ECE 463/663 - Homework #9

Calculus of Variations. LQG Control. Due Wednesday, April 7th

Soap Film

1) Calculate the shape of a soap film connecting two rings around the X axis:
   - Y(0) = 6
   - Y(2) = 5

2) Calculate the shape of a soap film connecting two rings around the X axis:
   - Y(0) = 6
   - Y(1) = free

Hanging Chain

3) Calculate the shape of a hanging chain subject to the following constraints
   - Length of chain = 4 meters
   - Left Endpoint: (0,6)
   - Right Endpoint: (2,5)

Ricatti Equation

4) Find the function, x(t), which minimizes the following funcional

$$J = \int_0^{10} (x^2 + 9\dot{x}^2)dt$$

x(0) = 6

x(10) = 4

5) Find the function, x(t), which minimizes the following funcional

$$J = \int_0^{8} (4x^2 + 9u^2)dt$$

$$\dot{x} = -0.2x + u$$

x(0) = 6

x(10) = 4

LQG Control

6) Cart & Pendulum (HW #6): Design a full-state feedback control law of the form

$$U = K_r R - K_x X$$

for the cart and pendulum system from homework #6 using LQG control so that

- The DC gain is 1.00
- The 2% settling time is 6 seconds, and
- There is less than 10% overshoot for a step input.

Compare your results with homework #6

- Where are the closed-loop poles with pole placement and with LQG control?
- Are the feedback gains larger or smaller with LQG control?
- Which one works better?

7) Ball and Beam (HW #6): Design a full-state feedback control law of the form

$$U = K_r R - K_x X$$

for the ball and beam system from homework #6 using LQG control so that

- The DC gain is 1.00
- The 2% settling time is 6 seconds, and
- There is less than 10% overshoot for a step input.