ECE 463/663 - Test #3: Name ______________________

Calculus of Variations. Optimal Control. Spring 2020

Open Book, Open Notes. Calculators, Matlab, Internet all permitted - just not other people.

Please sign if possible (i.e. you worked alone):

No Aid Given Received Observerd: ______________________

Calculus of Variations: Fixed endpoints

1) Find the function which minimizes the following funcitonal:

$$J = \int_0^2 \left(2x^2 + 3\dot{x}^2\right) dt$$

subject to the constraints

$x(0) = 6$

$x(8) = 4$

2) Calculus of Variations: Free Endpoint. Find the function which minimizes the following funcitonal:

$$J = \int_0^2 \left(2x^2 + 3\dot{x}^2 \right) dt$$

subject to the constraints

$x(0) = 6$

$x(8) = \text{free}$

3) Calculus of Variations: You are to build a road from point a) to point b). The cost per unit distance of the road is proportional to the distance from origin

$$J = \int_a^b \sqrt{x^2 + y^2}\,\sqrt{1 + \dot{y}^2} \cdot dx$$

Determine the differential equations that the solution must satisfy (don't solve - it's kind of nasty).

4) Optimal Control: Given the following cost function and constraing:

$$J = \int \left(x^2 + 4u^2 \right) dt \qquad \dot{x} = -2x + 3u$$

$$F = x^2 + 4u^2 + m(-2x + 3u - \dot{x})$$

Determine the three differential equations that the optimal solution must satisfy:

(1) $F_x - \frac{d}{dt}(F_{\dot{x}}) = 0$

(2) $F_u - \frac{d}{dt}(F_{\dot{u}}) = 0$

(3) $F_m - \frac{d}{dt}(F_{\dot{m}}) = 0$

Determine the optimal path for x(t), subject to

x(0) = 6

x(8) = 4

5) Find a feedback control law so that the following system:

$$sX = \begin{bmatrix} -2 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -1 \end{bmatrix} X + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} U$$

$$Y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} X$$

has the following step response: (your pick which method you use)

Give

- The method you used (pole placement, LQR, LQG/LTR)
- Your resulting cotrol law, and
- The step response of your closed-loop system

Note: matlab is encouraged for this problem.