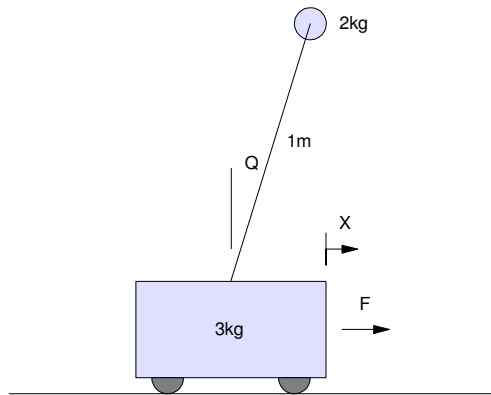


# ECE 463/663 - Test #2: Name \_\_\_\_\_

Due midnight Monday, March 30th. Individual Effort Only (no working in groups)

No Aid Given, Received, or Observed: (sign if possible) \_\_\_\_\_



The dynamics for an inverted pendulum system (homework #4) is:

$$\begin{bmatrix} 5 & 2 \cos \theta \\ 2 \cos \theta & 2 \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 2 \sin(\theta) \dot{\theta}^2 \\ 2g \sin \theta \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} F + \begin{bmatrix} 1 \\ 1 \end{bmatrix} d$$

The linearized dynamics with a disturbance (d) are

$$s \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -6.533 & 0 & 0 \\ 0 & 16.333 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0.333 \\ -0.333 \end{bmatrix} F + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.5 \end{bmatrix} d$$

## Feedback Controller Design

1) Design a servo compensator to

- Track a sinusoidal set point

$$R = \sin(0.5t)$$

- Reject a constant disturbance

$$d = 1$$

Give the resulting state-space matrices {A, B, C, D} for the closed-loop system.

Step 1: Add a servo compensator with poles at {0, +j0.5, -j0.5}

$$sZ = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0.5 \\ 0 & -0.5 & 0 \end{bmatrix} Z + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} (x - R)$$

Create the augmented system (plant plus servo)

$$s \begin{bmatrix} X \\ \dots \\ Z \end{bmatrix} = \begin{bmatrix} A & \vdots & 0 \\ \dots & \dots & \dots \\ B_z C & \vdots & A_z \end{bmatrix} \begin{bmatrix} X \\ \dots \\ Z \end{bmatrix} + \begin{bmatrix} B \\ \dots \\ 0 \end{bmatrix} U + \begin{bmatrix} 0 \\ \dots \\ -B_z \end{bmatrix} R$$

Find full-state feedback gains to stabilize the plant and servo

Input the plant

$$\begin{aligned} A &= [0, 0, 1, 0; 0, 0, 0, 1; 0, -6.533, 0, 0; 0, 16.333, 0, 0]; \\ B &= [0; 0; 0.333; -0.333]; \\ C &= [1, 0, 0, 0]; \\ B_d &= [0; 0; 0; 0.5]; \end{aligned}$$

Input the servo compensator

$$\begin{aligned} A_z &= [0, 0, 0; 0, 0, 0.5; 0, -0.5, 0] \\ B_z &= [1; 1; 1]; \end{aligned}$$

Create the augmented (7x7) system

$$A_7 = [A, \text{zeros}(4, 3); B_z * C, A_z]$$

$$\begin{bmatrix} 0 & 0 & 1.0000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0000 & 0 & 0 & 0 \\ 0 & -6.5330 & 0 & 0 & 0 & 0 & 0 \\ 0 & 16.3330 & 0 & 0 & 0 & 0 & 0 \\ 1.0000 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1.0000 & 0 & 0 & 0 & 0 & 0 & 0.5000 \\ 1.0000 & 0 & 0 & 0 & 0 & -0.5000 & 0 \end{bmatrix}$$

$$B_7 u = [B; 0 * B_z]$$

$$\begin{bmatrix} 0 \\ 0 \\ 0.3330 \\ -0.3330 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

```
B7r = [0*B;-Bz]
```

```
0  
0  
0  
0  
-1  
-1  
-1
```

```
t = [0:0.01:20]';
```

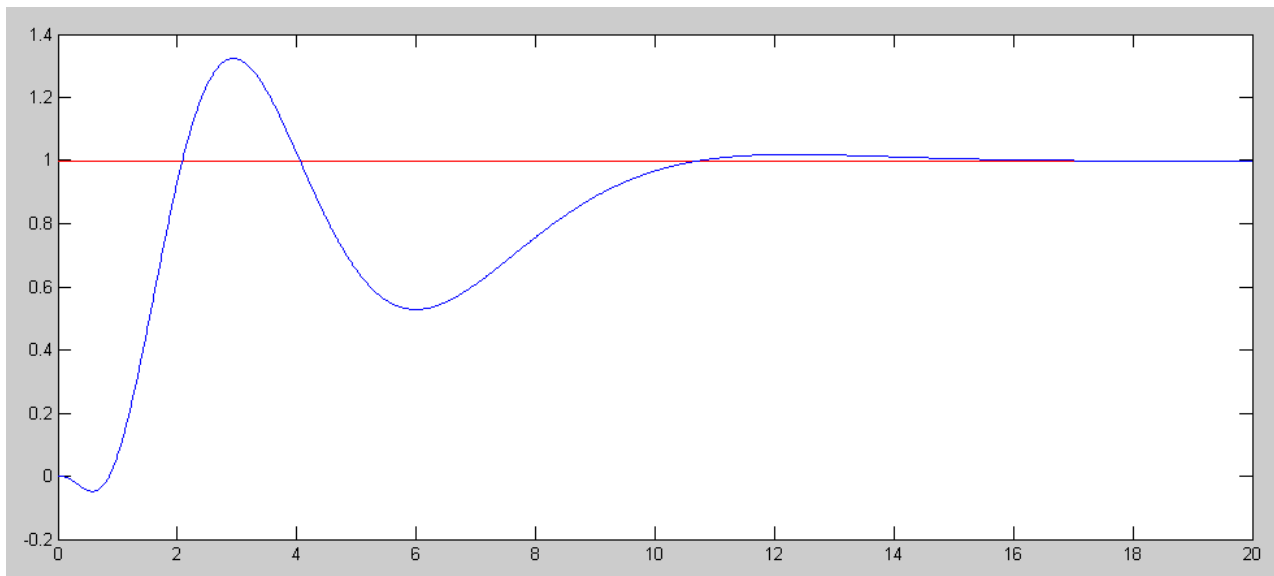
```
K7 = ppl(A7, B7u, [-0.5+j*0.5,-0.5-j*0.5,-1,-1+j,-1-j,-4,-4.1])
```

```
-54.1610 -271.5274 -43.9111 -80.2474 -20.1017 -17.7229 0.3642
```

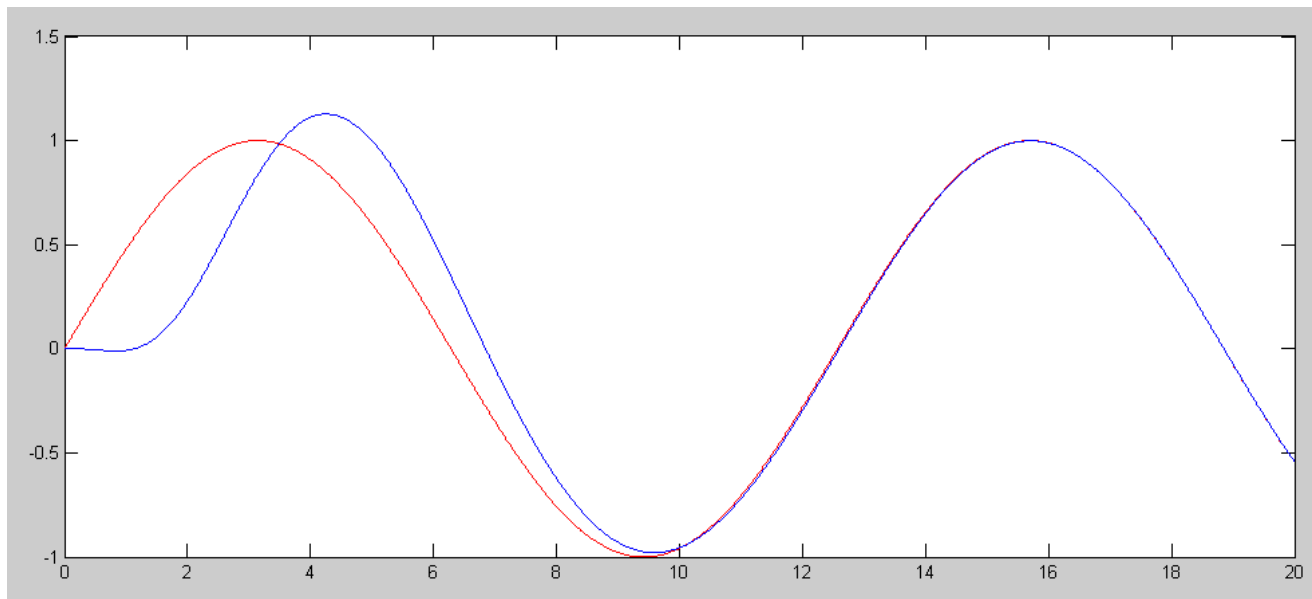
**note: The pole location is somewhat arbitrary. Various pole locations were tried until the gains were reasonable (took some time)**

## 2) Verify your control law on the linearized system

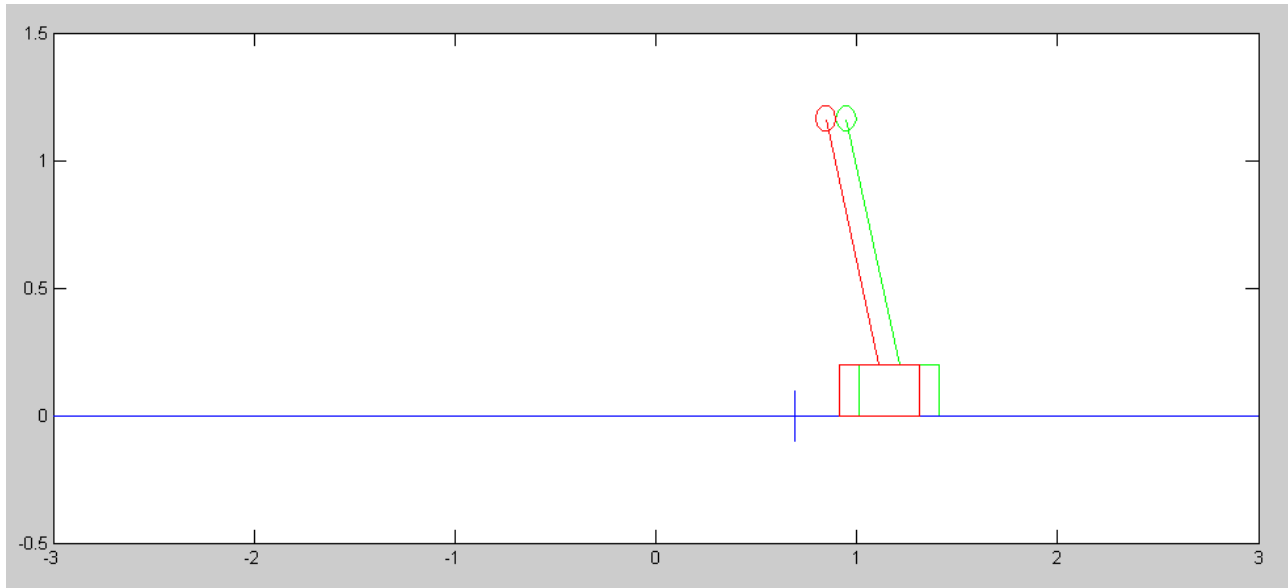
```
>> R = 0*t+1;  
>> y = step3(A7-B7u*K7, B7r, C7, D7, t, X0, R);  
>> plot(t,R,'r',t,y,'b');
```



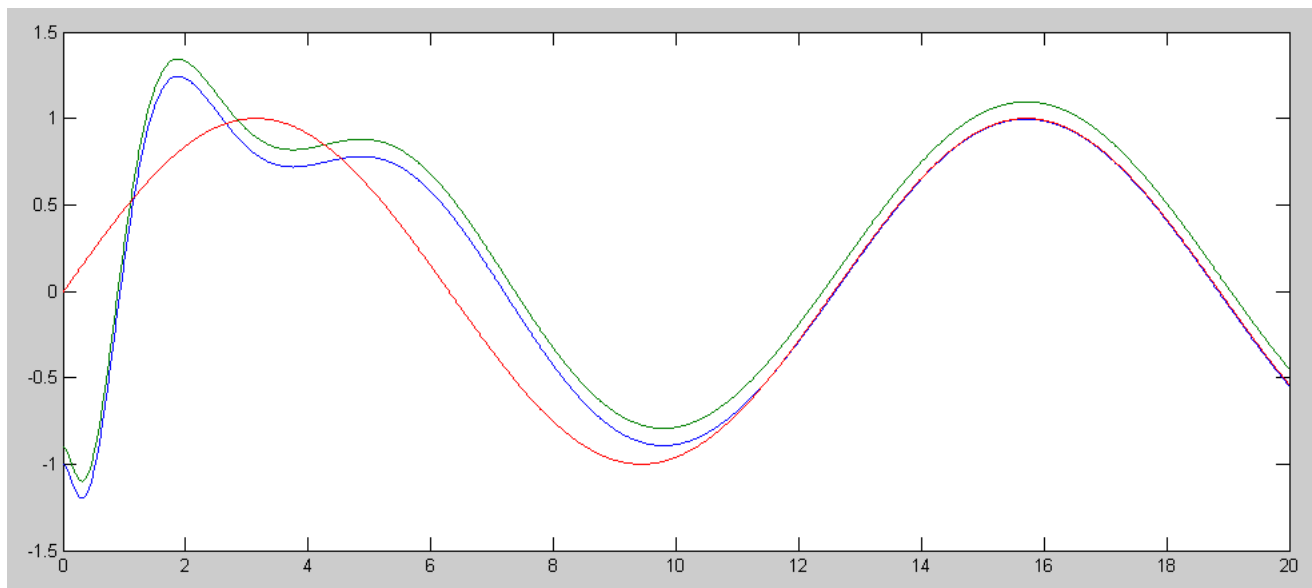
```
>> R = sin(0.5*t);  
>> y = step3(A7-B7u*K7, B7r, C7, D7, t, X0, R);  
>> plot(t,R,'r',t,y,'b');
```



3) Verify your control law on the nonlinear simulation (assume all states are measured).



Dynamic simulation of an inverted pendulum (green = observer estimate)



Plant (blue), Set Point (red), and Observer (not used yet - green)

## Code:

```
% Cart and Pendulum (sp20 version)
% Full Order Observer
X = [-1; 0 ; 0 ; 0];
Xo = X;
dX = zeros(4,1);
Ref = 1;
dt = 0.01;
U = 0;
t = 0;
Kx = [ -54.1610 -271.5274 -43.9111 -80.2474];
Kz = [-20.1017 -17.7229 0.3642];
% disturbance
Bd = [0;0;0;0.5];

% Servo Comp
Az = [0,0,0 ; 0,0,0.5 ; 0,-0.5,0];
Bz = [1;1;1];
Z = zeros(3,1);

y = [];
while(t < 20)
    d = 1;
    Ref = sin(0.5*t);
    U = -Kz*Z - Kx*X;

    dX = CartDynamics(X, U) + Bd*d;
    dZ = Az*Z + Bz*(X(1) - Ref);

    X = X + dX * dt;
    Z = Z + dZ * dt;

    % Observer. Cheating for now
    Xo = X + [0.1;0;0;0];

    t = t + dt;
    CartDisplay3(X, Xo, Ref);
    y = [y; X(1), Xo(1), Ref];
end

clf
t = [1:length(y)]' * 0.01;
plot(t, y);
```

## Observer Design

Assume you can measure both position and angle.

4) Design a full-order observer to estimate the states and the disturbance,  $d$ . Feed back the state estimates rather than the actual states.

$$s \begin{bmatrix} X_o \\ d_o \end{bmatrix} = \begin{bmatrix} A & B_d \\ 0 & 0 \end{bmatrix} \begin{bmatrix} X_o \\ d_o \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} U$$

$A_o = [A, B_d ; \text{zeros}(1,5)]$

```

0         0         1.0000         0         0
0         0         0         1.0000         0
0        -6.5330         0         0         0
0        16.3330         0         0         0.5000
0         0         0         0         0
    
```

$B_o = [B;0]$

```

0
0
0.3330
-0.3330
0
    
```

$C_o = [C,0]$

```

1     0     0     0     0
    
```

$H = \text{pp1}(A_o', C_o', [-2, -2.1, -2.2, -2.4, -2.6])'$

```

11.3000
-45.8005
67.2930
-187.9365
-17.6512
    
```

Give the state-space matrices for the resulting plant, servo compensator, observer, and full-state feedback  $\{A, B, C, D\}$

Open-Loop (no observer gains or feedback gains)

$$s \begin{bmatrix} X \\ d \\ Z \\ X_o \\ d_o \end{bmatrix} = \begin{bmatrix} A & B_d & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ B_z C & 0 & A_z & 0 & 0 \\ 0 & 0 & 0 & A & B_d \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} X \\ d \\ Z \\ X_o \\ d_o \end{bmatrix} + \begin{bmatrix} B \\ 0 \\ 0 \\ B \\ 0 \end{bmatrix} U + \begin{bmatrix} 0 \\ 0 \\ -B_z \\ 0 \\ 0 \end{bmatrix} R$$

Closed-Loop (with H, Kz, Kx)

$$s \begin{bmatrix} X \\ d \\ Z \\ X_0 \\ d_o \end{bmatrix} = \begin{bmatrix} A & B_d & -BK_z & -BK_x & 0 \\ 0 & 0 & 0 & 0 & 0 \\ B_z C & 0 & A_z & 0 & 0 \\ HC_{1:4} & HC_5 & -BK_z & A - BK_x - HC_{1:4} & B_d - HC_5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} X \\ d \\ Z \\ X_0 \\ d_o \end{bmatrix} + \begin{bmatrix} B \\ 0 \\ 0 \\ B \\ 0 \end{bmatrix} U + \begin{bmatrix} 0 \\ 0 \\ -B_z \\ 0 \\ 0 \end{bmatrix} R$$

or, treating the plant as the plant plus disturbance:

$$s \begin{bmatrix} X \\ d \end{bmatrix} = A_5 \begin{bmatrix} X \\ d \end{bmatrix} + B_5 U$$

$$s \begin{bmatrix} X_5 \\ Z \\ X_{5o} \end{bmatrix} = \begin{bmatrix} A_5 & -B_5 K_z & -B_5 K_x \\ B_z C_5 & A_z & 0 \\ HC_5 & -B_5 K_z & A_5 - B_5 K_x - HC_5 \end{bmatrix} \begin{bmatrix} X_5 \\ Z \\ X_{5o} \end{bmatrix} + \begin{bmatrix} B_5 \\ 0 \\ B_5 \end{bmatrix} U + \begin{bmatrix} 0 \\ -B_z \\ 0 \end{bmatrix} R$$

Assembling it all together in Matlab...

Create the 5x5 plant (plant plus constant disturbance)

```
A5 = [A, Bd ; zeros(1,5)]
```

```

0         0         1.0000         0         0
0         0         0         1.0000         0
0        -6.5330         0         0         0
0        16.3330         0         0         0.5000
0         0         0         0         0
```

```
B5 = [B;0]
```

```

0
0
0.3330
-0.3330
0
```

```
C5 = [C,0]
```

```
1 0 0 0 0
```

Append 0 to the end of Kx (you don't feed back the disturbance state in Kx)

```
>> Kx = [Kx, 0]
```

```
-54.1610 -271.5274 -43.9111 -80.2474 0
```





```

D13 = zeros(4,1);
X0 = [0;0;0;0;0;1;0;0;0;0;0;0;0;0;0];
R = sin(0.5*t);

```

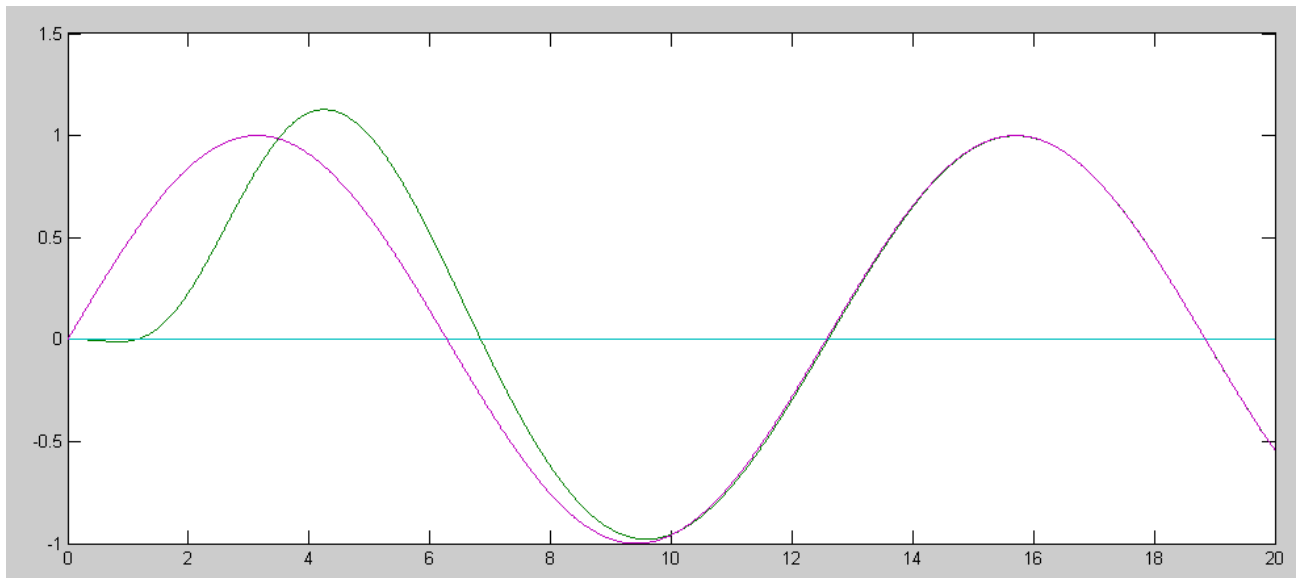
5) Simulate the response of the linear system (full-order observer with the servo compensator) for

A sinusoidal set point ( $R(t) = \sin(t)$ ,  $d=0$ )

```

y = step3(A13, B13r, C13, D13, t, X0*0, R);
plot(t,y,t,R);

```

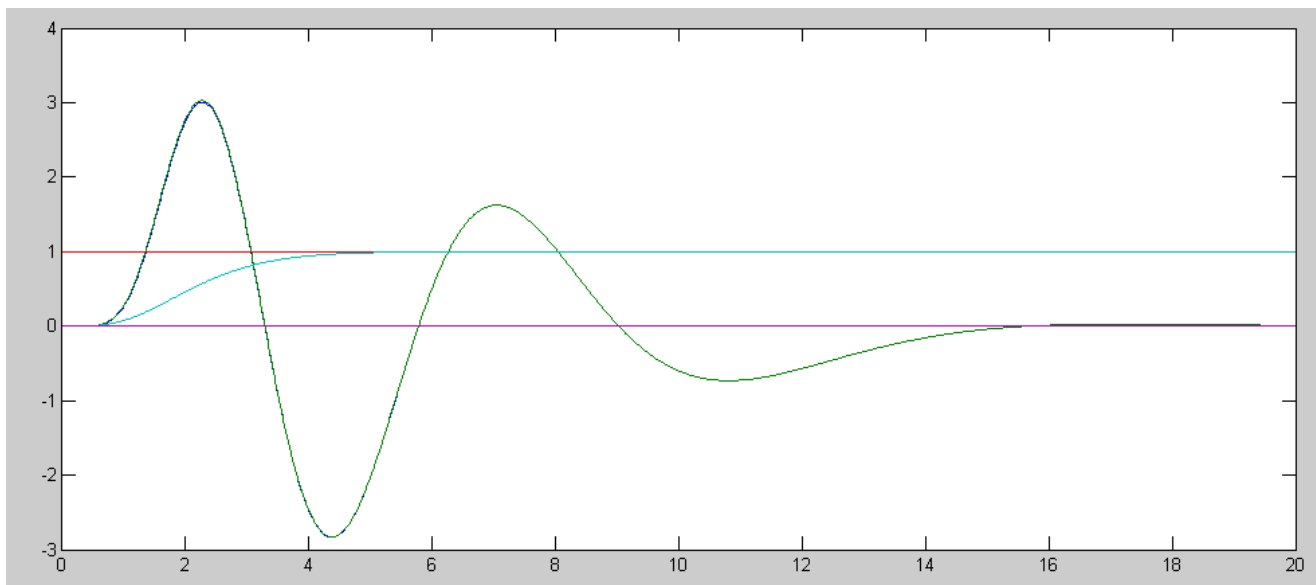


A step change in the disturbance ( $d=1$ ,  $R=0$ )

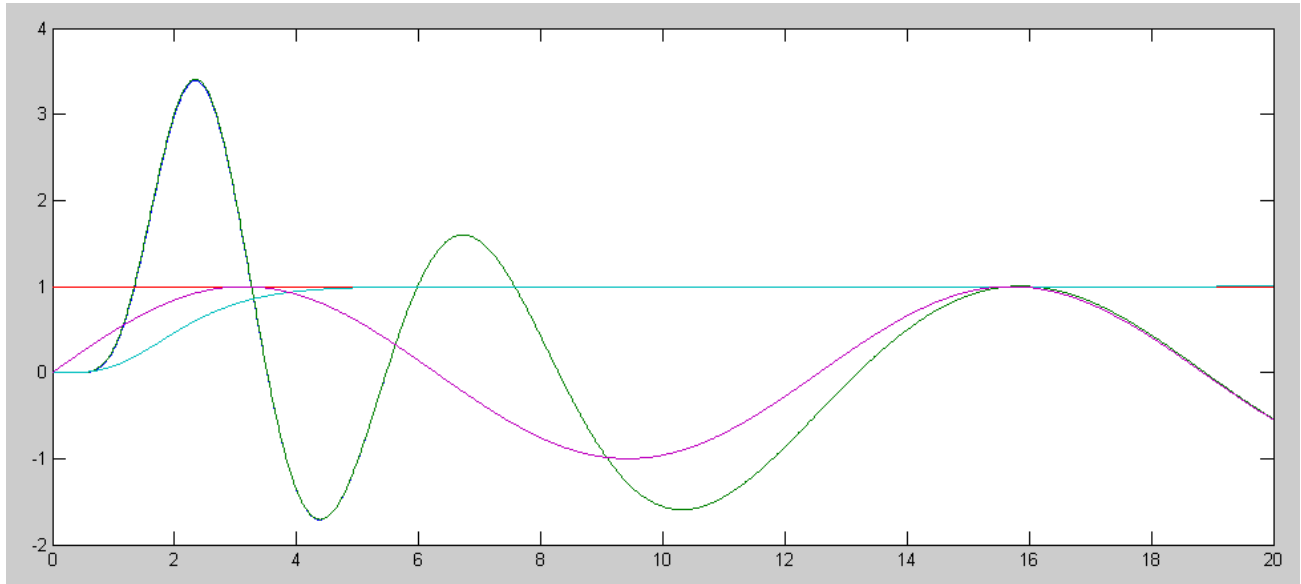
```

y = step3(A13, B13r, C13, D13, t, X0, R*0);
plot(t,y,t,R*0);

```



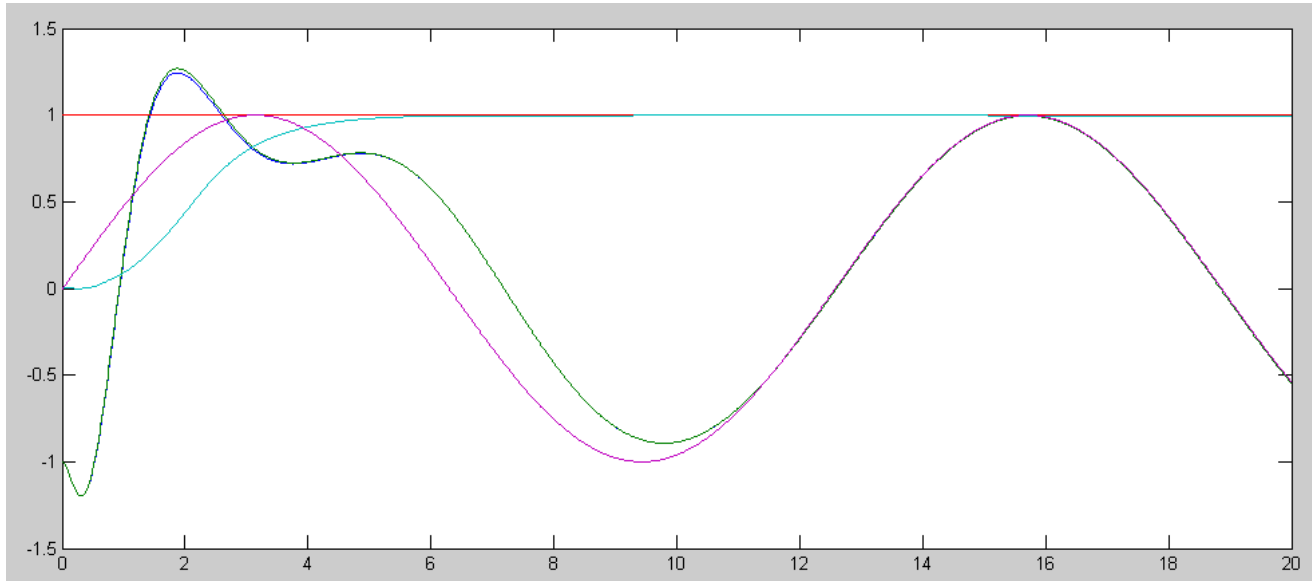
Both:



6) Simulate the response of the nonlinear system (full-order observer with the servo compensator)

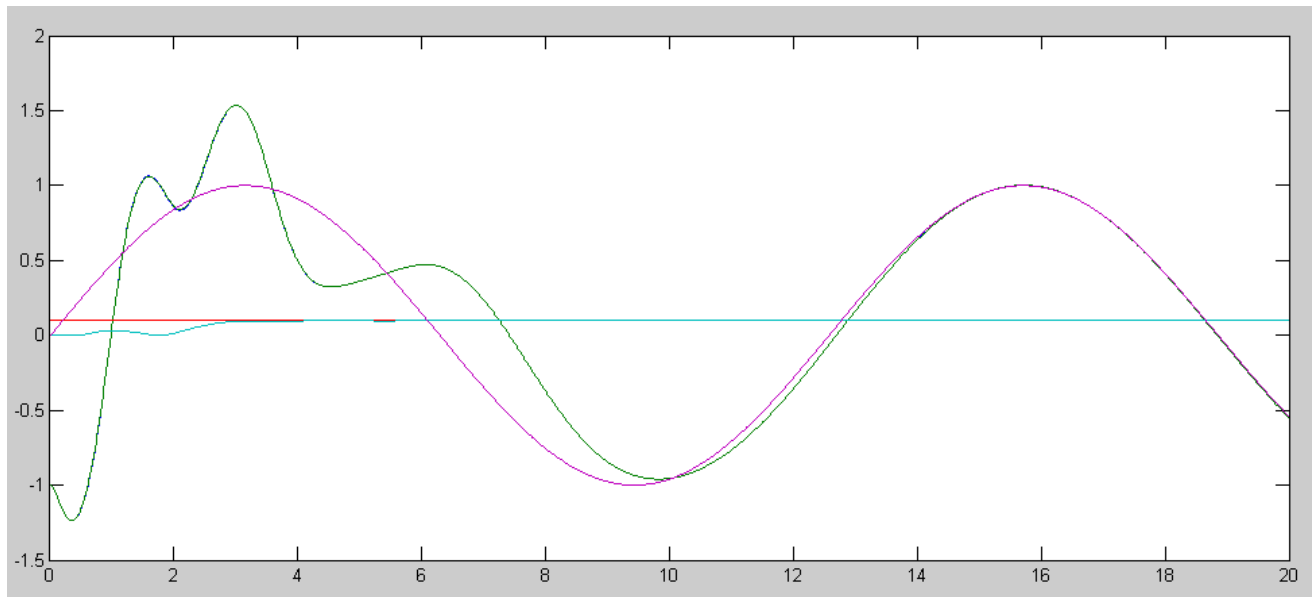
If you feed back the plant states, it works:

```
while(t < 20)
  Ref = sin(0.5*t);
  U = -Kz*Z - Kx*X;
```

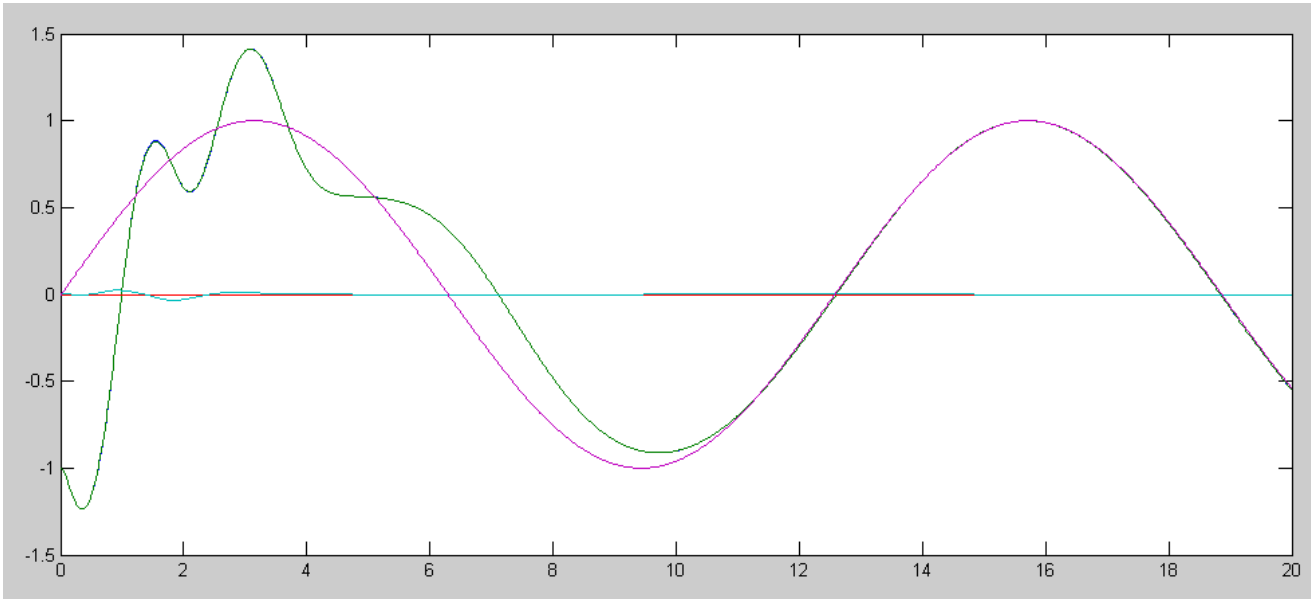


Nonlinear system response with  $R = \sin(0.5t)$ ,  $d = 1$ . Feedback uses actual states.

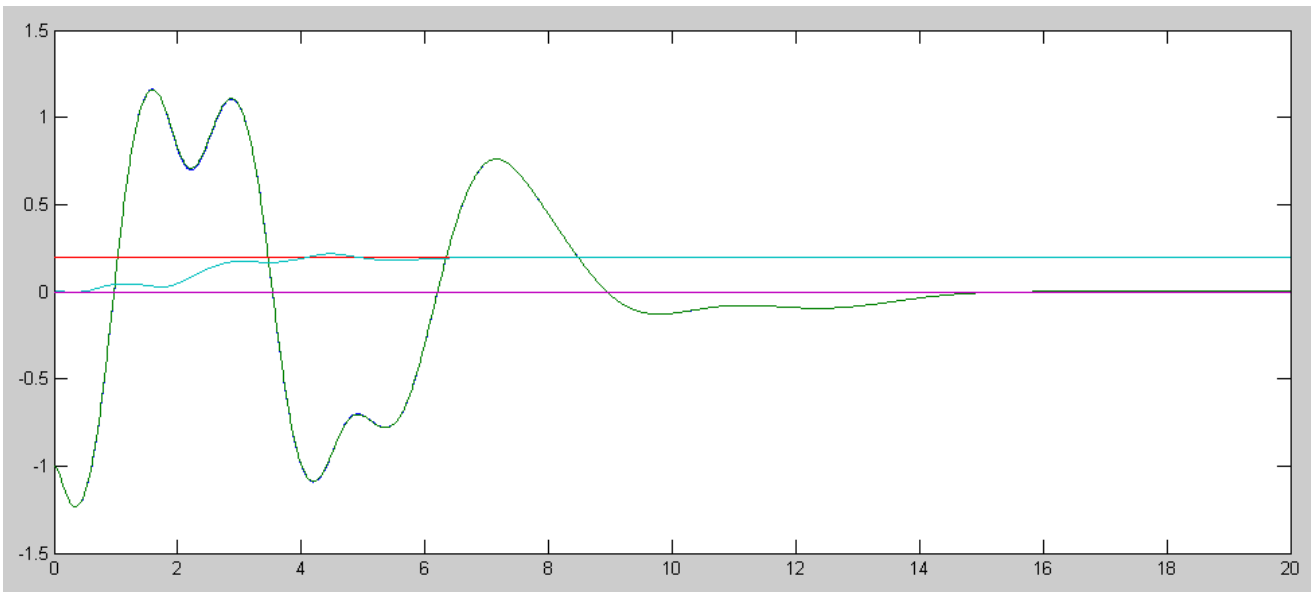
If you feed back the observer states, it doesn't (falls over). Reducing the disturbance to 0.1 does work though (the state estimator has to be closer)



Nonlinear system response with  $R = \sin(0.5t)$ ,  $d = 0.1$ . Feedback uses observer states.



$R = \sin(t)$ ,  $d = 0$



$R = 0$ ,  $d = 0.2$

## Code:

```
% Cart and Pendulum (sp20 version)
% Full Order Observer
X = [-1; 0 ; 0 ; 0];
Xo = X;
dX = zeros(4,1);
Ref = 1;
dt = 0.01;
U = 0;
t = 0;
Kx = [ -54.1610 -271.5274 -43.9111 -80.2474];
Kz = [-20.1017 -17.7229 0.3642];
% disturbance
Bd = [0;0;0;0.5];
d = 0;

% Servo Comp
Az = [0,0,0 ; 0,0,0.5 ; 0,-0.5,0];
Bz = [1;1;1];
Z = zeros(3,1);

% Observer
H = [ 11.3000 -45.8005 67.2930 -187.9365 -17.6512]';
A = [0,0,1,0;0,0,0,1;0,-6.533,0,0;0,16.333,0,0];
B = [0;0;0.333;-0.333];

Ao = [A, Bd ; zeros(1,5)];
Bo = [B; 0];
Co = [C, 0];
Xo = [X ; 0];

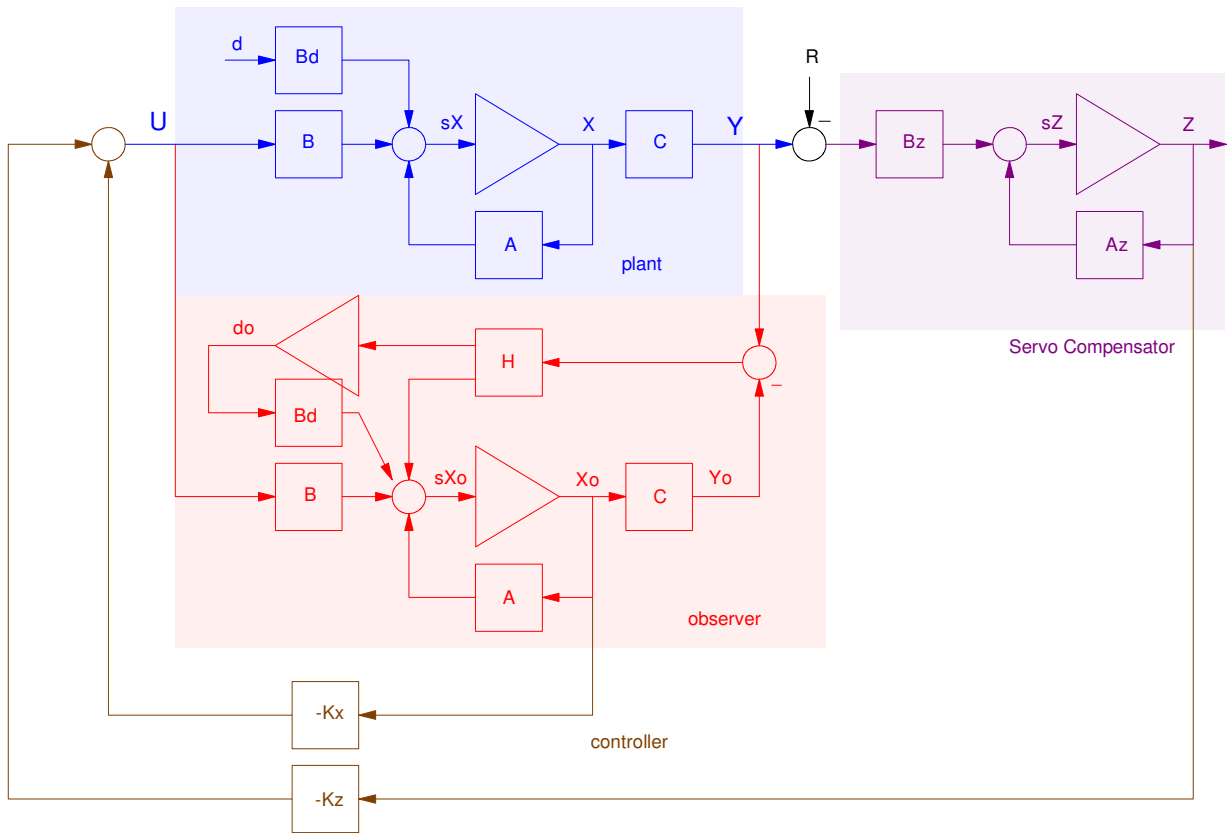
y = [];
while(t < 20)
    Ref = sin(0.5*t);
    U = -Kz*Z - Kx*Xo(1:4);

    dX = CartDynamics(X, U) + Bd*d;
    dZ = Az*Z + Bz*(X(1) - Ref);
    dXo = Ao*Xo + Bo*U + H*(X(1) - Xo(1));

    X = X + dX * dt;
    Z = Z + dZ * dt;
    Xo = Xo + dXo * dt;

    t = t + dt;
    CartDisplay3(X, Xo, Ref);
    y = [y; X(1), Xo(1), d, Xo(5), Ref];
end

clf
t = [1:length(y)]' * 0.01;
plot(t, y);
```



Block diagram for the Plant, Servo Compensator, Disturbance, Observer, and Full-State Feedback