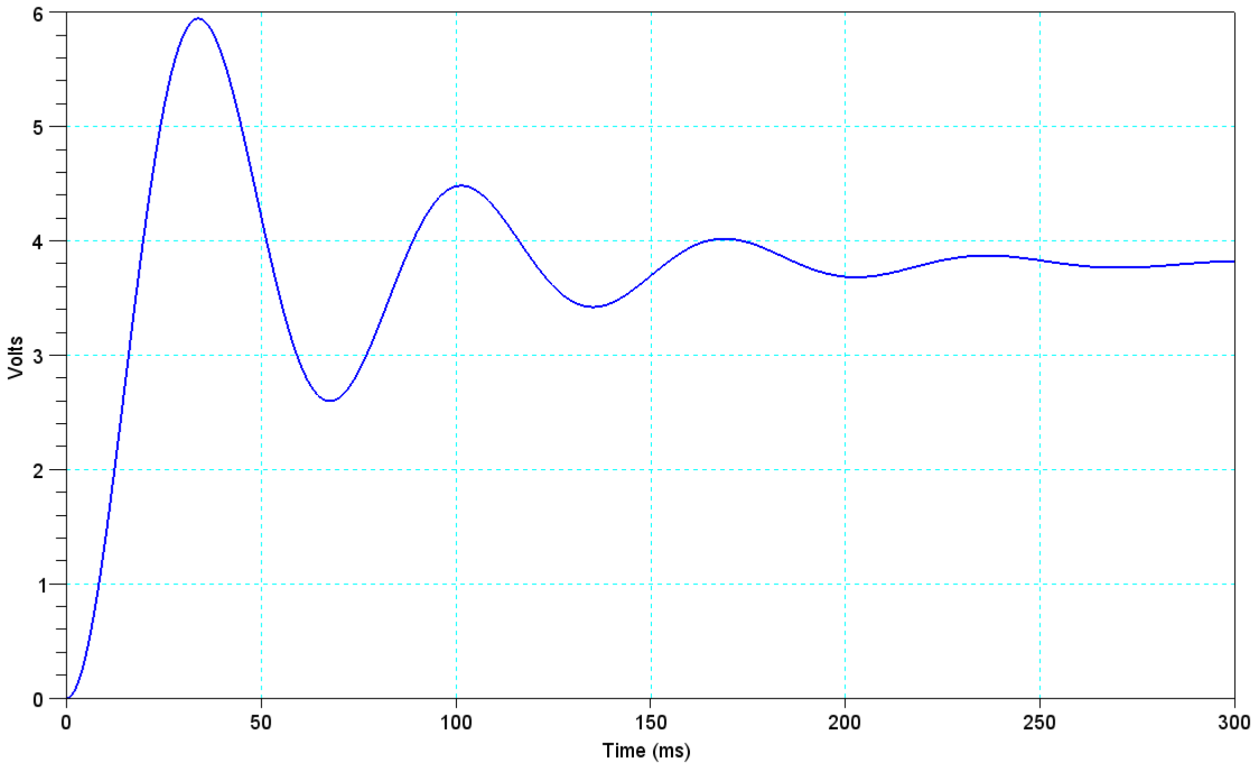
ECE 463/663: Test #1.   Name ______________

Spring 2020

1) Find the transfer funciton for a system with the following step response



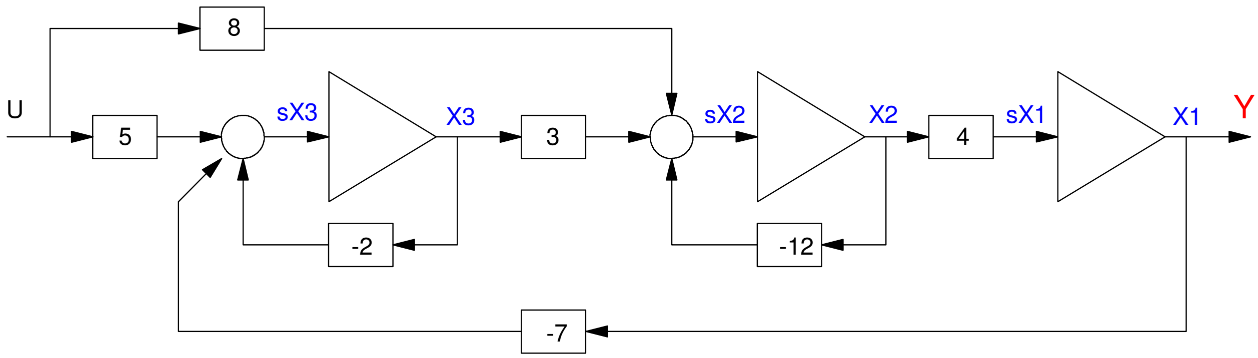
DC gain = 3.9

$$\omega = \left(\frac{2 \text{ cycles}}{130ms}\right) 2\pi = 96.66 \frac{rad}{\text{sec}}$$

$$\sigma \approx \frac{4}{250ms \text{ settling time}} = 16$$

$$G(s) \approx \left(\frac{36{,}480}{(s+16+j96.66)(s+16-j96.66)}\right)$$

2) Give the state-space model for the following system

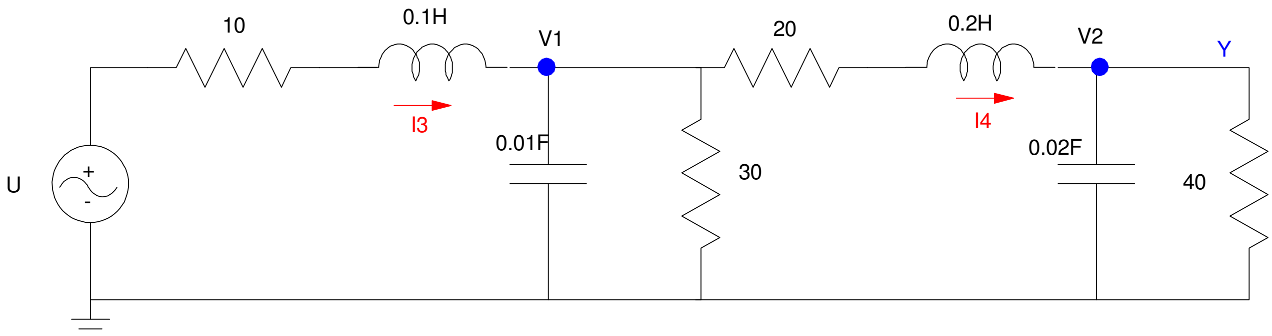


$$\begin{bmatrix} sX1 \\ sX2 \\ sX3 \end{bmatrix} = \begin{bmatrix} 0 & 4 & 0 \\ 0 & -12 & 3 \\ -7 & 0 & -2 \end{bmatrix} \begin{bmatrix} X1 \\ X2 \\ X3 \end{bmatrix} + \begin{bmatrix} 0 \\ 8 \\ 5 \end{bmatrix} U$$

$$Y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} X1 \\ X2 \\ X3 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix}$$

Problem 3) (work either this problem or the mass-spring problem)

3a) Write four coupled differential equations to describe the following circuit



$$0.1\dot{V}_1 = I_3 - I_4 - \frac{V_1}{30}$$

$$0.02\dot{V}_2 = I_4 - \frac{V_2}{40}$$

$$0.1\dot{I}_3 = U - 10I_3 - V_1$$

$$0.2\dot{I}_2 = V_1 - 20I_4 - V_2$$

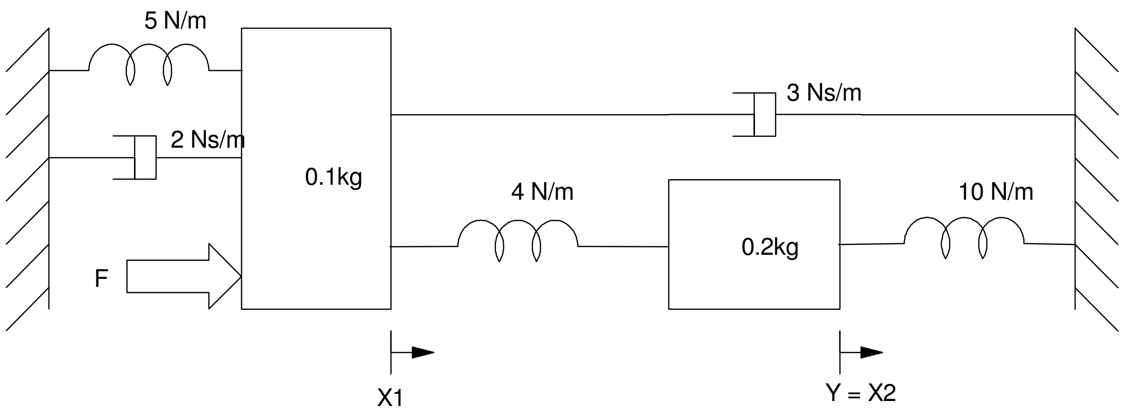
3b) Express these dynamics in state-space form

$$\begin{bmatrix} sV1 \\ sV2 \\ sI3 \\ sI4 \end{bmatrix} = \begin{bmatrix} -3.33 & 0 & 100 & -100 \\ 0 & -1.25 & 0 & 50 \\ -10 & 0 & 100 & 0 \\ 5 & -5 & 0 & -100 \end{bmatrix} \begin{bmatrix} V1 \\ V2 \\ I3 \\ I4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 10 \\ 0 \end{bmatrix} Vin$$

$$Y = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} V1 \\ V2 \\ I3 \\ I4 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} Vin$$

Problem 3) (work either this problem or the circuit problem)

3a) Write two coupled 2nd-order differential equations to describe the following mass spring system



$$0.1\ddot{x}_1 + 5\dot{x}_1 + 9x_1 - 4x_2 = F$$

$$0.2\ddot{x}_2 + 14x_2 - 4x_1 = 0$$

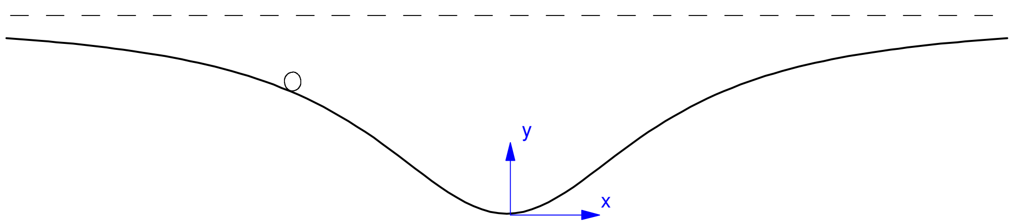
3b) Express these dynamics in state-space form

$$s\begin{bmatrix} X1 \\ X2 \\ sX1 \\ sX2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -90 & 40 & -50 & 0 \\ 20 & -70 & 0 & 0 \end{bmatrix} \begin{bmatrix} X1 \\ X2 \\ sX1 \\ sX2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 10 \\ 0 \end{bmatrix} F$$

$$Y = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} X1 \\ X2 \\ sX1 \\ sX2 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} F$$

4) A ball with a mass of 1kg is rolling in a bowl with the shape

$$y = 2 - \frac{2}{1+x^2} \qquad \text{note: } \frac{d}{dt}\left(\frac{1}{f(t)}\right) = \frac{-1}{f^2(t)} \cdot \frac{df}{dt}$$



Dettermine the potential and kinetic energy of the ball in terms of x:

4a) $PE = mgy = f(x)$

$$PE = g\left(2 - \frac{2}{1+x^2}\right)$$

4b) $KE = 0.7m\left(\dot{x}^2 + \dot{y}^2\right) = f(x, \dot{x})$

$$y = 2 - \frac{2}{1-x^2}$$

$$\dot{y} = -\left(1 - x^2\right)^{-2}(-2)(-2x\dot{x})$$

$$KE = 0.7m\left(\dot{x}^2 - \left(\frac{4x\dot{x}}{\left(1-x^2\right)^2}\right)^2\right)$$

$$KE = 0.7\left(1 - \frac{16x^2}{\left(1+x^2\right)^2}\right)\dot{x}^2$$

5) Assume the LaGrangian is:

$$L = 0.1\dot{\theta}^2 + 0.7\dot{x}^2 + 0.5x^2\dot{\theta}^2 - gx\sin\theta$$

Determine

$$T = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \left(\frac{\partial L}{\partial \theta}\right)$$

$$T = \frac{d}{dt}\left(0.2\dot{\theta} + x^2\dot{\theta}\right) - (-gx\cos\theta)$$

$$\boxed{T = 0.2\ddot{\theta} + 2x\dot{x}\dot{\theta} + x^2\ddot{\theta} + gx\cos\theta}$$

Green New Deal Bonus! One proposal for energy storage is to lift 2000lb rocks 10m in the air (to store energy) and then lower them (to recover the energy). How many 2000lb rocks would you need to produce 100kWh? (the energy used by a single residential house in one day)

$$PE = mgh = (2000lb)\left(9.8\frac{m}{s^2}\right)(10m)$$

$$PE = mgh = (907.81kg)\left(9.8\frac{m}{s^2}\right)(10m) = 88,904 \text{ Joules}$$

$$(88,904\ W\cdot s)\left(\frac{1hr}{3600s}\right)\left(\frac{1kW}{1000W}\right) = 0.02469kWh$$

$$N = \frac{100kWh}{0.02469kWh/rock} = 4,049 \text{ rocks}$$ per household

SWINGING STONES

This gravity-powered battery could be the future of energy storage

By MATTHEW MARANI • November 12, 2018

Energy Vault's storage tower consists of a six-craned tower capable of storing 35 MWh. (Courtesy Energy Vault)