ECE 463/663 - Homework #10

Kalman Filters, LQG/LTR Control. Due Monday, April 27th

Kalman Filters

Cart and Pendulum (HW #5): Use a previously deisgn control law for the cart and pendulum system. Add noise to the system as

$$s\begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -6.533 & 0 & 0 \\ 0 & 16.333 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0.333 \\ -0.333 \end{bmatrix}(F+n_u)$$

$$y = x + n_y$$

where there is Gaussian noise at the input and output

$n_u \sim N(0, 0.02^2)$ *mean zero, standard deviation 0.02*

$n_y \sim N(0, 0.01^2)$ *mean zero, standard deviation 0.01*

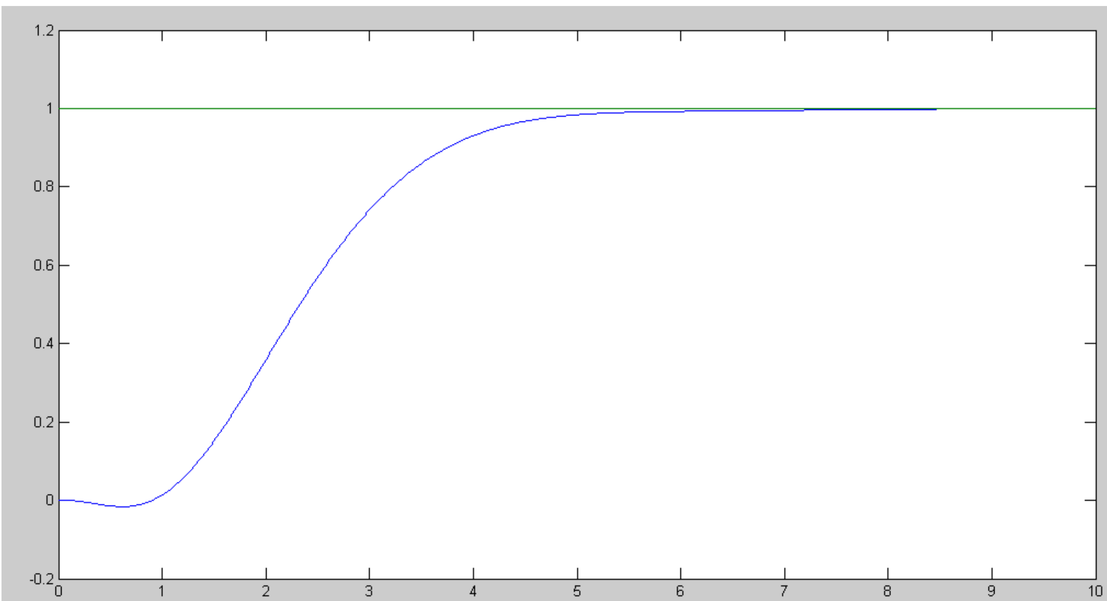
1) Use a servo-compensator to force the DC gain to one (i.e. use the servo compensator from homework set #9).

Plot the step response

- Without noise (same as homework set #9)
- With noise

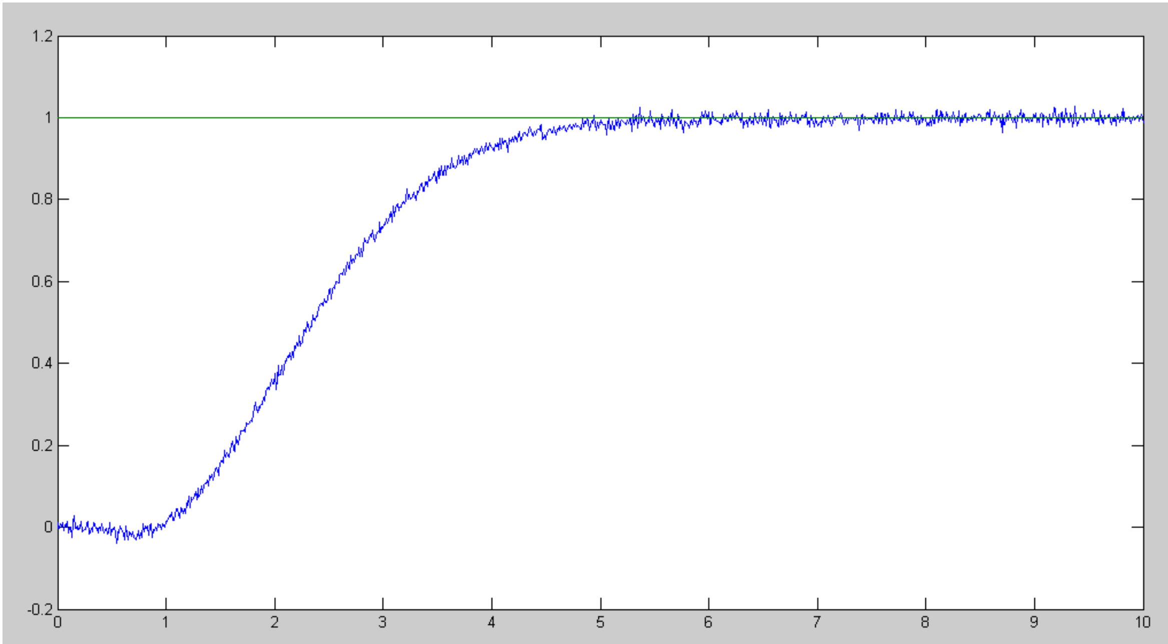
```
A = [0,0,1,0;0,0,0,1;0,-6.533,0,0;0,16.333,0,0]

         0         0    1.0000         0
         0         0         0    1.0000
         0   -6.5330         0         0
         0   16.3330         0         0

B = [0;0;0.333;-0.333]

         0
         0
    0.3330
   -0.3330

C = [1,0,0,0]

     1     0     0     0

A5 = [A, zeros(4,1) ; C, 0]

         0         0    1.0000         0         0
         0         0         0    1.0000         0
         0   -6.5330         0         0         0
         0   16.3330         0         0         0
    1.0000         0         0         0         0
```

```
B5r = [0*B;-1];
B5u = [  B; 0];
B5y = [0*B; 1];

K5 = lqr(A5, B5, diag([100,0,0,0,100]),1);
C5 = [C,0];
D5 = 0;

X0 = zeros(5,1);
t = [0:0.01:10]';

Ref = 0*t+1;

Nu = randn(size(t)) * 0.02;
Ny = randn(size(t)) * 0.01;

y = step3(A5-B5*K5,[B5r,B5u,B5y],C5,[0,0,0],t,X0,[Ref,Nu*0,Ny*0]);
plot(t,y,t,Ref)
```



No Noise

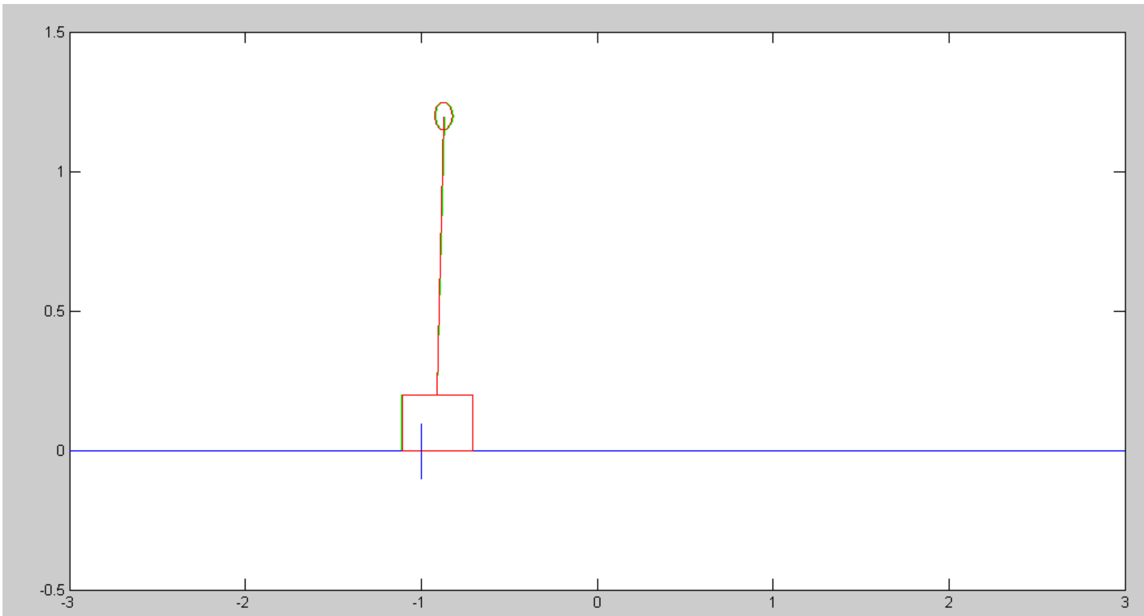
```
y = step3(A5-B5*K5,[B5r,B5u,B5y],C5,[0,0,1],t,X0,[Ref,Nu,Ny]);
plot(t,y,t,Ref)
```

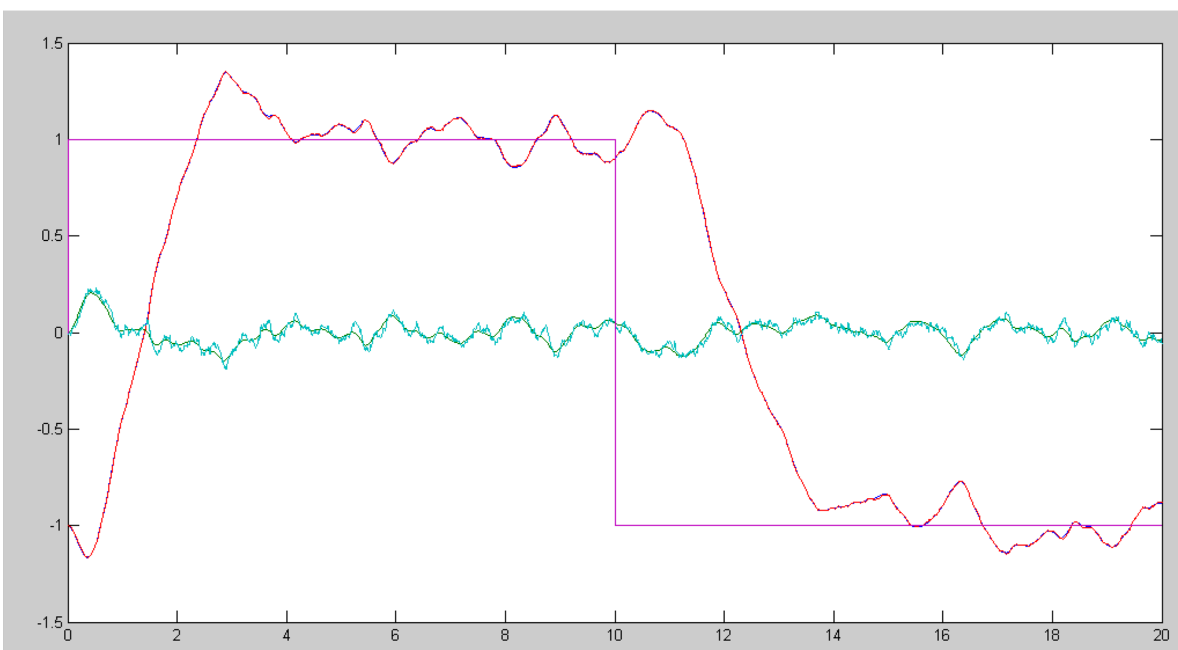


With noise

2) Design a full-order observer using pole-placement to place the observer poles at {-3, -4, -5, -6}

```
H = ppl(A',C',[-3,-4,-5,-6])'

   18.0000
  -97.3510
  135.3330
 -393.4477
```

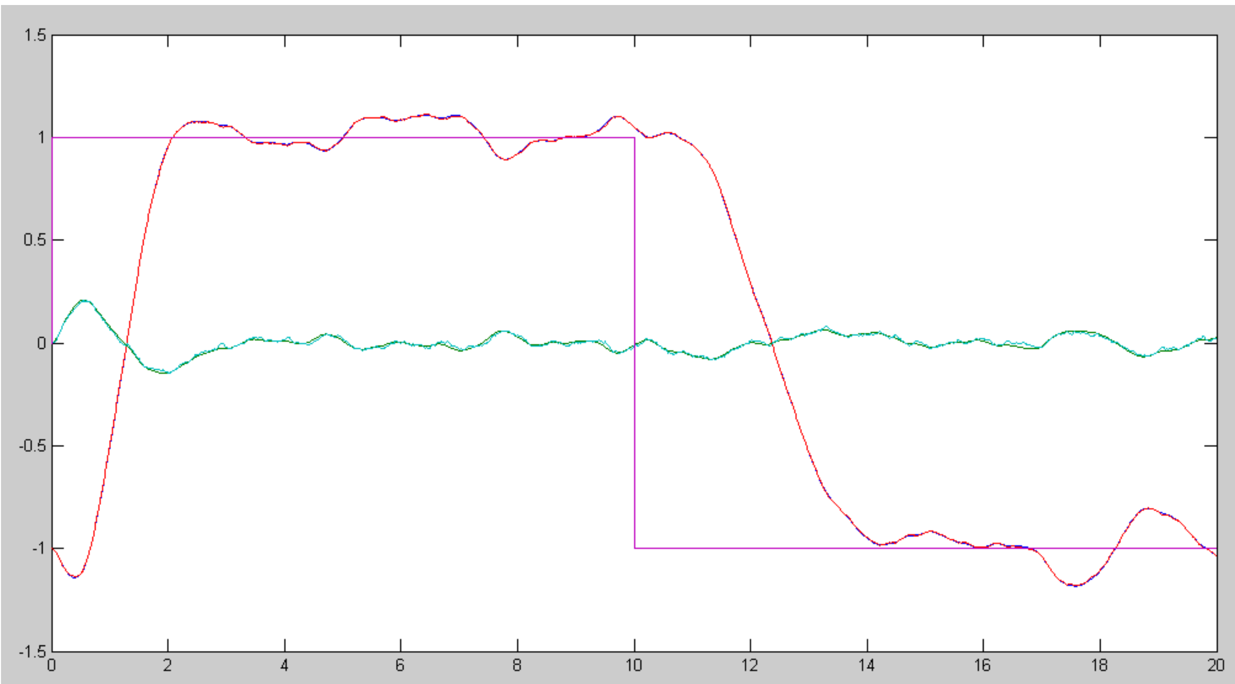




3) Design a Kalman filter (i.e. a full-order observer with a specific Q and R)

- Simulate the response of the cart with noise added at the input and output.
- Plot the states of the plant and the observer with noise,.

```
Q = F*F' * 0.02^2

  1.0e-004 *

         0         0         0         0
         0         0         0         0
         0         0    0.4436   -0.4436
         0         0   -0.4436    0.4436

R = 0.01^2;
H = lqr(A', C', Q, R)'

    8.9828
  -25.2052
   40.3458
 -101.8666
```



Matlab Code

```
% Cart and Pendulum (sp20 version)
X = [-1; 0 ; 0 ; 0];
dX = zeros(4,1);
Ref = 1;
dt = 0.01;
U = 0;
t = 0;
Kx = [ -24.8289 -204.2766  -25.8237  -58.6530];
Kz = -10;
Z = 0;

% H with pole placement
% H = [   18.0000  -97.3510  135.3330 -393.4477 ]';
% H with LQR (Kalman Filter)
H = [    8.9828  -25.2052   40.3458 -101.8666]';

Ao = [0,0,1,0;0,0,0,1;0,-6.533,0,0;0,16.333,0,0];
Bo = [0;0;0.333;-0.333];
Co = [1,0,0,0];
Xo = X;
Yd = [];

y = [];
while(t < 20)
 Ref = sign(sin(0.1*pi*t));
 U = -Kz*Z - Kx*Xo;
 y = X(1) + 0.01*randn;
 yo = Xo(1);

 dX = CartDynamics(X, U + 0.02*randn);
 dZ = X(1) - Ref;
 dXo = Ao*Xo + Bo*U + H*(y - yo);

 X = X + dX * dt;
 Z = Z + dZ * dt;
 Xo = Xo + dXo * dt;


  t = t + dt;
 CartDisplay3(X, Xo, Ref);
 Yd = [Yd; X(1), X(2), Xo(1), Xo(2), Ref];
end

clf
t = [1:length(Yd)]' * 0.01;
plot(t, Yd);
```

LQG / LTR

4) Design a control law so that the cart and pendulum behaves like the following reference model:

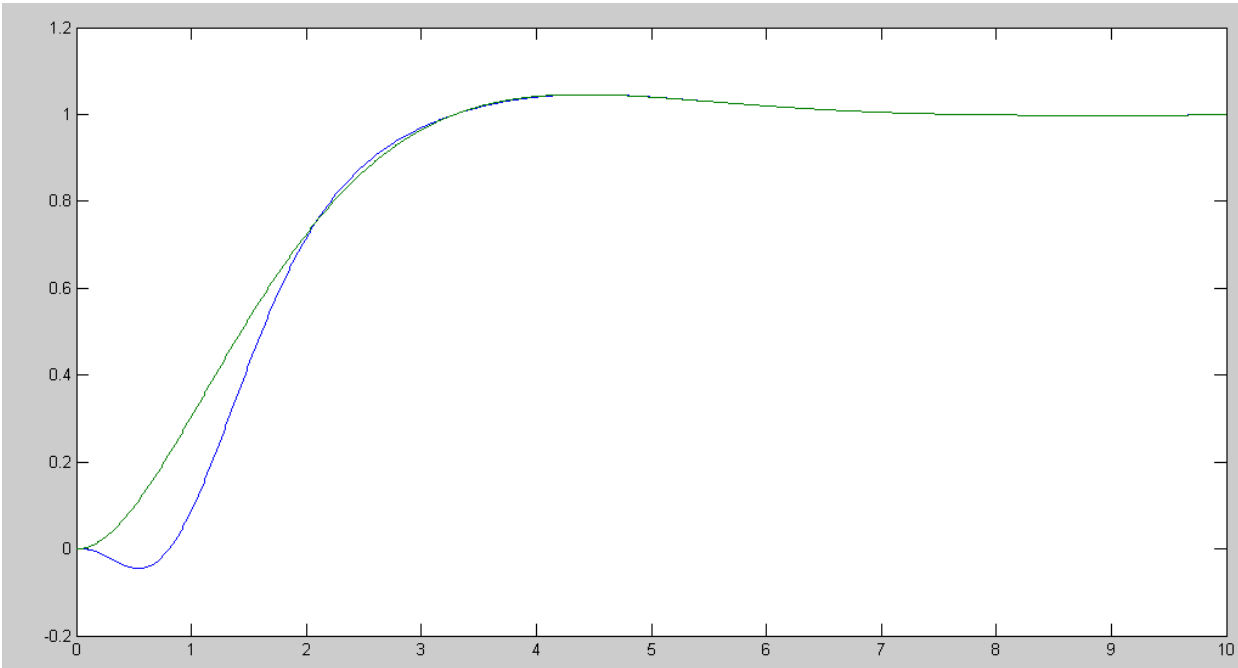
$$y_m = \left(\frac{1}{s^2+1.4s+1}\right) R$$

4a) Give a block diagram for your controller

4b) Give the resulting control law

```
Am = [0,1;-1,-1.4];
Bm = [0;1];
Cm = [1,0];

A7 = [A, zeros(4,1),zeros(4,2) ; C,0,-Cm ; zeros(2,4),zeros(2,1),Am]

         0         0    1.0000         0         0         0         0
         0         0         0    1.0000         0         0         0
         0   -6.5330         0         0         0         0         0
         0   16.3330         0         0         0         0         0
    1.0000         0         0         0         0   -1.0000         0
         0         0         0         0         0         0    1.0000
         0         0         0         0         0   -1.0000   -1.4000

B7u = [B;zeros(3,1)];
B7r = [zeros(5,1);Bm];


K7 = lqr(A7,B7u,diag([0,0,0,0,1e4,0,0]),1);

                 Kx                          Kz              Kxm
 -129.3873 -410.8648  -83.7054 -124.8218 -100.0000  106.7014   43.5891

eig(A7-B7u*K7)

  -1.5687 + 2.4197i
  -1.5687 - 2.4197i
  -4.4881
  -3.7084
  -2.3578
  -0.7000 + 0.7141i
  -0.7000 - 0.7141i

C7 = [1,0,0,0,0,0,0;0,0,0,0,0,1,0];
D7 = [0;0];
X0 = zeros(7,1);
t = [0:0.01:10]';
Ref = 0*t+1;
y = step3(A7-B7u*K7,B7r,C7,D7,t,X0,Ref);
plot(t,y)
```

4c) Plot the step response of the model and the linearlized plant for yor control law.


1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
1
2
3
4
5
6
7
8
9
10