t/20) > 0);

 U = -Kz*Z - Kx*X;

 dX = BeamDynamics(X, U);
 dZ = (X(1) - Ref);

 X = X + dX * dt;
 Z = Z + dZ * dt;

 y = [y ; Ref, X(1)];

 t = t + dt;
 BeamDisplay(X, Ref);
 end

t = [1:length(y)]' * dt;


plot(t,y(:,1),'r',t,y(:,2),'b');
xlabel('Time (seconds)');
ylabel('Ball Position');
```